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Staging Arithmetic: a Suggestion for the Start of Mathematics Instruction

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In elementary arithmetical instruction numbers and other symbols such as "=", ">", "+", "−" are illustrated by *material*. This is available in many varieties: blocks, rods, picture cards with numbers of objects — usually in attractive colours. The teacher writes a sum on the blackboard: 3 + 2. The pupils lay it down in rods even though they know that it is 5. They must count the total and note down the result. They are trained in actions they must carry out with with rods, blocks, and so on. In this activity, however, the material has only an *illustrative* function. Indeed blocks and rods are a building rather than reckoning material.

Moreover blocks arithmetic isn't as easy as one might superficially think. If one does 3 + 2, the number of the last block added provides the sum. In 5 − 2, however, the last block taken away does *not* provide the difference, which must be determined by counting the remainder. Subtracting by means of blocks leads its own life, different from that of addition. In blocks arithmetic addition and subtraction, considered as activities, are not inverses of each other. This is the reason why in traditional arithmetic textbooks they were practised for a long time separately. When I went to school, the school for retarded children in our quarter was called "the little blocks school" because they did not stop calculating with blocks, or so people believed.

Perhaps this was exaggerated. But it is an indication that arithmetic instruction relying on material unrelated to other imaginative representations ends up in meaningless rote activities with no impact outside arithmetic itself.

Dramatising

Another approach towards elementary arithmetic instruction is *playing arithmetic theatre*. Of course here, too, material is an important factor, for dressing and make-up. The leading idea is that calculating is primarily a *happening*.

The material need not be beautiful. What matters is the *performance*; the material evokes images in the children's minds in order to stimulate activities. These may be, but need not be, arithmetic activities. Children's imagination is broader.

Context and situation-bound numbers and arithmetical operations should not be considered as illustrating arithmetic activities. They may be subservient to the contexts and situations rather than dominating them. [Van den Brink, 1984] "Bus numbers" serve to count passengers, "bowling numbers" to count shuttles, and so on, depending on the particular situation. Events taking place in such a context (getting in and out of the bus) are documented by numbers and symbols, but they keep their meaning, bound as they are to the context. Here calculating means *registering the course of events or their performances*.

To be sure, not all arithmetic problems can be understood as happenings. They may be descriptions of static situations. For instance if I say, "John has 5 marbles and Peter has 8; how many has Peter more than John?", one can hardly say that something has happened. The same holds for problems like, "There are 11 children in the class, 7 are girls, how many are boys?" These numbers describe static situations rather than happenings. To solve them, however, one has to transform them into happenings. Otherwise there is no way of calculating.

We advocate *happening-bound numbers* in elementary arithmetic instruction rather than situation-describing numbers because they are more akin to what we demand of children doing arithmetic. In the literature attention is also paid to calculating as a dynamic process and to happening bound numbers [Ginsburg, Nesher, Herscovics, Moser, Vergnaud].

Children's (arithmetic) world

What contexts are most appropriate under this view? And what should arithmetic instruction at this level look like? To answer these questions we should first think about what young children's world looks like

— First, it seems that children's world is to a higher degree defined by all that has to do with *playing* (their toys, their playing rules, their play as an activity) than by their experiences of our adult world. For instance I was astonished that kindergarten children in Arnheim didn't know what a trolley bus was though these vehicles dominate the Arnheim city picture, while they did know the double-decker although there is only one in Arnheim — which, as it happens, may be used for a wedding party. Yet the double-decker is among their toys while the trolley bus is not. Playthings and playing situations influence children's imagination more than the adult world does, or so I think.

— A second experience: The first time I posed Kim (6;7) a bus problem, "Three people get into ...", she reacted immediately by "Aha, Pete, Tonny and Lizzy." I did not know about that trio, which, it appeared, were the main characters in a story told by the teacher. *Stories keep reverberating,* even in arithmetical problems.

— A third experience, in the second grade: Jerome (7;6) explains to Guiseppe (8;0) how to perform a column addition on the abacus: "Do you remember, Guisep, how Miss stood at the blackboard ...", and then he pictures the class situation as it was when this subject was introduced in arithmetic instruction.

Children are able to *evoke complete arithmetical situations* provided they have been lived through at a moment in the past. We should consider that children's image of arithmetical instruction tends to be sharply phased into periods, which are not meant as such by us. *In each phase arithmetic looks different.*

First the children learn, for instance, about "more" and "less" Then we speak about "5 more" and "5 less" with the same words "more" and "less", as though these were problems of the first phase. Firstgraders can be misled by an instruction like "Make a tower 5 more than these 2 blocks". They may build a tower of 10, because "this is more than 2". Arithmetic proceeds in phases, or so they are made to believe, which are isolated from each other, though the use of the same words suggest otherwise.

In arithmetical instruction we should properly be obliged to describe new things by new words — "5 onto" avoids the confusion of "5 more". Resuming, we may say that children's world is *imagined* and *played* and that lived-through stories are easily recollected.

Language of arrows

For elementary arithmetic instruction this means that you have to look for *lived-through* contexts and situations. Besides using *various kinds* of models and material (chips, blocks, number line, counting frame, abacus, fingers), arithmetic theater should be staged in the classroom.

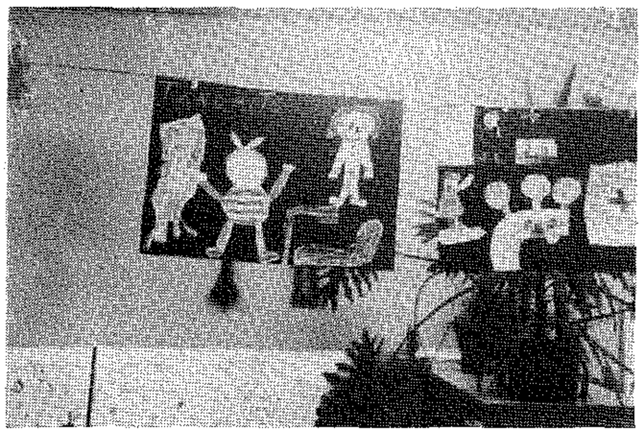
For instance, one tells a story about Pete, Tonny, and Lizzy going by bus. The course of the story is put on the blackboard by means of *arrows* and *numbers*. Then the children dramatise this story, supported by the arrow notation.

There are several kinds of arithmetic theatre:

1. Stories told around certain characters in certain situations

Midgest, giants, fairies and other fairy tale characters; but also like this:

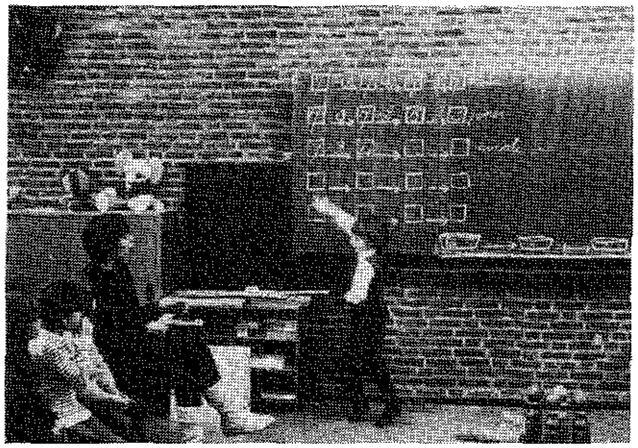
> Uncle John is sitting in the waiting room. He has tooth- and stomach-ache. Aunt Mary is also in the waiting room. She has tooth-ache. (The children dress themselves up). "Eelco, will you play the dentist?") Teacher keeps account of the people in the waiting room in arrow language on the blackboard.



2. Genuine games

Bowling — the course of the game is described in arrow language in order to find out the winner.

Other games of this kind.



Even in the more technical kind of game, children like it to represent some character. The leitmotif may be some story involving a number of people and animals (Pete, Tonny, Lizzy, Grandpa, Miss Kath, the Lord Mayor, Barkey, Pussy — characters in a continuing story told by the teacher, played by the pupils) who live together. Sometimes they travel by train, or Tonny climbs a steep ladder, or on Grandpa's 90th birthday they go bowling.

Such situations are dramatised and the course of events is accounted for in arrow language:

$$\textcircled{0} \xrightarrow{+4} \textcircled{4} \xrightarrow{-3} \textcircled{1} \xrightarrow{+1} \textcircled{2}$$

This language is quite useful for describing *happenings* because it registers changes in the situation. Appropriate contexts are:

> Stories and theatre around bus, train, waiting room, shop, post office,....
> Games: playing dice, game of "goose," bowling, pushing counters on the counting-frame, stepping across the classroom, playing marbles.

Traditional arithmetical notations

What about the problems that might arise when one passes from the arrow language of the dynamic arithmetic theatre to notations like $3 + 2 = 5$? In our experimental classes the transition from the arrow language to the "official" notation has been tried out in two ways:

(1) The equality sign is first introduced together with "$>$" and "$<$" as a relation symbol in a situation where towers are compared. This is dramatised: Grandpa is looking through a telescope describing towers, which are built according to the description. After the arrow language period such notations as $3 + 2 = 5$

are taught as a "neater notation on squared paper" of such arrow sums as:

$$\textcircled{3} \xrightarrow{+2} \textcircled{5}$$

(2) The equality sign is put at the end of the bus travel story in order to account for the final state:

$$\textcircled{0} \xrightarrow{+4} \textcircled{4} \xrightarrow{-3} \textcircled{1} \xrightarrow{+2} = \textcircled{3}$$

Afterwards the same hybrid notation is used in simple sums such as:

$$\textcircled{3} \xrightarrow{+2} = \textcircled{5}$$

and finally the arrow is dropped:

$$3 \ +2 = 5$$

This second approach has proved to function very well. For children who have troubles with calculation the hybrid notation is quite helpful.

Advantages

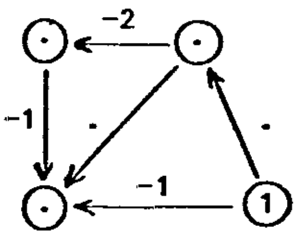
Why start with arrow language and arithmetic theatre?

Calculating is in itself dynamic: something happens Nothing happens in a book. Pictures are static. The children must contrive happenings themselves. To this end arrow language can be helpful.

Arithmetic theatre, however, means that, rather than looking at pictures, calculating aims at the child's real life: *non-arithmetical facts are important in arithmetic theatre* and this gives the possibility of integrating arithmetic into the child's life of playing and fantasy.

Our investigations prove that in the second half of the first grade the arrow language is still helpful:

— problems in arrow language can more easily be solved because they refer back to the bus scenes. Such happenings keep on living in the children's minds.
— Difficult indirect problems that resist solution are solved by translation into arrow language.
— Problems in arrow language invite the invention of solving strategies. How shall we start to solve the following problem?



— The arrow language can be used to symbolise one's own thought, that is, as a notation of reflexion on one's arithmetical activity.
— But, most important of all: calculating is not any more an isolated activity but by its relation to arithmetic theatre becomes arithmetic for life.

Reference

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In every field, deepening of insight into basic concepts eventually is bound to affect its presentation to beginners; and without comtemplation and clarification of the fundamental ideas, didactics is threatened with stagnation.

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