Does Mathematics Education Really Exist?

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Taking the question simplistically, the answer seems to be trivial Thousands of people would say in answer to a question about their profession that they are math educators. There are also the millions of students who currently study mathematics within the educational systems. Yet, the essence of David Wheeler's question* is, as I understand it: Is there a discipline, unique and autonomous in nature, which is not mathematics *per se*, nor education *per se*, but that concerns both mathematics and education and adds to the state of our professional knowledge and to our existing theories?

A discipline is a special way of looking at the world. One who observes the world through the spectacles of physics sees it differently than the one who looks at the same world through poetry, or through botany. The different disciplines are different ways of approaching the world and different means of accumulating knowledge about the world.

If this is so, what new sort of knowledge, what new spectacles does math education contribute? If mathematics may be considered to be a discipline that enriches our knowledge with formal descriptions of the world and with rules of the game of deduction, then mathematics education certainly does not do that. Its subject matter is not in the realm of creating new formal outlooks on the world, but rather lies in the realm of the acquiring of knowledge by the young generation, and as such falls within the broader realm of learning and knowing. There are already several disciplines that deal with the acquisition of knowledge: philosophy, psychology, pedagogy, etc

Is there something in the science of pedagogy which is unique to the learning of mathematics? I think this is the case. Knowledge about the learning of mathematics relies partly on general knowledge about learning and teaching, but to a large extent it depends on knowledge that is quite specific to the subject of mathematics, and to the phenomena that may be observable when mathematics is taught and learned This knowledge is epistemological in nature and has to do with the acquisition of that special kind of knowledge whose "content" is mathematics. This epistemological knowledge deals with specific mathematical notions and the challenges encountered in acquiring them. For example, in order to understand how to approach children who need to learn the values of decimal numbers, one must fully understand the relationships between integers, rational numbers, and decimals Yet understanding the relationships among these sets of numbers as mathematicians understand them will not suffice for teaching these notions One should also understand what constitute the potential pitfalls for those starting to learn decimals. What are the erroneous overgeneralizations of their knowledge of integers and fractions that one can expect young learners to carry over into their learning of decimals? This kind of question belongs to the field of mathematics education, it can be studied with the methods developed by researchers who have specialized in attending to such questions, and the knowledge that results can be made available to mathematics teachers for use in their classrooms

Beyond the general knowledge that pedagogy can give us about, say, the need for pupils to be actively involved in the process of learning; the need to connect new learning to old knowledge, the need to reflect on what one has learned in order to understand it and see how to adapt it; etc., there are other parts of pedagogy which are specific to a certain content. Learning is always learning about something and it makes a difference whether that "something" is mathematics or biology or auto repair. When the something is mathematics the study of its learning lies in the realm of mathematics education

I would like to consider two main activities within the realm of math education:

> a) Pursuing investigations that could improve the practice of learning mathematics, and
> b) Defining (and redefining) the boundaries of mathematics as a subject matter to be learned in school by the general population.

a) Studies in math education over the last twenty years or more, have revealed facts that were not known before, and which have been introduced into schools by teachers who were informed about them. I will mention some notable findings without mentioning the math educators who brought them into focus.

It is well known today that all word problems calling for addition or subtraction are not of the same kind or the same level of difficulty. It is now known that the long-held belief that addition word problems are always easier for the young learner than subtraction word problems is not true It rather depends on whether they are "combine", "change", or "compare" problems and of what variety. Thus it is not the nature of the numbers nor the operations in their symbolic form that cause the learning difficulties, but rather other factors which have been brought to light and are now better understood. Research on the difficulties and misconceptions that children exhibit concerning various elementary topics, multiplication, fractions, decimals,

* The term 'mathematics education", adopted a few decades ago in English-speaking circles, seems to claim the existence of an autonomous field of study I put the question in the title to a few people and invited them to write briefly about it. Only Pearla Nesher accepted the challenge directly, though its influence can be felt in a couple of other contributions to this issue —David Wheeler

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ratio, etc., is very instructive for those of us who want to understand and improve math learning. This kind of research usually illuminates issues that do not concern mathematicians and were of no interest to anyone until people began to study what happens when these elementary notions are taught to students—people motivated particularly by a wish to discover why such elementary notions could apprently give some students such difficulty. Research findings in this area bring into focus aspects of human cognition that are relevant to everyone's learning of mathematics They are of special interest when they counter the intuitions of well-trained mathematicians and experienced teachers.

b) While mathematicians are occupied with findings at the frontiers of mathematics, in the realm of math education we need to study and try to produce guidelines for determining what should constitute the mathematical knowledge to be taught at schools. I illustrate this role by referring to our present dilemmas. Technological advancements have made many topics that used to be taught in schools obsolete. It is no longer necessary or sensible to teach students in mathematics classes how to behave like calculators now that so many classrooms have the real thing What topics can we risk dropping from the curriculum because the new technology can cover them? What should we teach instead? Will the nature of learning mathematics be substantially changed when classrooms are saturated with computers? Will the role of teachers change, and how should we prepare them to approach the new modes of learning?

I believe that no one yet has the answers to the above questions. And while students continue to "waste" their intelligence on drills that machines can do better, many math educators are now working hard on experiments trying to anticipate the consequences of various approaches, exploring the potential of technological tools in facilitating future learning.

Is there something about the study of the teaching of mathematics which is shared with other disciplines? My reply is "Yes". Math educators should know that what they study lies within the framework of general human behavior, that the motivation to learn mathematics, for example, just as much as to behave according to some social or ethical code, is derived from the fact that students are human beings obeying human rules of behavior. From this point of view, it is important for math educators to be knowledgeable about human behavior in general. This part of the story, in my view, does not reside within the discipline of mathematics education *per se*, but has to be studied from within other theoretical disciplines such as psychology and sociology.

In trying to draw a line to show where mathematics education begins and where it ends (a fuzzy line, of course), I would like to make the point that: (1) not every psychological study that employs mathematical tasks contributes to knowledge in the field of math education (as when the study shows a lack of understanding of the mathematics involved), and (2) when math educators deal with aspects of general human behavior, whether social, motivational, cultural, etc, their research should borrow from the best of the research traditions in those other fields in all their depth Some current fashions in math education research seem to be fascinated by other disciplines but employ intellectual shortcuts that would not be accepted within those disciplines and would disqualify studies using them from being considered sound pieces of research

While I think that the community of math educators can proudly write a valuable chapter on the learning of mathematics, deeply rooted in research, this is not the case when it comes to research situating mathematical knowledge within the broader and more general knowledge of human behavior, well covered by other disciplines. Here, occasionally, the research published in journals of math education seems on marshy ground

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The last words (of Shakepeare's play, "A winter's tale"), "Hastily lead away", summon us like a beckoning to a new and impossible world, and our cue is to say, like Antipholus of Syracuse when confronted by a wife he never saw before, "I'll entertain the offered fallacy"

Northrop Frye
