A Week has Seven Days. Or does it? On Bridging Linguistic Openness and Mathematical Precision*

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To many, educators as well as laymen, it would probably appear to be a considerable advantage if everyday concepts had the same degree of precision as do scientific ones. Concepts that are used in what Berger and Luckmann [1966] refer to as "finite provinces of meaning", such as science, are refined and specified to refer in an as explicit way as possible to a particular class of phenomena. This process of achieving referential exactness is an essential ingredient of scientific progress and a prerequisite for theorising and increased conceptual mastery of reality. However, as Rommetveit [1974] and others have pointed out, the more open semantic structure of everyday linguistic expressions is not to be conceived as a defect in our everyday language. In fact, it is an essential asset making possible communication across social, cultural, and professional boundaries and between individuals and groups that do not share experiential backgrounds. Everyday, spoken, language thus serves many purposes and is structured to make possible communication in situations where the communicative priorities are different from those that apply to scientific work.

However, in specific situations, the openness of everyday linguistic expressions causes difficulties. Many words, concepts, and even sentences, have to be given a context before they become intelligible. The famous example within cognitive psychology of the sentence, "The notes were sour because the seams were split", appears to have been generated at random by a computer from a corpus of the English language until one is informed that the remark refers to a bagpipe. In the context of learning to quantify and to solve work problems, much research has pointed to the significance of the fact that solving word problems "draws upon the processes of language understanding and strategy seeking as well as upon basic computational skills" [Resnick & Ford, 1984, p. 95]. Consequently, manipulation of wording, sentence order, problem context, and many other aspects that, from a strictly formal mode of thinking, do not affect the mathematical structure of the problem at hand, have been shown to influence problem difficulty [cf. e.g. Resnick & Ford, 1984, p. 22 ff] Similarly, Nesher [1982; cf Nesher & Katriel, 1978] divides "the semantic component" into two different aspects; "the contextual constituent" (referring to how a verbal problem as a whole is understood) and "the lexical constituent" [p. 30] (referring to how isolated lexical items are understood) and has found that "when the logical structure of the word problem is controlled, the semantic variable is most influential, and that the lexical component of isolated words can be fully appreciated only in interaction with the more general contextual semantic variables" [ibid. p. 36]

When quantifying in everyday situations we use expressions that in context rarely create any communication problems since their meaning is effectively co-determined by the context and the speakers' joint concern for understanding each other. One could take as examples the quantifications of time that people make in everyday communication. Words such as "hour", "day", "week" may appear to be rather clearly defined, but in fact, to be correctly interpreted, one requires information about the specific context in which they are used. Alongside the formal definition of an hour as sixty minutes, we can point to contexts such as the school where, in many countries, this linguistic expression refers to intervals of 40 or 45 minutes. A day can be a time unit of 8, 12 or 24 hours depending on whether one is talking about a working day, the daylight period, or a car rental. The concept of week, which is the one we will focus on in the following, is very ambiguous and is—we believe—becoming even more so. As opposed to the formal definition of one week equalling seven days, one can point to a number of contexts where this expression has acquired a different meaning. When referring to school and working life, a week nowadays, in many countries, refers to a time span of five days, but in some countries or professions four-and-a-half or six days would be the natural referent. The student coming to school, or the bank clerk entering his office on Monday morning telling his friends that they only have one week before the holiday begins, is only referring to the next five days. In more specific sectors of society, still other uses of the concept of week are taken for granted. The salesman or buyer of a charter tour (at least in a Scandinavian country) who talks about a one week holiday is referring to a period of eight days since international agreements prescribe that this is the minimum period for charter traveling

The examples could be multiplied, but the general point to be made is that the solving of word problems including quantifiers of this kind also requires, besides the necessary computational skills, a sensitivity to the meaning that is presupposed in a particular situation that is embedded in a specific culture [cf. D'Ambrosio, 1985]. The purpose of the

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present study is to investigate to what extent contextual changes in the meaning of the concept of week causes problems for children. The concept of week is thus used as a prototype for terms whose meanings vary in complex societies and the issue we are attempting to address concerns the types of skills that are necessary to possess in order to master such problems in everyday settings in schools as well as outside.

Method

The empirical data we have collected compares how children deal with the concept of "week" in two contexts where the referent differs and where the children are familiar with this difference. Three problems were used:

1. A cow produces about 18 litres of milk per 24 hours. How much milk does the cow produce during one week?
2. Kalle goes to school and on average he has 7 lessons a day. How many lessons does he have per week?
3. Stina goes to primary school in grade 3 and has on average 6 lessons per day. How many lessons does she have a week?

As can be seen, in the first problem, "week" refers to a period of seven days, while in the second and third case, the correct interpretation—known to the participants—is five days.

In all, 334 pupils were tested on one of the problems. The participants were primary school pupils in the 5th and 6th grades, 12 or 13 years of age. The problems were presented within the context of regular teaching by the class teacher. They appeared on various forms of work sheets normally used and they were presented alongside various other problems.

In the analysis of the results, attention will be given to how children at various achievement levels in mathematics deal with the tasks. The placing of individuals in one of the three performance groups, low, average and high, was done by the class teacher on the basis of previous achievement in mathematics.

Results

The first problem about the cow producing 18 litres of milk per day presented the participants with very few difficulties. In Table 1, the results are given indicating what solution model pupils used. As can be seen, nine out of ten pupils solve the task as a straightforward multiplication: 7×18.

When scrutinizing the performance of the three achievement groups, we find that all the classifications in categories 2, 3 and 4 except one come from the group of low achievers. This can be seen in Table 2.

One conclusion obtained from the answers to this initial problem is that there is only a small group among the low achieving pupils who do not handle that problem as a multiplication, 7×18. Even in these 8 cases, where alternative solutions are suggested by this group, there is no evidence that there were any difficulties in realizing that the relevant contextual specification of "week" was 7 days.

Table 1 Solution used when solving the problem: A cow produces about 18 litres of milk per day. How much milk does the cow produce during one week

Solution model [1]	Grade 5 n	Grade 5 (%)	Grade 6 n	Grade 6 (%)	Total n	Total (%)
1 7×18	42	(88)	40	(91)	82	(89)
2 Incorrect mathematical procedure but where "week" = 7 days	2	(4)			2	(2)
3 Other incorrect	2	(4)			2	(2)
4 No answer	2	(4)	4	(9)	6	(7)
Total	48		44		92	

[1] Computational errors, which are disregarded in this table as well as in those below are very few and evenly distributed over various solution models.

Table 2 Performance on the task: A cow produces 18 litres of milk per day. How much milk does the cow produce during one week?

Solution model	Grade 5 Achievement group						Grade 6 Achievement group					
	High		Average		Low		High		Average		Low	
	n	(%)	n	(%)	n	(%)	n	(%)	n	(%)	n	(%)
1 7÷18	17	(100)	15	(94)	10	(67)	14	(100)	16	(100)	10	(76)
2 Incorrect mathematical procedure but where "week" = 7 days			1	(6)	1	(7)						
3 Other incorrect procedures					2	(13)						
4 No answer					2	(13)					4	(29)
Total	17		16		15		14		16		14	

The situation changes, however, when we turn to problem two where the relevant specification of a "week" is 5 days. In this case, the problem was given to pupils in the 6th grade and the results show that the outcome can be described in terms of two solution models, 5×7 and 7×7 respectively, and a "no answer" category The distribution can be seen in Table 3.

Table 3 Solution model used in solving the problem: Kalle goes to school and on the average he has 7 lessons a day How many lessons does he have a week?

Solution model	Achievement group							
	High		Average		Low		Total	
	n	(%)	n	(%)	n	(%)	n	(%)
1 5×7	56	(81)	55	(73)	34	(55)	145	(70)
2 7×7	13	(19)	20	(27)	25	(40)	58	(28)
3 No answer					3	(5)	3	(1)
Total	69		75		62		206	

In spite of the fact that (a) the problem does not present any mathematical difficulties for this group, and (b) that all pupils are well aware of the fact that they only go to school 5 days per week, as many as 28 per cent fail to solve the problem correctly As is evident, there is a steady increase in the percentage of incorrect specifications of the meaning of the concept of "week" as the performance level of pupils drops. 40 per cent of the pupils in the low achieving group suggest that the problem should be solved 7×7, while in the two other groups the corresponding figures are 27 (average) and 19 (high achievers) per cent respectively Excluding the no answer category, this difference is statistically significant ($Chi^2_{obs} = 8\ 758$; $p < .02$; df 2)

As a check on the general validity of these findings, the structurally analogous problem 3 above was given to a smaller group of grade 5 pupils under the same conditions as has been described The results, presented in Table 4, confirm the general pattern indicated, although some new ways of dealing with the task appear.

Table 4 Solution model used when solving the problem: 'Stina goes to primary school in grade 3 and on the average she has 6 lessons per day. How many lessons does she have per week?

Solution model	Achievement group				
	High	Average	Low	Total	
	n	n	n	n	(%)
1 5×6	11	7	6	24	(67)
2 7×6	1	5	3	9	(25)
3. 6×6			1	1	(3)
4 3×6			2	2	(6)
Total	12	12	12	36	

Only one of the pupils in the high-achieving group makes an incorrect specification of the meaning of the lexical item "week". Among the low achievers, half the group does so, and also among the average achievers difficulties of this kind are obvious

Discussion

The idea behind the present study has been to illustrate the type of difficulties in solving word problems that appear to result from problems in handling linguistic openness rather than a lack of mathematical skills in a narrow sense. In fact, in our view, it is essential to make clear distinctions between mathematical difficulties and the problems of handling semantically open concepts when attempting to improve the performance of children. Thus, an effective teaching strategy aimed at developing the skills necessary to establish the exact meaning of quantitative expressions in everyday language in mathematical terms may not necessarily imply increasing formal training. On the contrary, in order to increase learners' awareness of the difficulties involved in solving word problems and in applying mathematical precision to everyday life, discussions in the form of mathematical dia- and multilogues are probably vital ingredients. Learning about these issues at a general level implies being made aware of the importance of consistently considering what is being referred to when quantifications are introduced. As Carraher, Carraher and Schliemann [1985] conclude in their study of the diverse strategies for solving mathematical problems in everyday life and in formal settings, results such as these "seem to be in conflict with the implicit pedagogical assumption of mathematical educators according to which children ought first to learn mathematical operations and only later to apply them to verbal and real-life problems," [p. 25] and instead we should "seek ways of introducing these (mathematical) systems in contexts which allow them to be sustained by human daily sense." [p 28]

At an even more fundamental level, the problems of people not using the information they de facto possess when solving problems of this kind reflect the particular mode of reproducing intellectual skills that dominate societies with a strong reliance on the literate knowledge tradition. As Ginsburg and Allardice [1984] point out, "on entrance to school, young children are faced with the necessity of assimilating and accommodating to written culture", meaning that they "are taught symbolic, codified arithmetic" and "encounter written symbolism, algorithms, and explicitly stated mathematical principles" [p 203]. To be successful in an arena where such abstract entities are targets for learning, "children ... must develop a new set of intellectual skills" [loc cit] that differ in nature from the intuitive and informal knowledge that functions well in everyday settings [cf. D'Ambrosio, 1985]

Our findings, that it is the poor performing children in particular who have difficulties in finding the proper semantic referent are interesting when considering the nature of the difficulty introduced into the problems used here In our view, the results could be interpreted as suggesting that the problems that these pupils have in dealing successfully with this type of task reflects a fundamental unfamiliarity with the literate knowledge which becomes particularly visible in the context of learning mathematics In this sense, these problems may be *perceived* as problems in learning mathematics but may be indicative of a broader type of difficulty in relating to the abstract type of knowledge valued in the formal context of the school

Studying in modern societies thus takes place in a paper world where cognitive activities are performed as decontextualised activities in particular settings divorced from daily action To use Leont'ev's [1981] suggestive terminology, the activities in such institutions become "energized" by motives other than those that characterize everyday life In other words, the finding made by Lave, Murtaugh and de la Rocha [1984] revealing a "virtually error-free arithmetic performance by shoppers who made frequent errors in parallel problems in the formal testing situation" [p. 83] is understandable if one recognizes that the two situations—making formal computations with pencil and paper on the one hand and quantifying when deciding what to buy on the other—are genuinely different "activities" to use Vygotskian language [Vygotsky, 1978]. In fact, the tendency to view situations as identical that in a formal, abstract, sense have certain similarites, is probably per se a typically literate assumption that may appear natural and self-evident to the person who is used to the abstract type of knowledge characteristic of "literate thinking", but which may cause difficulties for the person relying on more informal, less explicit, knowledge. In this perspective, questioning traditions of teaching and learning implies questioning the premisses and "ways of worldmaking" [Goodman, 1978] on which our abstract knowledge about teaching and learning builds and on which our pedagogics rely Using this intellectual strategy as a tool for unlearning what we as academics and teachers tend to take for granted about what characterizes knowledge, and at the same time learning about alternative modes of construing knowledge that are valid in everyday life, is a difficult, but—we are inclined to argue—fascinating task. In this perspective it should be the role of the educator to have access to these two knowledge bases that are valid in complex societies and to attempt to bridge the gap by pointing at critical differences rather than enforcing the one and forgetting or, worse, neglecting the other.

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