

Are Theories in Mathematics Education of any Use to Practice?

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1. The gap between theory and practice

Some years ago, I took part in arranging a seminar on activity forms and student engagement in Danish mathematics instruction in the 10th-12th grade (*gymnasium*). The organising committee intended the participants to leave the course with an outcome which they could connect to the problems they experienced in their daily teaching practice. So we started by asking the participants to write down what they found to be the greatest problems in connection with students' involvement and activity forms. One participant wrote:

When I 'throw' a question to the class, 1-2 of the students raise their hand – and of course always the same two. How do you get a good show of hands?

How do you *efficiently* overhaul the class?

If you finally get someone to go to the board, you often feel that it is not worth the time – it takes a *long* time. And do the rest remember any better? Hardly.

Can you – using games, quizzes or the like – get proofs and definitions to stick?

How does one motivate the students to learn the material *efficiently* at home? I feel they should learn in a fun way but it must also be well hammered in. How does one, for instance, motivate them to 'overhaul themselves'?

On the one hand, the members of the planning group could easily recognize the frustration of the participant and the mentioned problems. On the other hand, the questions also induced a slight head-shaking amongst us. There definitely was a certain view of teaching and learning behind these questions – a view we found hard to work within. The result was that we did not feel in a position to answer this sort of question. We stuck with presenting theories as well as concrete ideas whose basis lay in quite a different view of teaching.

Perhaps this particular participant changed his view of teaching over the duration of the course. Or perhaps he experienced an implicit standard in our organisation and presentations for relevance and views of teaching. The latter is perhaps far more likely. But then it must have limited how much real communication took place among us. The question is whether such communication is possible at all.

2. Should the teacher become a constructivist?

The views of learning as stocking up on knowledge and of teaching as transferring such knowledge to empty vessels (alias students) have been thoroughly criticised, and the notion of constructivism and Piaget's ideas are often mentioned in this connection. There have been many objections to several elements in Piaget's theories, but that is beyond the scope of this article. I just want to mention activity theory which to a larger extent takes into consideration the learner's goals with her or his activity, as well as the role of language in learning (see the works of Vygotsky). Mellin-Olsen (1987) and Skovsmose (1994) have stressed that all activity, and thus all learning, is intentional. Skovsmose does not address whether the intention has to be to learn or whether it could be directed towards other types of goals. However, the latter should probably be included as a possibility, since this could be seen to be true of much learning in everyday practices.

Though constructivism has gained solid ground in the educational community, it has also been criticised extensively. Kilpatrick (1987) has pointed out that it is but one possible metaphor for learning. O'Loughlin (1992) points to the limitations of constructivism when it comes to taking into consideration: "issues of culture, power, and discourse in the classroom" (p. 791). Furthermore, he finds that constructivism stresses the ability to adapt to rather than to be critical towards or change society (see also Christiansen, Nielsen and Skovsmose, 1997). Another strong critique comes from theories of situated learning or learning viewed as legitimate peripheral participation (Lave and Wenger, 1991).

The literature in mathematics education is full of papers which argue for constructivism and on the basis of this refer to some forms of teaching as more constructivist than others. Therefore, one could expect that I (based on such considerations) could give some good advice to the above mentioned teacher: you do this like that, and then you create a learning environment which encourages the students' active construction of knowledge. However, the teacher apparently did not have a constructivist view of learning, so it was problematic to use as a starting point an understanding of learning which is not shared between the teacher and myself.

But then is it not as clear as an unclouded sky how I as a mathematics educator should have acted in the situation with the questioning teacher? I could simply use the ideas of constructivism on challenging the existing knowledge of the learner, but in this situation with the *teacher* as the learner!

For instance, I could have used as my starting point some experiences which I could reasonably expect the teacher to have – namely the experiences of sudden insight (‘aha!’, ‘I see!’, ‘Of course!’). These flashes of insight are hard to explain within the tank-filler view of learning, while the constructivist could claim that they are the subjective experience of accommodation. Furthermore, it is difficult to imagine that the students would have great flashes of insight through answering questions which concern the next step in specific calculations, the correct definition, and the like. There must be something extra taking place, before this type of insight occurs.

With a little luck, the teacher could have his views on learning revised sufficiently for him to have a new basis from which to reflect on his teaching practice. But now, is that a fair way to treat the teacher?

We do not know what actually takes place within a person who learns. Whether we have a view of learning as piled-up information on inner shelves or as cognitive structures, and whether we view learning as stocking up on knowledge or as active construction of connections between experiences, these are only metaphors for the actual learning process. Both metaphors are useful in describing aspects of what takes place in a learning or teaching situation, but they can never capture the entire situation (Kilpatrick, 1987, p. 13).

Using a notion coined by Fasheh, Khuzwayo (forthcoming) has substantiated the claim that the so-called ‘fundamental pedagogic’ underlying the apartheid schooling system in South Africa can be accurately described as an *occupation of the mind*. Such an occupation ‘victimises’ both those who seem to gain from the system and those who become less privileged, because both are left with no alternatives. I will say that whenever there is no seeing of alternatives, an occupation of the mind is at work. Khuzwayo warns that the new Curriculum 2005 in South Africa may be top-down and expert-driven (and thus with little respect for practice), with the risk of constituting another form of occupation of the mind.

I will allow myself to draw a similar parallel to the widespread insistence on constructivism within the mathematics education community. It is a problem within the research community, because it is limiting when – as it has happened to me on more than one occasion – the acceptance of a paper depends on stating connections to the constructivist position. It is problematic in relation to practice also. By insisting on a constructivist perspective, teachers’ experiences may be denied and teachers silenced; the respect for practice and teachers’ (tacit) knowledge is limited and the basis for fruitful collaboration with the possibility of mutual learning disappears. Instead, we must encourage the seeing of alternatives in both theory and practice.

Therefore, it is unreasonable to palm off onto the teacher one view of learning rather than another. But what then can the educator do?

3. Is a catalogue of techniques useful?

Perhaps the educator should simply do what one could easily be led to think the quoted teacher wanted – namely present a catalogue of techniques to handle the mentioned problems.

One gets a good show of hands by asking many ‘little’ questions, by counting to ten before one picks on a student, by praising those that put their hand up as well as those who are willing to offer an answer, whether correct or wrong, ...

(Project SEED, an outreach program which functions in several of the major cities in the USA, has worked extensively with developing the use of such techniques in the classroom.)

The problem is that the instructional situation is very complex and influenced in numerous ways by the participants’ perceptions of mathematics, of each other, of being in school, of knowledge and learning, of abilities, of the purpose of mathematics instruction, etc. (See the extensive categorisation of views on mathematics instruction depending upon these ideological elements in Ernest, 1991.)

Therefore, the same concrete advice could be used very differently by different teachers, and rarely with the desired result. In this connection, I emphasise that even though teaching is an intentional activity there exists no means which will sufficiently ensure that goals are reached or even furthered (Christiansen, 1989, p. 36)

Exactly for this reason it must be made clear, that *it is not the task of educational and pedagogical theory to offer guidelines or methods for the teacher’s practice*. There are quite different goals within educational and pedagogical theory, on the one hand, and actual teaching, on the other. Educational theory is descriptive and explanatory, but not prescriptive. This is in opposition to even the theoretical elements of the teacher’s professional knowledge, which is directed towards continuous development of his or her own practice, and hence will always serve a prescriptive function. Thereby it is also said that one field is not to be seen as superior to the other (see Steinbring, 1989).

In other words, one should not be tempted. The teacher should not be tempted to think that the educator can provide practical suggestions to her or his problems and the educator should not be tempted to offer such suggestions. Instead, we must strive for collaboration and communication between educational theory and practice, a collaboration and communication which has as its starting point the equality of the two ‘domains’

I see two ways in which the practice of instruction can be developed through an interplay with educational theory. [1] I will discuss both ways below in the next two sub-sections

4. Inspiration of the theoretical knowledge of the teacher

First, theoretical educational knowledge can inspire the teachers’ theoretical knowledge about learning, among others, which again may inspire the teacher to develop her or his practice.

Here, I find it worthwhile to apply the reflections of a practicing teacher on the continuous development of his own

practice. Simon (1995) has formulated a useful – in my opinion – model of the teaching cycle of a reflecting teacher (see also Steffensen, 1996, but also Steffe and D'Ambrosio, 1995). He says that the teacher must formulate a hypothesis concerning how the learning and teaching will take place. This she or he does on the basis of her or his goals for the students' learning, planned teaching activities and hypotheses about the concrete learning processes.

Clearly, this formation of hypotheses will be influenced by the teacher's hypotheses about the students' existing knowledge, by the teacher's theories about learning and teaching, by the teacher's experiences with students' ways of handling the topic at hand, by the teacher's mathematical knowledge, by the teacher's knowledge of educational theories, and not the least by the teacher's evaluation of how the students reacted to her or his last teaching activities. This is exactly why it becomes relevant to talk of a *cycle*.

In this cycle, there are several times when the teacher can make use of educational theories. First, in his formulation of goals and plans. Therefore the teacher's formation of hypotheses about the course of the teaching will be influenced by these theories and considerations. Second, in his evaluation of students' activity and thereby his evaluation of the course of the teaching.

In this connection, the constructivist view of learning can be a useful acquaintance for teachers, as an alternative metaphor for the actual learning processes. It can serve to make a point of the necessity of connecting with learners' existing cognitive structures; it can point to places in the learning process where the teacher can contribute most actively; and it can offer ideas about how teaching best furthers development of concepts with students.

But though Simon starts from constructivism, in my view his model can also be used with other views of learning. I consider this a strength in the model. Thus, a teacher, independently of her or his view of learning, will be able to form hypotheses about students' prerequisites and potentials, and on the basis thereof revise instructional goals.

5. Collaboration within the practice of teaching

Second, the teacher and the educator can collaborate on the development of teaching, but this must be at a concrete/practical level, for instance working on analysing teaching and finding alternative possibilities for actions in such concrete situations.

When it comes down to basics, perhaps this was the way we should have addressed the questions from the teacher mentioned in the introduction. Without ever having put foot in his classroom, we could ask questions which would have the potential to promote reflection:

Why are you so interested in a good show of hands? Which types of answers would you like to hear? What would you do with a good show of hands? Why do you think the students do not put up their hands?

How would you overhaul the class? What is your purpose? What do you expect the students to gain from it? Are there other ways to achieve the same end?

Once one has first started to think about one's own teaching, and not least one's ideas about teaching in such a way, then there are almost no limits to the potential for development. And, the reflecting teacher is the acting person, holding the responsibility for her or his own process of development – also in the view of the educator.

But the process cannot end there, of course, because then it would not be a equal collaboration, and it would not be respectful towards the experiences and knowledge of the practicing teacher. Thus, the teacher must come back to the educator and ask:

I try to use the students' knowledge as a starting point and when possible put the mathematics into a context. But I cannot recognise what you said about intentionality or curiosity in my students. They do not seem to accept a challenge; instead, they ask for algorithms and answers. How do you explain that?

Sometimes, I experience that a student talks of a sudden insight and suddenly can explain connections that she clearly has not been aware of before. But two days later, it seems as if she has forgotten everything. How can you explain that within constructivism?

Which role does doing standard exercises play in the students' construction of knowledge?

Then perhaps the educator too will also have some food for thought.

The only way to further such development is through dialogue. Following Mellin-Olsen (1993), I will distinguish between two types of dialogue:

It is time to explore the nature of a dialogue. Here we shall in particular analyse one understanding of it. That is the dialogue as a *confrontation about ideas and understandings* [... Diderot] considers the dialogue as a way to gain insight and knowledge. The power of this tool lies in its method of confrontation which is based on the idea that consensus is not the goal.

This understanding of a dialogue is in opposition to the common understanding that dialogue is a tool for consensus. In politics, the notion of dialogue usually makes one think about two opposing sides which approach each other in order to reduce contradictions. A headline in a newspaper, 'Dialogue between East and West' or 'Dialogue between North and South' usually provokes hope and relief with the reader. (p. 245)

If we apply the notion of dialogue as a tool for consensus, we may lose a lot of vitality, seeing of alternatives and potential for development. I argue for the promotion of a dialogue of the former type between theory and practice, between teachers and educators, with the purpose of exploring different perspectives directed by an epistemic interest in furthering communication and understanding.

Taking the initiative towards such a dialogue would not only be in accordance with constructivism, it would also fit well into activity theory, and it would open the seeing of alternatives in theory as well as practice. One can only wonder why this idea did not occur to us on the planning committee.

Note

[1] Obviously, educational theory will also be developed in a such interplay. This approach is being pursued in several research projects. Here are brief accounts of two:

- (i) A Danish research project, involving the mathematics teachers at N. Zahle's College of Education in Copenhagen, the psychologist Mette Geldman from the same college and myself, aims to describe the practice of the mathematics teachers and links between their practice and their understanding of teaching, mathematics, etc. Actual teaching sessions have been video-recorded, but the research material consists of the conversations between participants in the research team about the teaching and the teacher's thoughts about it. On the basis of these conversations, we hope to have a foundation for development of theories in mathematics education which will be based on a respect for practice

For further information on the project, contact Anna Jørgensen at N. Zahle's College of Education, Linnésgade 2, 1361 København K, Denmark (annaj@image.dk)

- (ii) The project *Describing Teachers' Current Mathematics Classroom Practices* is a research project under the South African president's education initiative on classroom-based research: towards best practices in the teaching of mathematics. It is being carried out by the Primary Mathematics Project at the University of the Western Cape in Cape Town and aims to investigate and describe current practices for several reasons

- (a) To form a basis from which to assist teachers in seeing and using alternatives in terms of materials, teaching style and activities, content and organisation, etc
- (b) To determine what is working within practices, especially those which have hitherto not been well described
- (c) To develop methods for describing practices. This is necessary for further work on describing how practices change making it relevant in terms of determining the success of curriculum development initiatives.
- (d) To provide a basis for the planning, implementation and evaluation of actual initiatives in the in- and pre-service training of teachers

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