Economizing Learning: The Teaching of Numeration in Chinese

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In May and June of 1984, while conducting a series of mathematics teacher education workshops in Beijing, capital of the People's Republic of China, I was introduced to some pedagogical problems in Chinese numeration. They involve the teaching and learning of how to speak numerals with fluency in Chinese, using Hindu-Arabic written numerals. A salient feature of these problems manifests itself when Chinese students attempt to read numerals longer than four digits. For example, even graduates of senior middle schools find it necessary to read 6,721,394 by first pointing at and naming from right to left the place value of each digit before knowing how to read the "6" in the millionth place and the rest of the numeral.

In this paper I discuss what I consider to be the nature of these problems and offer an alternative approach to the teaching of numeration in Chinese. Finally, I discuss implications that these problems and the pedagogical alternative may have for mathematics education generally.

There exist two interrelated problems in the way numeration is taught in the People's Republic of China. The first of these has to do with the instructional process. China's national mathematics curriculum and textbooks outline the teaching of numeration over a three-year period. The reading and writing of numerals up to 100 are taught in the first grade; in the second grade, this is extended to all four-digit numerals and 10,000. Finally, this treatment is completed for numerals up to eight and twelve digits long by the end of the third grade [Yu Yuanging and Cao Feiyu, 1980].* Given the length of time of the instruction, the way numeration is treated in the textbooks for these grades, and the predominance of lecturing as the mode of instruction, it is evident that rote memorization is the dominant learning mode students use to retain knowledge of numeration. This is an instance of a more general problem — the lack of congruence between the cognitive powers of learners and the curriculum — which Professor Hu Monyü of Beijing Normal University has identified as one of the three significant problems (or contradictions) in the teaching and learning process of mathematical education in China [Hu, 1983]. As we shall see, the learning and teaching of numeration can be accomplished in considerably less time and without requiring rote memorization.

The second major problem, in my view, is more curious since, in addition to its pedagogical dimension, it points to a consequence of the cultural and political domination of so-called "developed" societies' scientific norms over those of developing nations. The difficulty that many students have in reading numerals longer than four digits is an example of this second problem. This difficulty seems to be caused principally by two factors:

1. During instruction, little attention is given to the linguistic regularity of naming numerals in Chinese. The dynamics of Chinese, as we shall see, allows one to "invent" the names of virtually all numerals with knowledge of only a few items of language.
2. This lack of focus on the dynamics of Chinese is further compounded by the adoption or imposition of a foreign convention of delimiting digits in a many-digit numeral. Though this convention is appropriate for the linguistic structure of certain Western languages, it causes confusion for Chinese learners since the convention is contradictory to the structure of Chinese numeration.

In English, Spanish, French, and Portuguese, for instance, commas or spaces are used as delimiters between groups of three digits in a many-digit numeral. For example, the numeral 3624795 is written 3,624,795 or 3 624 795, and in writing it so the decoding of this numeral is facilitated. At present, the same convention is employed in the Chinese system of numeration, using Hindu-Arabic numerals. However, the linguistic structure of naming numerals in Chinese is based on groups of four digits

The alternative approach to teaching numeration in Chinese presented here allows learners to become aware of the regularity of Chinese numeration. It also helps learners to develop strategies for by-passing reading difficulties caused by the employment of a convention of delimiting digits which is contradictory to the linguistic structure of Chinese. In contrast to learning to read and write numerals by rote, the approach requires that learners memorize an absolute minimum number of linguistic conventions. From this minimal set, learners construct knowledge of the rest of Chinese numeration, and, through this teaching approach, they are encouraged to be producers rather than mere consumers of knowledge

The approach is based on the notion that it is useful for educators to recognize the high cost, in terms of memorization, for learners to operate almost exclusively as consumers of knowledge. This notion raises two questions: Is it

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possible, then, to examine carefully a given educational situation or subject matter to determine what within it can be left to the dynamics of the mental functioning of learners to invent or discover, and what of it is convention? And, what pedagogical tools and approaches can be structured to engage learners in activities in which they can generate a maximal set of knowledge from a minimal set of conventions?

In the case of numeration in English, it has been demonstrated that only 24 or so items of language and other conventions are required for one to learn to read numerals of any length [Gattegno, 1972]. For example, if one knows only how to read numerals from 1 to 9 and is given how to read, say, 100, then one can "discover" how to read all numerals of the form $N \times 100$ and $N \times 100 + K$, where $N, K \in \{1, \ldots, 9\}$. Given a minimal set of 10 linguistic items, then, a learner can, through combinations, maximize them to discover how to read 99 other numerals in English.

The pedagogical tools employed in this approach are a number array [Gattegno, 1967, 1974; Gattegno and Hoffman, 1976; Hoffman and Powell, 1981] and a blank space. The blank space is used as a reading device. Numerals of the same order are arranged in columns. Since Chinese numeration is based on groupings of four digits, the number array in Figure 1 contains four columns, unlike the three used for teaching numeration in English, for instance. (For reference purposes, the columns in Figure 1 are labeled zeroth, first, and so forth, from right to left, according to the order of the numerals in them. It is not suggested that these headings be used in teaching numeration.)

Number Array for Reading and Writing Numerals in Chinese

THIRD	SECOND	FIRST	ZEROTH
9000	900	90	9
8000	800	80	8
7000	700	70	7
6000	600	60	6
5000	500	50	5
4000	400	40	4
3000	300	30	3
2000	200	20	2
1000	100	10	1

Figure 1

Children in the People's Republic of China begin school at the age of six. Through songs, games, and the like, sung and played at home and in day care centers, most children become acquainted with the names of certain numerals as descriptive entities. This acquaintance with numerals is not necessarily linked to their written form.

In Chinese, numerals are read with virtual linguistic regularity, unlike the irregularities which exist in English in saying numerals from 10 to 19 and 20, 30, and 50. The numerals from 1 to 9 in the zeroth column of the number array (Figure 1) are read: yī, èr, sān, sì, wǔ, liù, qī, bā, jiǔ.

Each of these nine, distinct words must be retained by a learner. To retain each requires effort, some amount of mental energy, on the part of a learner. To make it possible to estimate and compare the energy expenditures required by different pedagogical approaches to retain knowledge, I borrow from Gattegno the idea that each distinct item of knowledge requires the hypothetical expenditure of a "unit" of mental energy, which he calls an "ogden." [Gattegno, 1972] Therefore, a learner must expend nine ogdens to retain the names of these first nine numerals and, at this juncture, is primarily a consumer of knowledge.

Another ogden is required for 10 in the first column, "yī shí."† Afterwards, however, a learner can "discover" how to read any numeral formed with "yī shí" and a figure from the zeroth column. For example, 14 is read as the concatenation of the sounds for 10 followed by those for 4, "yī shí sì." A teacher can elicit this response by pointing quickly first to the 10 in the first column and then to the 4 in the zeroth column. Each of the other numerals in the first column, as well as ones formed by combining these with those in the zeroth column, can be known without expenditure of additional ogdens. "Sì shí jiǔ" is said for 49. The cost in ogdens is few (ten) to know how to read 99 numerals.

In the second column, 100 is "yī baǐ," which to retain requires one ogden for "baǐ." This one ogden suffices then to read all the other numerals in this column. New numerals formed by selecting one numeral from each of the three columns studied can be read without expending extra ogdens; they come free of charge. For instance, 493 is read "sì baǐ jiǔ shí sān." Special attention needs to be given when a numeral from the second column is combined with one from the zeroth column omitting any from the first, such as 507. In such a case, a word is given to the omission, "líng," thus requiring an ogden to be paid. "Wǔ baǐ líng qī" would be said for 507. In total, so far, twelve ogdens are required to learn to read 999 numerals, and a learner can enjoy operating as a producer of knowledge.

With the addition of the word "qiān," the numerals in the third column can be read. The numeral 7000 is read "liù qiān," and 7013 is read "liù qiān líng yī shí sān." No ogden is required for "líng;" payment for it has already been extracted. Therefore, for only thirteen ogdens 9999 numerals can be read. Many, many more ogdens are expended when one is required to learn these through rote memorization under traditional methods.

In this approach, a learner who has paid thirteen ogdens for a collection of thirteen distinct words can generate knowledge of the reading of the other 9986 numerals from a teacher's use of the number array. Thirteen sociolinguistic conventions are all that a teacher needs to present for learners to invent the rest for themselves. With sufficient practice for fluency and mastery of the reading of numerals

† Actually, numerals from 10 to 19 are read without "yī;" for example, 15 is read simply as "shí wǔ." However, when a numeral from the second column is combined with 10 and one from the zeroth column, then "yī" is read before "shí." The purpose in using "yī" before "shí" here is pedagogical. Later, "yī" can be dropped so that the reading of numerals from 10 to 19 conforms to cultural standards.

from 1 to 9999, learners are empowered now to read numerals of any length in Chinese. Just a few more conventions remain.

If 3469 and 7042 can be read then the eight-digit numeral 3469 7042 can be read by "reading the blank." The blank space between the two groups of four digits can be vocalized as "wan." The left-hand group of digits is read as if the others did not exist, then the blank is read, and the final group of digits is read. Hence, 3469 7042 is "sān qiān sì baĭ liù shí jiŭ wàn qī qiān líng sì shí èr." An ogden is required for "wàn;" once it is expended, numerals up to eight digits long can be read fluently.

For numerals between 9 and 12 digits in length, another ogden is required for the word "yì." It corresponds to the second blank, counting from right to left, in a numeral such as 904 3469 7042. Numerals between 13 and 16 digits long would require another ogden to be expended for the word "zhào" represented by a third blank. With the payment of only 16 ogdens any numeral from 1 to 9,999,999,999,999,999 can be read masterfully and at once. This empowers one with the ability to read numerals longer than one might care to confront. The point, nevertheless, is that by recognizing what are conventional and inventable items in the study of numeration, learners can be given the opportunity to use their mental functionings to harvest a lot of knowledge from a small number of seeds.

The sequence of numerals from 1 to 9 is ordered. It may require one or more ogdens to learn this additional structure associated with the set of utterances for these numerals. The same ogdens, however, can be applied to the sequence after 10. Finally, another ogden may be required to read 200 and 2000 in the alternative, mostly colloquial, way as "liăn baĭ" and liăn qiān" respectively.

The number array can also be used for learning and teaching the writing of numerals. Space does not allow a presentation of this here. [For an example of how this can be done in English, see Gattegno, 1974.] It suffices to note that a learner needs to expend the same number of units of mental energy or ogdens to master the task of writing numerals as expended in learning to read them and, therefore, can operate almost exclusively as a producer of knowledge. Thus, the reading and writing of numerals of any length can be "taught" in a fraction (less than 1) of the time it presently takes.

Before commenting on the possible implications for mathematics education that this alternative approach provides, I offer two suggestions for handling the problem many Chinese encounter when reading many-digit numerals grouped by three's using commas (or spaces). The first is political and the second pedagogical.

The logic of reading numerals in Chinese, as we have seen, is based on groups of four digits, not three as in French and other Western language. Therefore, China could choose to place commas or some other delimiter after every group of four digits, counting from right to left, and require others to learn this structure if desiring to communicate with China. In this case, the technique of "reading the blank" becomes "reading the comma" or whatever delimiter is chosen. This solution would give recognition and respect to the linguistic structure of Chinese numeration. The purpose of delimiters inserted judiciously in a many-digit numeral is precisely to facilitate decoding the numeral without fuss.

In lieu of the adoption of this alternative, after the reading of numerals has been fully integrated, activities can be devised in which the digits of a numeral are grouped in varying configurations. The digits, for instance, could be placed in groups of two, three, five, or with the length of a group varying within a numeral; 4592 1345 could be shown as 459 213 45. Learners can be challenged to read a numeral grouped in such a manner. The purpose would be to develop a learner's ability to transform mentally, say, 30 317 69 into 303 1769 and say "sān baĭ líng sān wàn yī qiān qī baĭ liù shí jiŭ." These challenges can then lead into the use of the comma for delimiting every group of three digits in a numeral as an international standard, one which plays no functional role in reading a numeral such as 64,108 in Chinese. Essentially, one can learn to ignore the presence of delimiters, and to do this may require the expenditure of an ogden or two.

The difficulty that Chinese have reading many-digit numerals exemplifies two problematics in mathematics education. First, the instructional method for teaching numeration, requiring the expenditure of lots of time and energy, outlined in China's national curriculum relates to a particular epistemological conception. Second, the problem which results from the use of a Western convention for placing delimiters in numerals is related to problems in the interaction between language and mathematics learning. This problematic is particularly found in many Third World nations.

Now that we have seen that numeration in Chinese can be taught with minimal expenditure of mental energy or ogdens on the part of learners, what are some implications that this might have for mathematics education in general? Its primary significance is in demonstrating that if the structure of a subject is known, then it is possible to minimize the number of items that one needs to memorize. These aspects of the subject which cannot be invented, namely conventions, would be the only elements requiring presentation to learners. The rest could materialize consequent on a learner's involvement with objects and the dynamics of relationships and patterns observed between and among the objects.

Finally, the particular difficulty, referred to earlier, that many Chinese experience reading many-digit numerals is embedded in the second problematic: the interaction between language and mathematics. Some research exist on the nature and causes of mathematics learning difficulties manifested when curricula are adopted for use in a society whose cultural and linguistic milieu is distant from the one for which the curricula were developed [Philp, 1973; Berry, 1985]

In his analysis of problems in second language mathematics learning in Botswana, Southern Africa, Berry [1985] develops a general theory of types of language associated learning problems. He suggests that there exists two categories or types of such problems It is Berry's "Type B" which is of interest here Berry describes Type B problems as those which "result from the 'distance' between the cognitive structure natural to the student and implicit in his mother tongue and culture, and those assumed by the teacher (or designer of curriculum or teaching strategies)" [p. 20].

By the term "distance," Berry refers to the linguistic distance between languages which are "semantically and culturally" [p. 19] different. This category could be widened to include those problems of the kind described in this paper which demonstrate the existence of a linguistic distance which is *syntactical* in nature. Our example exemplifies yet another learning difficulty of the nature Berry and Philp observed in Botswana and Papua New Guinea, respectively. These difficulties often occur when Western curricula are adopted, without critical evaluation and appropriate adjustments, by developing societies which fail to respect the structure of their own language, culture, or logic/reasoning systems.

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For, to talk frankly with you about Geometry, is to me the very best intellectual exercise: but at the same time I recognise it to be so useless that I can find little difference between a man who is nothing else but a geometrician and a clever craftsman. Although I call it the best craft in the world, it is after all only a craft, and I have often said it is fine to try one's hand at it but not to devote all one's powers to it.

Pascal to Fermat, 1660