TOWARDS A SEMIOTICS OF MATHEMATICAL TEXT (PART 3*)

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In this article, the last of three in a series, I focus primarily on the social contexts of mathematical text. In particular, I use the social semiotics of Michael Halliday (1975, 1985) to structure my account. In this final part I explore what Halliday's interpersonal and textual metafunctions might mean in elucidating the semiotics of mathematical text.

The interpersonal function

The second metafunction of mathematical text according to Halliday's scheme is the interpersonal function. This concerns the positioning of the 'speaker', that is the author of the text, the positioning of the reader, and the relationships between the author and addressees as embodied in the text (Morgan, 1996). [1]

From the perspective of semiotic systems, there are two levels of language and text. First, there are the signs or texts of a semiotic system, which is where mathematics is constructed, utilized or otherwise enacted. Second, there is the metalanguage employed within the social context in which the semiotic system is utilized. This meta-level corresponds to actors discussing activities related to the semiotic system or its social context, rather than enacting the mathematics itself. Rotman (1993) makes a comparable distinction between the Code and the MetaCode of mathematics.

In the school mathematics context, both these levels of text are involved in the setting of tasks, their performance by the students, and commentary on and evaluation of the texts produced. Classroom texts spoken by the teacher, often in conjunction with other supporting modes of representation, position the actors in a number of reciprocal and pairwise defined roles, including task setter – task performer, work manager – work producer, assessor – assessee, knowledge giver – knowledge applier, and knowledge owner – knowledge requester. [2] In each case, the student is in the second of the two roles, the less powerful position. This analysis reflects that the teacher has two overlapping roles – namely as director of the social organisation and interactions in the classroom (*i.e.,* social controller) and as director of the mathematical tasks and work activity of the classroom (*i.e.,* task controller). This distinction corresponds to the traditional separation between being 'in authority' (social regulator) and being 'an authority' (knowledge expert) (Amit & Fried, 2005; Lloyd, 1979).

In written classroom texts only some of these listed personal positions and roles are embodied in the text, including task setter – task performer, knowledge giver – knowledge applier, with the student/addressee adopting the second of the two roles, as before. Most of the roles prescribed for students, whether in spoken or written texts, are to a large extent implicitly embodied and encoded at the level of semiotic system texts (*i.e.*, at the Code level).

Halliday (1975) has argued that positionings in the text become a surrogate for social regulation. They stand in for and reproduce social structures and power differentials as experienced by children. Thus there is a "chain of dependence such as: social order – transmission of the social order to the child – role of language in transmission of the social process – functions of language in relation to this role – meanings derived from these functions" (Halliday, 1975, p. 5). So social structures and power relations are embodied in language uses (in discursive practices), and in particular, in the uses of texts.

Post-structuralists like Henriques *et al.* (1984) assert the potency of the constitutive triad of power-knowledge-subject. They challenge the concept of the unitary human subject and argue that, through the confluence of power and knowledge embodied in socially located texts, not only positions but subjectivities are formed. In place of the human subject as unitary agent, they see the "subject as a position within a particular discourse" (Henriques *et al.*, 1984, p. 203). [3]

The formative import of text and discourse in the construction of subjects and selves can be traced back to the works of G. H. Mead and Vygotsky, via such processes as are described in the previous paper in this series and its Figure 1 (Ernest 2008b). Such views are stressed by discursive psychologists including Gergen (1999), Harré (1979), Harré and Gillett (1994), and Shotter (1993), who see distinct identities being constructed for an individual within differing discursive practices, according to the linguistic and social positionings in play.

More broadly, the strong impact of attitudes and beliefs on the formation of mathematical identity is well known through a variety of studies, although commonly drawing on more traditional theorizations. The social construction of mathematical ability (and inability) is multiply theorized by psychological, sociological, educational and feminist researchers (see, *e.g.,* Burton, 1988; Buxton, 1981; Diener & Dweck 1978; Ernest, 1995; Evans. 2000; Fennema & Leder, 1990; Walkerdine, 1998). There is currently a growth of interest in research on mathematical identity within the mathematics education research community (*e.g.,* Boaler, 2002). [4] But little of this work treats the role of mathematical text in positioning its

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readers (and writers) and the impact of this on identity construction. [5]

Semiotics offers some tools that further this project. In analyzing the role of texts and their effects on readers Eco (1984) theorizes the Model Reader presupposed by and produced by the text.

> The author has to foresee a model of the possible reader (hereafter Model Reader) supposedly able to deal interpretatively with the expressions in the same way as the author deals generatively with them.
>
> At the minimal level, every type of text explicitly selects a very general model of possible reader through the choice (i) of a specific linguistic code, (ii) of a certain literary style, and (iii) of specific specialization-indices. …
>
> Many texts make evident their Model Readers by implicitly presupposing a specific … competence. … But at the same time text … creates the competence of its Model Reader. (Eco, 1984, p. 17)

This quote raises questions. How do mathematical texts impact on their readers and what is specific about these texts (and their readers) in the context of mathematics? What are the characteristics of the model reader(s) of mathematical texts? What assumptions are made of the model reader in order to engage with the text and what are the constitutive impacts on the model reader of these engagements?

According to Radford (n.d.) the model reader's interpretation of a text falls within a system of knowledge, which, when applied to mathematics, comprises:

1) *systematic knowledge:* knowledge of language as a vocabulary and set of grammatical rules, and in particular, the signs and rules of the semiotic system(s) of mathematics in play;

2) *encyclopedia:* encyclopedia of cultural knowledge and conventions, including general knowledge of mathematics, its meanings and domains of interpretation and application, both within and outside mathematics; and

3) *experience:* history of previous interpretations and engagements with other texts, primarily mathematics texts of various forms, but also including some popular texts or others unrelated to mathematics.

Thus the reader of a mathematical text both draws on and develops their systematic knowledge (of semiotic systems), their broader personal encyclopedia of knowledge, and their experience of earlier engagements with mathematical tasks and other texts. (The development of the reader's knowledge in this way constitutes the major goal of education, namely learning.)

Theories of the model reader can be compared to models of mathematical thinking and problem solving, including that of Charles and Lester (1982) that specifies three factors: experience, affect, and cognitive factors. This model does not distinguish systematic from encyclopedic knowledge, and introduces the novel factor of affect (interest, motivation, pressure, anxiety, *etc.*). Schoenfeld (1992) offers a more elaborate model with five components: knowledge base, strategies, metacognition, beliefs and affects, and practices (knowledge of real-world contexts, experience). This again introduces affect (including belief). It also distinguishes two types of systematic knowledge: knowledge base and strategies. However, the version of encyclopedia is limited and is put with experience into one category. However, a further novel component introduced in this model is metacognition, the self-knowledge and self-regulation of the mathematical reader/enactor. These are typical of such models in mathematics education research (*e.g.,* Charles, 1985), in adding further psychological details but underestimating the significance of the encyclopedia.

Eco (1984) distinguishes between the model reader (a theorized and generalized addressee) and the empirical reader (an actual reader of a text). Clearly Radford's three types of knowledge (systematic, encyclopedic, experiential) will vary between model and across the range of actualized empirical readers. This variation across reader types is especially important in mathematics, where levels of competent engagement with mathematical text vary greatly. In addition to drawing on and impacting on readers' knowledge, engagement with mathematical text results in the development of different selves or identities within the reader. [6] I assume these to be different from the selves or identities developed in extra-mathematical contexts and practices.

The deepest analysis of the different identities developed within mathematics is provided by the semiotic theory of mathematics due to Rotman. As part of his project Rotman (1988, 1993) analysed the language of published research mathematics texts. He identified sentences to be the main linguistic units, and these to be made up of symbols, terms (nouns) and verbs. Following the traditions of literary and grammatical analysis, he takes the type of verb case in mathematical sentences to be the main indicators of the roles of author and addressee. Drawing on literary theory, most notably Berry (1975), he finds these verb cases to be of two main sorts.

First, there are verbs in the indicative mood, concerning the communication or indication of information. In this case, "the speaker of a clause which has selected the indicative plus declarative has selected for himself the role of informant and for his hearer the role of informed" (Berry, 1975, p. 166). Thus the speaker/author asserts to the hearer/addressee some state of affairs that obtains, or, more commonly in mathematics where texts describe mathematical actions and processes, the outcomes of these processes. Such sentences not only describe the outcome of past, contingent sequences of actions and procedures, a particular transformation of signs, but also claim that when operating within the rules of the language game (the semiotic system), the outcome described is what always must happen. The descriptions of these outcomes resemble logical predictions, taking place in a timeless realm but describing the logical outcomes of the processes involved. Thus indicative propositions might be said to describe thought experiments which persuade us to accept the validity of their assertions (Peirce, 1931, p. 58).

Second, there are verbs in the imperative mood, requesting that an instruction or action be carried out. There are two forms:

1) the inclusive imperative (*e.g.* "Let us define … .", "Consider a language *L* …"), in which the addressee is required to cooperate or collaborate in following the speaker or carrying out the instruction in some imposed shared realm of discourse jointly, and

2) the exclusive imperative (*e.g.,* "Add …", "Count the cases …", "Integrate the function …") which demands that an action be carried out by the hearer alone in a presupposed shared frame.

In both cases, "the speaker of a clause which has selected the imperative has selected for himself the role of controller and for his hearer the role of controlled. The speaker expects more than a purely verbal response. He expects some form of action" (Berry, 1975, p. 166).

These findings are echoed in Shuard and Rothery's (1984) analysis of school mathematics texts. They found several types of text, each with its own purpose: exposition, instructions, examples and exercises, peripheral writing, and signals. The number of types of text goes beyond that in Rotman's analysis, presumably because the function of school texts goes beyond that of describing mathematical results. They found expository writing utilizing the indicative mood in school mathematics texts, although these typically provided exposition of concepts and methods, including explanations of vocabulary, notation, and rules, rather than fully fledged proofs.

Another type of language in school mathematics texts utilizes the imperative mood. Such texts include instructions to the reader to write, draw or to perform some action, typically utilizing direct imperatives. They also include examples and exercises for the reader to work on. Often these examples are routine problems involving the application of specific predetermined procedures to mathematical expressions (*e.g.*, 'Calculate 134 – 79'), but they can also include word problems, non-routine problems and investigative work requiring the use of general heuristics to guide solutions. These tasks may be expressed with direct imperatives, but can also utilize implied imperatives if they are in question form or contain the substance of imperatives without the appropriate verb.

Beyond these two types of text, Shuard and Rothery also found peripheral writing, including introductory remarks and meta-exposition encouraging the reader, giving clues, and so on, that utilized the indicative mood but at the metalinguistic level. Lastly they found signals, including headings, letters, numbers, boxes and logos. These signals are not assertions but meta-signals to the reader to give the text structure and emphasis.

There is thus a good correspondence between the types of language employed in school mathematics and research mathematics, as indicated by the verb forms. However, the roles of reader and writer and the social power relations they embody are more sharply distinguished in school texts than in research texts, reflecting the clear cut role and power relation differentiation in the social context of schooling, where the teacher and pupil roles are not interchangeable. The teacher is almost invariably the writer of the task text (or their surrogate, its presenter), while the pupil is the reader/enactor of the text. In research mathematics texts although the author role controls that of the reader, it is expected that mathematicians will assume the roles of both reader and writer according to whose text is being read. [7] Furthermore, such readers can also adopt the meta-role of critic of research mathematics texts in a way that is not normally encouraged in the context of the mathematics classroom. However, Walkerdine argues that classroom indicators of mathematical success include both rule-following and rule-challenging:

> To be successful, children must follow the procedural rules. However, teachers perceive breaking set as the challenging of the propositional rules. They read it as 'natural flair'. … To challenge the rules of mathematical discourse is to challenge the authority of the teacher in a sanctioned way. Both rule-following and rule-breaking are received – albeit antithetical – forms of behaviour. (Walkerdine, 1998, p. 90) [8]

Thus the positioning of the reader of school mathematics text is a complex and contradictory one if the reader is to both develop the powers that are available through engagement with the text (including acting on its imperatives) and be perceived by the teacher as so doing.

Rotman (1993, 2003) continues his account of the semiotics of mathematics with a theory of the mathematician as both an author/writer and addressee/reader of mathematical text, based on the linguistic features of such text. He posits three aspects of the identity of the mathematician, three 'actors', each with a corresponding and increasing level of abstractedness, but with diminishing agency and subjectivity, in the sequence: Person, Subject and Agent.

The Person is embedded in the material world and has full access to voluntary human activities including the uses of language, metalanguage, and meaning-making in general. [9] In the classroom, the student as Person hears the teacher's instructions to engage in a specific mathematical task. The Person is the only one of the three agentic roles that has full access to metalanguage as well as the language of semiotic systems, to indexical signifying resources, which are typically excluded from the signs of mathematical semiotic systems, and to the meaning structure associated with the semiotic system.

A subsidiary, abstracted and restricted identity is that of the mathematical Subject. The Subject has a restricted agency corresponding to the mathematical languages that Rotman (1993) terms the Code (as opposed to the MetaCode) of mathematics, corresponding to the signs and texts of the semiotic systems of mathematics. The Subject is circumscribed by sign systems that lack indexical markers for time, physical place and other personal modes of expression. Thus the Subject lacks the Person's capacity for self-reference or self-expression (not to mention feeling), and is only able to read and write within the semiotic system itself. A student evokes/assumes the identity of mathematical Subject in carrying out a task within the semiotic system, although she may punctuate this performance by assuming the full identity of Person in making meta-remarks about the task, or in attending to events or other personal matters and breaking out of the restricted functional identity of Subject (or of Agent).

The Agent is a minimal representative of the reader or writer, with no voice, no subjectivity, only the power to carry out imagined instructions as defined in the text. The Agent is a 'skeleton diagram' of self (Peirce, 1931-58, 2.227), like the moving fingertip on a map tracing out an imagined or past journey, but in the realm of signs. The Agent is

> the actor associated with the domain of procedure who functions as the delegate for the Person through the mediation of the Subject; the Agent executes a mathematically idealized version of the actions imagined by the Person and it does so formally since it lacks the Subject's access to meaning and significance. (Rotman, 2003, p. 3).

The Agent represents an unconscious agency that can only follow fully specified procedures without any decision making. Because of these features, it is possible to fully mechanize the agency involved, which is what underpins digital computing. The Agent in mathematics could only be imagined until electronic computing was developed, with its electronic mechanization of the mathematical agent. A semiotic realm has been created, namely digital information space, where a surrogate agent can act out its instructions mechanically.

In the context of schooling, when a learner has developed automaticity in a certain range of rule-based transformations of text, the Agent is the sub-identity of the Person that enacts them. The Agentic self carries out mechanical procedures such as those involved in '27 + 91', 'Simplify $3a + 5b - 2a + b$', or 'Solve $39x - 42 = 17$', once they have become automatic for the student. In carrying out such tasks there are three stages. First, the Person acknowledges or recognises a task and its goal structure. Second, the Subject identifies the procedures to apply. Third, the Agent applies the rules of the semiotic system involved to the text and generates the sign sequence to which they give rise. The person monitors the activities of the Agent (and Subject) to ensure the chosen actions fulfil the task goals. Thus, movement between and alternation of the three roles during the working of tasks is not only a possibility but, in many cases, is a necessity.

To be successful a student of mathematics needs to be able to make transitions between these roles/positions. Ultimately a range of powers needs to be developed for each of the three roles/positions, including the following.

The Agent must be able to:

- perform routine text transformations;
- obey basic imperatives in mathematical text.

The Subject must be able to:

- read mathematical texts and make sense of them as tasks, computations, derivations;
- access a repertoire of text transformations and apply them in completing tasks;
- write mathematical problems and tasks;
- judge whether mathematical texts follow the appropriate rules (*i.e.,* read them critically). [10]

The Person must be able to:

- regulate the activities of the Subject (and Agent) through planning, monitoring, and control;
- regulate the identity assumed and the subjectivity accessed, according to the social context.

Although these are the powers of the different roles/positions making up the model reader in mathematics, they do not form a fixed developmental sequence. Clearly what comes first in the emergence of the empirical reader is the Person, although the development of its powers is an accomplishment achieved over several or many years, and for students its successful completion is by no means guaranteed. In Ernest (2003) I discuss how the Agent may develop before or at the same time as the Subject, in response to the types of mathematical texts/tasks with which the learner engages, subject to and constituted by the specific contexts of schooling and pedagogies in which it takes place. I also point out the existence of pathologies in the empirical reader, with some students stuck with only an Agent and not a properly formed Subject in mathematics, as evidenced by their ability to perform limited routine tasks but not able to decide what processes and knowledge to apply to less routine or not recently practiced tasks. This is a limited version of Skemp's (1976) instrumental understanding.

Schoenfeld (1992) studied the patterns of work of novice and expert non-routine problem solvers in mathematics and categorized it into seven stages: Read, Analyze, Explore, Plan, Implement, Verify, Self-question. He found that many novices' attempts comprised a minute or two of reading the question, followed by an unbroken sequence of unreflective exploration of the problem, seeking to solve without plan, typically through the generation of specific example texts. This is commonly observed, and when persisted in, it usually leads to failure due to the lack of regulation and monitoring of outputs and their poor match with task goals.

In contrast, experienced mathematician problem solvers typically work through all of the first 6 levels or stages, cyclically, often more than once. Furthermore, they typically ask themselves questions privately or out loud throughout the process, suggesting metacognitive self-monitoring and self-regulation of their overall problem solving processes. Success in mathematical problem solving is typically associated with such self-questioning behaviours. This suggests that the regulatory powers of the Person as listed above are an essential part of creative and non-routine problem solving in mathematics, if not all mathematical tasks.

The powers of these mathematicians resemble those intended for the model reader of mathematical texts, as well as those at the powerful end of the spectrum of the actualized capacities of empirical readers. In contrast, the capacities of the novices constitute those near the other, weaker end of the spectrum of powers of empirical readers, as in the pathological cases mentioned above.

Although Eco's concept of the model reader is a useful theorization to apply to the interpersonal metafunction of mathematical text, there are two notable ways in which it falls short. Mathematical identity, even just in theory, includes quite a lot more than the model reader. Model readers of any genre of text engage in interpreting and actively constructing meanings for texts (Corner, 1983). However, in mathematics the model reader usually also writes and draws, constructing novel texts as well as interpretations of given texts (Rotman, 1994). Thus in mathematics we need to include writing powers in theorizing the model reader/(writer).

It is relevant to mention here Hall's (1980) theorization of three modes of reader response/sense making arising from engagement with text, based on the assumption that texts encode a dominant ideology or reading. In brief, these modes are 1. *dominant (or 'hegemonic') reading:* the reader fully shares the text's code/ideology, which may seem 'natural' and 'transparent'; 2. *negotiated reading:* the reader broadly accepts the preferred reading, but sometimes modifies it in a way which reflects their own position, experiences and interests; 3. *oppositional ('counter-hegemonic') reading:* the reader, who adopts an oppositional relation to the dominant code, understands the preferred reading but does not share the text's code and rejects this reading, bringing to bear an alternative frame of reference. The extent to which mathematical texts encode dominant ideologies and to which model readers can construct oppositional readings needs further development.

However, note that in Part 1 (Ernest, 2008a) I critique the dominant Platonist ontology that constitutes the received ideology of mathematics, and in Ernest (1998) I offer a model of mathematical development (the Generalised Logic of Mathematical Discovery) in which critical reactions to published mathematical texts can lead to a global restructuring of the overall research practice, including changed methods, informal theories, proof paradigms, criteria and meta-mathematical views – many of the elements that make up an ideological position. In Ernest (2005) I distinguish between two text-related roles:

1) the role of proponent (or friendly listener) presenting (or following sympathetically) a text, argument or thought experiment (attempting to 'share' the constructor's meaning, rather than looking for grounds on which to dismiss it). The role of proponent/friendly listener can be at two levels 1a, reflective or higher order, or 1b, at a lower, passively attentive level (corresponding approximately to Hall's first two roles – reversed).

2) the role of critic, in which a text is examined for weaknesses and flaws. This can be at two levels 2a, local critique, and 2b, global critique, as indicated in Generalised Logic of Mathematical Discovery (Ernest, 1998). The latter partially corresponds with Hall's oppositional reading role, in the way that it offers an analogue of Kuhn's (1970) scientific revolution within mathematics. Thus the mathematical reader / writer through their mathematical identity, potentially brings much more to bear in engagement with mathematical texts, even in the case of simple mathematical tasks, than a collection of mechanical and higher level skills and knowledge.

Secondly, continuing in this vein, as discussed above mathematical identity has an important affective component. The mathematical model reader/writer does not just have cognitive/epistemological capacities, but also emotions, feelings, attitudes, beliefs, and values that play a central role both in the self-perceptions of subjective identity, and in facilitating or blocking the functioning of the mathematical model reader/writer. Enlarged in this way, the concepts of model reader/writer and empirical reader/writers have real potential for theorizing differing mathematical identities and linking them to different levels of mathematical performance.

The textual function

Halliday's third category, the textual function, is about what sort of text a mathematical inscription/utterance is, how the text is created and structured, and how it uses signs and textual components to fulfil its purposes. This function concerns issues such as what the mathematical text is attempting to do, whether it is describing a process, communicating relationships between signs, setting out to prove a claim, specifying a task, and so on. In mathematical text, as in all subject domains, this function overlaps with the ideational and interpersonal functions described above. In particular, the ideational, interpersonal and textual components of mathematical texts form a mutually constitutive triad in the closed and self-referential semiotic space of mathematics.

One of the special characteristics of mathematics is the range of signs and sign types utilized in constructing mathematical text. Typically in printed or written texts these are of three types, and Rotman (1994) emphasises the essential and ineliminable role of these last two categories for mathematics:

1) Alphanumeric signs used to make terms, words and sentences, encompassing written language (*e.g.,* French or English) and numeric and algebraic terms and propositions;

2) Special mathematical symbols for relations, functions, objects, either in the form of pictograms (*e.g.,* $\rightarrow$, $\subseteq$, ∞, $\sum$) or taken from a range of languages and scripts (*e.g.,* Gothic and Hebrew letters), and often employing added subscripts and superscripts;

3) Diagrams and pictures, typically two dimensional drawings and line diagrams with or without labelling.

The simplest mathematical texts utilize only the elementary signs of type 1, which, in addition to alphabetic letters (both lower and upper case) and punctuation signs also includes a small selection of simple and familiar pictograms (=, +, –, ×, ÷) as well as the numerals 0, 1, 2, 3, 4, 5, 6 ,7, 8, 9. A very large range of compound signs can be made with this set of basic signs, including all possible word problems and arithmetical tasks, and some elementary algebraic tasks. [11] Mostly, the texts written by students working on word problems or routine mathematical tasks will just employ a limited range of type 1 signs.

Texts that incorporate signs of type 2 are typically more complex and advanced mathematical texts, and it would be unusual to find advanced mathematical texts that do not use a significant number of such signs. Signs of type 3 (diagrams and pictures) typically occur throughout school texts, but will only be part of some specialized mathematical task presentations (*e.g.,* in geometry). [12] In advanced published (or equivalent) mathematical texts signs of type 3 are usually sparingly used, except in, say, topology or geometry texts, and will often be very abstracted and heavily labelled, such as in the arrow diagrams of category theory.

In contrast, the texts written by students working on non-routine mathematical tasks will often employ signs of types 1 and 3, and possibly also signs of type 2 in advanced mathematical studies. As indicated above, such texts may well utilize a broader range of sign types than those employed in the problem or task specification or in the final text, the answer.

The two most important functions of teacher presented mathematical texts in the social context of schooling are first, to present tasks for the reader to work, and second, to supply expository text providing explanations (including definitions and terminology), or demonstrations of procedures, calculations or deductive proofs. Putting to one side meta-expository text intended to help orientate, organise or instruct the reader, mathematical texts will typically fall within well defined semiotic systems, so that the reader has a good idea of the set of signs that is in play, which one is appropriate to use, and the sets of rules and methods to apply in completing mathematical tasks. The expository texts will be designed to add to the reader's meaning structure (*i.e.,* their understanding) and their knowledge of the rules and methods in play in the system (their competence in operating the semiotic system). However, as we have seen in the previous section, such texts and tasks both elicit and generate a complex array of reader/writer roles and positions in both utilizing and constructing mathematical identities.

Only advanced mathematical students or mathematicians will be expected to read extensive mathematical texts with elaborate demonstrations of procedures or deductive proofs, as these take specialized knowledge and reading skills (Mousley & Marks, 1991). Although such texts have epistemological significance – that is, they are knowledge claims and validations – they are fundamentally technical and non-narrative texts which are read to see if and how rules and transformations are correctly applied to derive their final signs. [13] Great ingenuity and imagination may have been deployed in their construction, and indeed considerable knowledge and imagination may be needed in their reading, but nonetheless such mathematical texts are technical inscriptions. Because of their dense and technical nature successful readers of advanced mathematical texts typically imagine a universe of mathematical objects that are described and acted on in the text. Although these correspond to objects in the meaning structure of the associated semiotic system, they are typically mythologised by mathematicians as entities in an ideal superhuman realm. Such realms are, needless to say, imaginary, but nevertheless fulfil an important function in facilitating human understanding and meaning making (Burton, 1999). [14] Thus ironically, given my critique in Part 1 (Ernest, 2008a), Platonistic conceptions of mathematical objects may be an effective means of comprehending mathematical signs via the meaning structures of semiotic systems even if they do not have the ontological significance they are often given (Rodd, 1998). Thus, in one sense, the textual function of advanced mathematical texts is to tell tales about these fictional mathematical objects. [15]

Conclusion

In this series of articles I have presented a tentative and evolving semiotic theory of mathematical text. The aim has been to give an account of how mathematical text 'works', in both the contexts of schooling (including university taught mathematics) and mathematical research. The theory of semiotic systems provides a simple structural device for understanding the mechanics of mathematical text and its symbolic functions, although despite this superficial simplicity there is a pool of ineliminable complexity in the underlying meaning structure of any semiotic system. A further area of complexity concerns the relations between semiotic systems. Repeated minor changes to semiotic systems results in a sequence of related, overlapping semiotic systems that are treated as the same in some contexts but must be treated as different in others. Yet all learners, to be successful in mathematics, must master such chains of semiotic systems, adding new properties to say, semiotic systems concerning number, and 'forgetting' other properties (*e.g.,* 'you cannot divide larger into smaller numbers'; Ernest, 2006).

This account also sketches elements of a theory of the mathematical subject, and its relations with mathematical activity and mathematical identity. The simplicity of the theory of semiotic systems, which looks at whole series of mathematical signs and derivations/transformations in the performance of a mathematical task, is not supposed to diminish the difficulties and real accomplishments of learners in being able to perform just a single transformation of text. [16] Nevertheless, even if such single acts are backgrounded in the quest for the 'big picture', there is a need for a single unifying theory for describing what goes on in the reading/writing of mathematical text, and within any such text itself. In my view such a theory is required for further progress in understanding the key role of mathematical text in the teaching and learning of mathematics, within mathematics itself, and in the discursive production of mathematical identity.

Notes

[1] Although I refer to the speaker, author, utterer, writer of mathematical text, if not interchangeably, without drawing firm distinctions between the spoken and written text, I fully appreciate that there are significant differences. As Derrida (1976) has argued, writing is not reducible to the spoken word. Rotman (1994) further argues that mathematical text is not reducible to alphanumeric text, let alone the spoken word (see the discussion of the textual metafunction below). Beyond this, in Part 2 (Ernest 2008b) I have argued that the texts I refer to are constituted by multimodal sets of signs, all of which (in token form) are physically embodied, some on the scribbled or printed page/computer screen, and some gestural, spoken or otherwise physically embodied. Viewed in this way, the spoken/written distinction loses some of its force.

[2] Morgan (1998) has also identified a number (8) of roles for teachers and students, indicated by the language used in assessment tasks, which overlap with, but are distinct from those given here.

[3] Henriques *et al.* (1984) go on to problematize the tensions between different subject positions and the production of human subjectivity. However, my concern takes for granted a temporarily unproblematized concept of a person to ask: How are new (subsidiary) subjectivities or identities constructed through engagement with mathematical texts/contexts?

[4] It is worth noting that these projects draw on differing theoretical bases.

P. Grootenboer, T. Smith and T. Lowrie (2006, *Researching identity in mathematics education: the lay of the land,* last accessed on 2008 June 18 at http://www.merga.net.au/documents/symp12006.pdf) contrast three different approaches: (1) the [cognitive] psychological/developmental, (2) the socio-cultural, and (3) the poststructural. L. Black, M. Brown, H. Mendich, M. Rodd and Y. Solomon (2006, *Mathematical relationships: identities and participation project*, last accessed on 2008 June 19 at http://www.lancs.ac.uk/fass/events/mathematicalrelationships/index.htm) draw upon three distinct theoretical bases in their group project investigating identity, participation and mathematical relationships, namely (1) socio-cultural theory, (2) discourse theory and (3) psychoanalysis, but view them as complementary. The present paper stems from a social constructivist/socio-cultural perspective, but also draws on discursive/poststructural and cognitive psychological concepts as tools for semiotic analysis.

[5] Walkerdine (1998) explicitly discusses the role of text in constructing subjectivities without detailed elaboration, but most references in the literature refer to the discursive production of selves without explicitly highlighting the role of written text in such processes See, *e.g.,* Evans *et al.,* 2006. See also Davies, B. and Harré, R. (n.d.) *Positioning: the discursive production of selves,* last accessed on 2008 June 14 at http://www.massey.ac.nz/~alock/position/position.htm.

[6] It is in the development of these different selves or identities that the affective factors discussed above importantly come into play.

[7] Indeed mathematicians, certainly in anglophone countries such as USA and UK, are loathe to bestow the title 'mathematician' on anybody who is not a writer of research mathematics texts, irrespective of whether they routinely read research mathematics texts, or work professionally with mathematics in other ways.

[8] Walkerdine goes on to argue that there are gendered presuppositions about the appropriateness of rule breaking, and that it was valorised by teachers for the boys that she studied in the 1980s and pathologised for the girls.

[9] It is the Person (and only the Person) who owns and experiences beliefs, affects and feelings, and has metacognition, subjectivity and voluntary agency.

[10] In some activities this capability might also or instead be part of the role of the Person.

[11] For very young children some texts include pictures of, *e.g.,* flowers or cars, to be counted, and multimodal texts for them can also employ arrays of material objects, *etc*.

[12] An exception is when diagrams, photographs or other pictures are attached to/included in mathematical tasks to signify the task context or 'real world' situation which the task is supposed to model. Although a widespread view is that such type 3 signs support the student's meaning-making for the task, researchers have indicated that this role is far from proven and indeed can impact differentially on learners according to social class, rendering some tasks less accessible (Boaler, 1993; Cooper, 1992; Dowling, 1998).

[13] Hanna (2000) has convincingly argued that mathematical proofs can have explanatory roles as well as their demonstrative (epistemological) functions. Although mathematical proofs are fundamentally technical, in the sense described, they nevertheless employ (and extend) a wide range of argumentative devices, including those of deductive logic and rhetoric, used in all types of texts intended to convince or persuade. Such devices include all of the traditional forms of argument, such as *modus tollens* and *reductio ad absurdam,* and add in further modes such as mathematical induction.

[14] Although imaginary, mathematical objects are no less real than any other cultural objects, and indeed might be said to be more so, in view of their distinct and clear properties specified in the texts that generate them.

[15] I find myself torn – on the one hand I strongly espouse nominalism in mathematical ontology, mathematical objects are nothing but signs (Ernest, 2004) – on the other hand I increasingly find myself a mathematical realist – mathematical objects are real, cultural entities (Ernest, 1998). Perhaps these traditionally opposed positions are but two sides of the same coin.

[16] I use the phrase 'being able to' with the caveat that, just as with other capacities and competences, such states are always unobservable theorizations inferred (induced) from observed sequences of performances. We can never know with certainty what another's true abilities are, because ability includes the promise of future behaviours as well as referring to past observations.

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The Lost Treasure

You find a treasure map – or rather, a set of treasure directions. The directions state that a great treasure lays buried on a nearby island in close proximity to a stand of three great trees (an oak, an elm, and an ash). To find the exact location of the treasure you are to start at the oak and walk directly towards the elm being careful to count your steps. When you get to the elm you are to make a precise 90˚-left turn and count out the same number of steps as you took to get to the elm. Mark this spot with a flag. Now, you are to return to the oak and walk towards the ash – again being careful to count your steps. When you get to the ash you are to make a precise 90˚-right turn and count out the same number of steps as it took you to get to the ash. Mark this spot with a flag. The treasure lays buried below the midpoint of the two flags.

Being a keen treasure hunter as well as being aware of the island that the map (er … directions) speaks you rent a boat and set off. Upon arriving on the island in question you quickly locate both the elm tree and the ash, but there is no sign of the great oak. It has undoubtedly long ago been struck by lightning and burned to the ground. Where is the treasure? (unknown origin; selected by Peter Liljedahl)