Communications

Counting in threes: Lila's amazing discovery

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On a Saturday in September 2012, Mellony's five year old (5¾ years) daughter, Lila Rose, came running excitedly into the kitchen with the television remote control in hand yelling repeatedly, "Mommy, I've worked out how to count in threes!" Interested in her excitement and its ability to get her away from the television Mellony asked her how she had figured this out. "Look mommy," she said, pointing to the buttons on the remote, "It's 3, 6, 9." Mellony quickly grabbed her cell phone and asked her if she would explain to her again. What follows is the transcript of the subsequent interaction.

Lila: I am going to count in threes look? [*Lila holds up the remote towards the camera*]

Mellony: But how did you work that out show me?

Lila: Cause everyone is 3. [*Lila lifts the remote up and starts pointing with her finger from left to right across the rows of buttons 1 2 3; 4 5 6; 7 8 9—see Figure 1*]

Mellony: So show me how you are counting?

Lila: Three, six, nine, twelve. [*Lila holds her three fingers over each of the number sets 1 2 3; 4 5 6; 7 8 9, then over the 3 buttons below the 789 as she calls out 3; 6; 9; 12*]

Figure 1. Remote buttons.

Mellony: And how did you know that's 12 my angel?

Lila: I don't know, because that's 10 [*Lila points to the unnumbered button under the 7*] and that's 11 [*points to the unnumbered button under the 8*] and that is 12 [*points to the unnumbered button under the 9*]

Mellony: And why do you think it is called counting in threes?

Lila: Because everyone you count in one threes.

Mellony: Show me.

Lila: Three, six, nine, twelve. [*Lila holds her fingers over each row of three buttons as she calls out 3; 6; 9; 12*]

Mellony: And what do you think comes after 12 if we are counting in threes?

Lila: Um, um 13, 13! [*Lila looks to the side, away from the remote, moving her lips mouthing the words one, two, three, … twelve, then aloud*] 13, 13!

Mellony: 13 is the number after 12. That is right, but that is one number after 12, what if you were counting in threes?

Lila: I'm not sure. [*Lila looks at her mom and puts her finger in her mouth seemingly thinking*]

Mellony: Have a look on the buttons [*Lila lifts remote up again and looks at the buttons*] and see if they can help you.

Lila: [*Lila, looking at her mom indicating to her with pointing that there are no more buttons*] There aren't any. Only these. [*She points to the bottom row of three non-numbered buttons*]

Mellony: Can we pretend that there are three maybe?

Lila: [*Looking at the remote and placing her three middle fingers over the four rows of buttons 1 2 3; 4 5 6; 7 8 9; * * * and as she does this says:*] Three, six, nine, 12 13 14 15. 15! [*15 declaratively with a huge smile*]

Mellony: My word my angel, that is brilliant! And what do you think might come after 15?

Lila: [*Rubs her eye and looks down at the remote again counting but this time pointing to the end number in each row only while counting*] Three, six, nine, 12, 15, 18. 18!

Lila was in preschool, in the reception year before formal schooling. When asked about whether she had done this type of counting at school, Lila explained that she had counted in 2s, but never in threes. The teachers at her school

explained that she had some experience of counting blocks in pairs (encouraged by a paired arrangement) and some experience of putting counters on every second block on a number chart.

The episode begins with Lila running in to share her discovery with her mother. Two possible motivators for this are her seeking affirmation for her discovery and initiating an opportunity for sharing her finding. Indeed, Lila is a child who enjoys verbally articulating her thinking and enjoys and seeks positive affirmation. Mellony responds in her typical affirming way, similar to if Lila showed her a picture she had drawn, although with added delight and surprise at the mathematical discovery. Recognising the opportunity for extending Lila's discovery combined with Mellony's sometimes teacherish [1] style of engaging with Lila leads to an extended dialogue between mother and child, each catching the other's attention with constant reference to the artefact (the remote) which stimulated the discovery.

Lila is very familiar with the remote having used it to change television channels at least since she was three. On this morning she had begun her day by watching television and would have entered the numbered buttons 3, 0, 5, in order to get her favourite channel. The remote had a range of buttons with words and letters and the numbers 1 to 9 structured in 3 rows of three with three non-numbered coloured buttons below (as shown in Figure 1).

Thus the organisation of the numbers into rows of three had the mathematical notion of groups of three embedded in it:

1	2	3
4	5	6
7	8	9
O	O	O

Lila's explanation involves noticing the 3, 6, 9 column and realising that each of these numbers indicates the number of buttons counting from left to right and top to bottom. The three buttons below the numbers, which are used to extend Lila's discovery by counting up to 12 are labelled "menu", "0" and "help". The numbers on a telephone or lift might similarly afford the notion of counting in groups of threes (or twos, *etc.*) and a similar "counting in" could be noticed by children in such contexts.

So what is going on here? How can we explain Lila's insight, her flash of inspiration? What theoretical tools will help us identify the process of her discovery in a manner that might help us explain what is happening in other similar situations (is the little boy's response in Duckworth, 1972, another instance of the same kind of event? [2]) There is an artefact, a tool; there is a child playing with the artefact/tool and suddenly she sees a number pattern on the buttons that catches her attention and gives her insight into the structure of numbers. There is nothing *specific* in her schooling up to that moment or in any intentional input from her parents that she might have been picking up or repeating.

We have been working with the notion of the zone of proximal development (ZPD) to analyse classroom data and we use this concept to explain Lila's discovery. Our intention in this short piece is to share this explanation and to invite comment and alternative explanations.

Lila, Mellony and the ZPD

Within newer conceptualisations of the ZPD as bi–directional and collaborative (Goos, Galbraith & Renshaw, 2002, p. 196) both mother and child have knowledge but require the other to move it. In Holzman's (1997) terms we see the emergence of a "life space that is inseparable from the we who produce it" (p. 61). From this perspective, several shifts can be noted during the episode. Initially Lila is the activator of the emergence of a ZPD. In this respect, and given that she is holding the artefact which mediated her discovery, Lila is the more knowledgeable other of the discovery and her mother is the learner connecting the relationship between the structuring of the numbered buttons on the artefact and Lila's counting in threes. Once Lila has shown and explained her discovery Mellony becomes the more knowledgeable other, affirming Lila's discovery and confirming her correctness of naming the arrangement as counting in threes. She then asks a series of questions that catch Lila's attention and extend the conversation with continued reference to performing actions (even if imagined) on the remote Lila holds.

In terms of the relationship between artefacts, tasks, talk and social relations (the four parameters of numeracy events described by Askew, Denvir, Rhodes & Brown, 2000), we might describe the episode as: perception of the artefact (the remote) by Lila; noticing and action on artefact by the child leading to the emergence of a "task"—counting in threes; the articulation (*i.e.,* talk) of perceptions and actions through sharing with another (Mellony); extended engagement between Lila and Mellony about task and tool; and abstracted imagined noticing; with the social relations between mother and child influencing all of the above.

From Davydov (1988), we have the idea that the ZPD does not exist prior to a learning activity. In what sense, however, was this event a learning activity? There was no teacher or informed peer, just the artefact. We might call this a self-generated learning activity. When more mature students sit alone and work from a mathematics textbook, the teacher or more informed peer is the absent author of the textbook, and the student may be motivated by impending exams, clarification of ideas from a lecture, or whatever. We might think of the remote as a kind of textbook, the absent author being the designer of the remote face. Lila did not choose to "study" the remote in the scholarly sense of studying for the purpose of learning, however, so the activities are not the same, but there may be some similarities.

From Meira and Lerman (2009), the ZPD emerges, or not, when the participants in the interaction catch each other's attention. We want to suggest that Lila's attention was caught by the layout of the numbers on the buttons, and a ZPD emerged. Using gestures such as holding three fingers over the rows of buttons first on the top row (1, 2, 3), then the second and third rows, and then on the three buttons below the numbers that performed other functions as a continuation, but also at another moment pointing to 3, then 6 below it, then 9 below that, and finally the stand-in button for 12, Lila demonstrated a clear idea that the button arrangement showed how one could count in threes. The ZPD was sustained because she rushed to her mother to show her what she had found and her delighted mother pursued her discovery and, as mathematics teachers would, pushed her further,

beyond the buttons with numbers and the three buttons below that could be taken as numbers and beyond, to 15 and 18.

We end by asking whether our account of a this incident extends in some small way the notion of ZPD into a new area that might resonate with experiences that colleagues have encountered, or whether current alternative learning theories might have useful things to say by way of explanation. We also want to consider whether there is anything from this episode that we can use as teachers. Mellony has already used the video clip to indicate to early years teachers that children, when they enter school, may know a lot more about mathematics than the teachers might think, and that their environments (even the TV room) are rich contexts for mathematical exploration and extension if parents choose to engage children in these contexts. How might we structure numeracy lessons to stimulate the emergence of such learning events in contexts where there is increasing prescription of what must be taught and when? Are there other insights that the incident might illuminate for teachers?

Notes

[1] Mellony's teacherish style relates to her identity as a mathematics educator. She taught mathematics for several years, is a mathematics teacher educator and runs a weekly mathematics club.

[2] Duckworth's paper tells of 7 year-old Kevin who, before being told the aim of an activity to put a set of different length drinking straws into order from smallest to biggest, says "I know what I am going to do" and proceeds to take the straws and order them by size himself. He was very proud of himself and Duckworth puts this down to the task having been self-set, as she calls it.

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Intercultural dialogue and the geography and history of thought

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First vignette: China and Italy

Mariolina (Bartolini Bussi) is talking at a conference with a Chinese colleague, Xuhua. She has just presented a report on fractions and is writing on the whiteboard with a felt pen. Suddenly Mariolina notices that Xuhua writes fractions in a strange order, first the denominator, then the fraction bar and eventually the numerator:

Mariolina:	Why do you write fractions in this way?
Xuhua:	What do you mean? How should I write them?
Mariolina:	I mean the order. We write them in the reverse order (top-down): first the numerator then the fraction bar and last the denominator.
Xuhua:	Very strange, indeed! How do you know how many pieces you wish, if you do not know in how many pieces you have cut the whole?

Second vignette: Italy and Burma (Myanmar)

Mariolina and Alessandro (Ramploud) are talking with two Burmese colleagues (Thein Lwin, a mathematician, and Ko Ta, a doctor and coordinator of a network of Monastic schools) who are visiting their department:

Mariolina:	How do you write fractions in Burmese? For instance two thirds.
Thein Lwin:	[*Is a bit surprised, writes 2/3 top-down*] Why?
Mariolina:	I have read in Wikipedia that the Burmese order is the same as the Chinese one: bottom-up.
Thein Lwin:	[*Shakes his head*] No, it's the same as yours!
Ko Ta:	[*Smiles*] I am not a mathematician!

Ko Ta closes his eyes, takes a pencil and traces gestures in the air. Alessandro has the impression that Ko Ta is looking for a kind of motion memory of the gesture used when he was a child in a primary school. After some seconds, Ko Ta smiles and shows a bottom-up process: first 3, then the fraction bar and eventually 2.

Thein Lwin:	[*Smiles and nods*] He's right. I agree!

These two vignettes tell us a simple story. Chinese and Burmese are in the same family of Sino-Tibetan languages. Hence, it is not surprising that their way of saying fractions (and the process of writing fractions) are similar. Yet in Chinese the traditional process of writing (order) and saying fractions is still the same as in the past, taught in the same way in textbooks, whilst in Burmese it seems that a "Western" habit is changing the tradition. It would be interesting to know whether this process depends on the effect of colonialism (that for decades designed the Burmese education system according to the British tradition) or on the effort to run after Western mathematics and mathematics education as a way to overcome the negative effects of military rule. This issue deserves further analysis; however, it helped the participants in the interaction to reflect on each other's own *un-thought*. Here we are quoting