Children's Representations of the Development of Solids

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Bloom [1988] stresses the importance of geometry in school mathematics. He emphasizes "the usefulness of geometry, not only in its role in day-to-day activities but also as a vehicle to promote visualization in mathematics and many of the sciences". In Greece, in the present primary mathematics curriculum, this aspect of geometry has been neglected. Especially, little attention has been given to the development of childrens' ability to visualize space.

Michelmore [1980] also argues that "It is of great value to be able to visualize and represent three-dimensional configurations and to comprehend the geometrical relations among the various parts of a figure". Howe and Yasu [1988] notice that the lack of emphasis on imagery is surprising when one considers that most basic science concepts are related in some way to a three-dimensional object or model whose understanding requires an act of imagination in the formation of a mental image.

This paper describes an attempt to explore how children conceive the nets of three-dimensional objects. This is closely related to children's ability to visualize the solids' characteristics in relation to their nets. Apart from Piaget's work (Holloway [1967]), not much has been done in this area. More recent studies concerning nets of solids are these of Bourgeois [1986] and Mariotti [1989].

We find the problem of the development of solids stimulating because it not only provides an opportunity for visualization, but it also has applications to many aspects of everyday life, for example to the design and construction of various industrial products (shoes, boxes, hats, etc.)

The particular issues which this study addressed were the following:

- Were there any commonalities in childrens' drawings of nets and, if so, what models were identified?
- Were there any differences in these models among children of different ages?
- Would whole class discussion, which provided opportunity for reflection on the models, improve children's performance?

Description of the experiment

The experiment was carried out in a state primary school in Patras. It was designed to include two phases: to identify children's ways of drawing nets of solids and to provide opportunities for children to reflect on their own models. 67 children consisting of 42 eleven year olds (two classes) and 25 nine year olds (one class) took part in the first phase of the study. Only the 42 eleven year olds participated in the second phase of the study.

First phase

Three physical objects were chosen: a matchbox (the internal and the external part), a toilet roll, and a sardine tin. The materials were presented to the children. Children were asked to draw on a piece of paper how these solids would look when cut and unfolded on their desk. They were free to use the objects but not to cut them. In addition it was explained to them that they should be able to reconstruct the original solids from their drawings. Children worked individually for as long as they needed to complete the project.

The tasks were administered in the same order to each child. The whole activity lasted 45 minutes.

Children were interviewed while they were tackling the activity. The interviews were conducted by both authors and audiotaped, and aimed to illuminate children's thinking and methods

Second phase

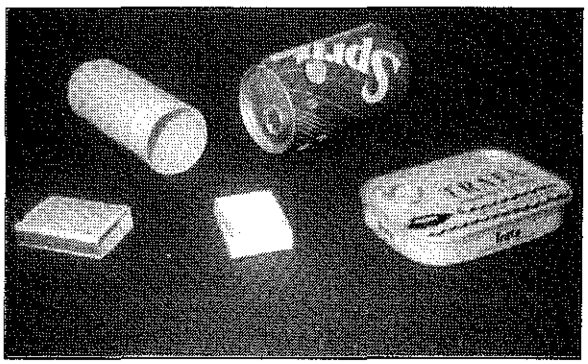
The results from the first phase were analysed and the drawings were grouped in categories.

One month later the older age group was shown some particular models of each category that had been produced. The models of the toilet roll were discussed in one class and the sardine tin in the other. Both classes discussed the external matchbox models. In a whole class discussion children were asked to comment on these models and reflect on their thinking process during the tasks.

The aim of this phase was not only to challenge children's initial approaches and promote conceptual change but to verify our interpretations and categorisation of the children's drawings.

An activity similar to the one described in the first phase was organised at the end of the 35 minutes discussion. This time a Sprite tin was given. It was intended to examine how the discussion affected children's models in a new situation.

Analysis of the materials



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The materials were real objects familiar to the children. The corresponding geometrical nets had not been studied in school mathematics. The materials varied in their physical construction and in their geometrical and topological characteristics The matchbox and the toilet roll were made of paper and were open solids whereas the tins were metal and closed. The toilet roll and the external matchbox were topologically equivalent. This was also the case for the metal tins The internal matchbox was topologically equivalent to a square. The Sprite tin differed from the sardine tin in its lateral surface and its height. The lateral surface of the Sprite tin was cylindrical, the sardine tin's surface consisted of both curved and plane parts. In the sardine tin the two flat bases constituted the largest part of the external area and the height was comparatively small. On the contrary in the Sprite tin the cylindrical surface was dominant.

A variety of nets could appear depending on the cuttings. The solids could be cut along the edges or by straight or curved arbitrary lines. The physical characteristics of the solids could also influence the nets.

Results from the first phase of the experiment

Classification of children's drawings

Analysing the children's drawings, the interviews and the researchers' observations, some commonalities seemed to appear. This led to a classification of the children's drawings in five categories: holistic models, models with elements of projection, incomplete geometrical models, complete geometrical models, and physical models

Holistic models: The children have difficulty representing in a conventional way the image they have of the solid and of its net. In the drawings, faces are not distinguished and dimensions are not taken into account Children with this model usually represent the net of the solid by a rectangle or a quadrilateral of arbitrary dimensions. Holistic models can be characterized as topological. Examples are given in Figures 1a, 2a, 3a, 4a.

Models with elements of projection: The children tend to represent the solids as they are or as opened out in a perspective way. Drawings of one or two bases of the solids are also included. In these cases the solids are viewed orthogonally. Particularly for the toilet roll, this perspective is expressed by projecting the front, or both front and back parts, of the cylindrical surface. Examples are given in Figures 1b, 2b, 3b, 4b.

Incomplete geometrical models: These models involve children starting to see the solids' nets, but considering only some of their elements or characteristics. For example, in the cases of the matchbox and sardine tin, some faces are left out, while the net of the toilet roll is represented by a parallelogram or a square with at least one arbitrary dimension. Examples are given in Figures 1c, 2c, 3c, 4c.

Complete geometrical models: The children are able to develop an acceptable net of the solid. They tend to see clearly, with the exception of the toilet roll, all the elements of the solids and how they can be developed. Nets with larger or smaller dimensions are also observed. Examples are given in Figures 1d, 2d, 3d, 4d.

In the case of the sardine tin, in both complete and incomplete models, two subcategories appear: a net with a continuous lateral surface and a net of a box. In the first, children see the development of the lateral surface as continuous and closed, while in the second they treat it as if it consisted entirely of (plane) faces.

Physical models: For the matchbox (external and internal) children are able not only to draw complete nets but to consider their physical construction as well. For the toilet roll, children use their imagination and experience of unfolding a real one to draw a non-conventional net. The curved parts of the lateral surface of the tin lead the children to make approximate geometrical models Although these models can be regarded as physical, such an idea is not expressed by the children Examples are given in Figures 1e, 2e, 3e, 4e.

Children's responses

Internal matchbox

The models met in each category are presented in Figure 1. The percentage of children at each age level who are functioning with each model is indicated in Table 1.

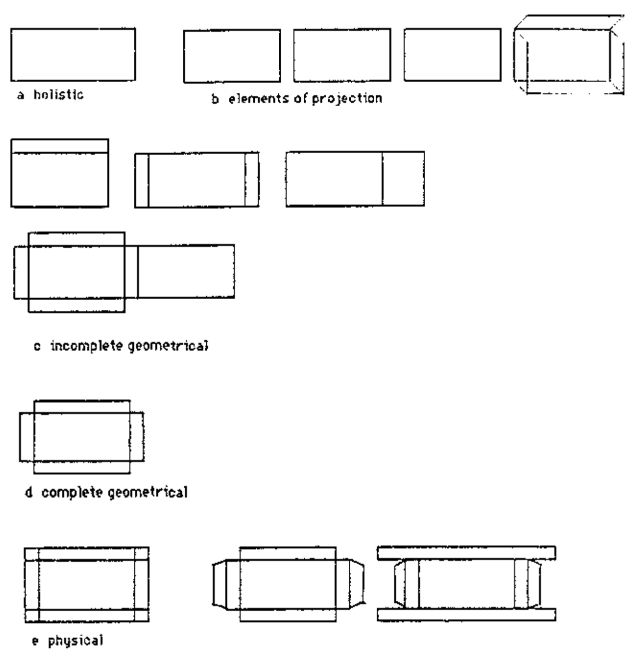

Figure 1
Children's figures of the internal matchbox.

	9 year olds N=25 (%)	11 year olds N=42 (%)
holistic	8	5
elements of projection	28	24
incomplete geometrical	8	10
complete geometrical	44	57
physical	12	5

Table 1
Children's models of the internal matchbox
(Percentage in each age group)

As may be seen, in both age groups, the majority of children draw a complete geometrical model About a quarter of the children construct a model with elements of projection Older children show a better appreciation of the geometrical characteristics of the developments. Physical models are found mostly in the younger children's drawings. About the same number of children in both grades adopt the holistic model.

Children whose drawings are classified under the model "elements of projection" (Figure 1b) more often tend to emphasize the base of the matchbox As some children admitted, the internal pieces which appear in some of the drawings represent either the lateral faces of the box or the two extra pieces used to seal the box.

Two children who clearly perceived the height did not draw the development but the open box in a perspective view One child's geometrical net with an extra base has been classified as an example of an incomplete geometrical model (Figure 1c).

Children adopt various strategies to draw complete models Often they use the box to stamp out its faces. sometimes children measure the dimensions of the box to draw the net. There are also some children who draw the net in arbitrary dimensions but in proportion to the real ones. Eleni (11 years old) recognises that her net is smaller than the actual solid but she has done this deliberately. When she was asked, "Why have you drawn it smaller?", she replied: "If I had measured and drawn it exactly as it was, I would have lost much time. So I made it like this, as quickly as possible." As appeared from the rest of the interview, Eleni was capable of drawing the net with the exact dimensions. On the other hand, Panagiota (11 years old) could not find any way to keep the same dimensions. "I don't know how to do it. Probably by eye".

Taking into account the degree of imagination and abstraction required to draw nets in proportional dimensions, these cases were allocated to complete geometrical models.

External matchbox

The models met in each category are presented in Figure 2. The percentage of children at each age level who are functioning with each model is indicated in Table 2.

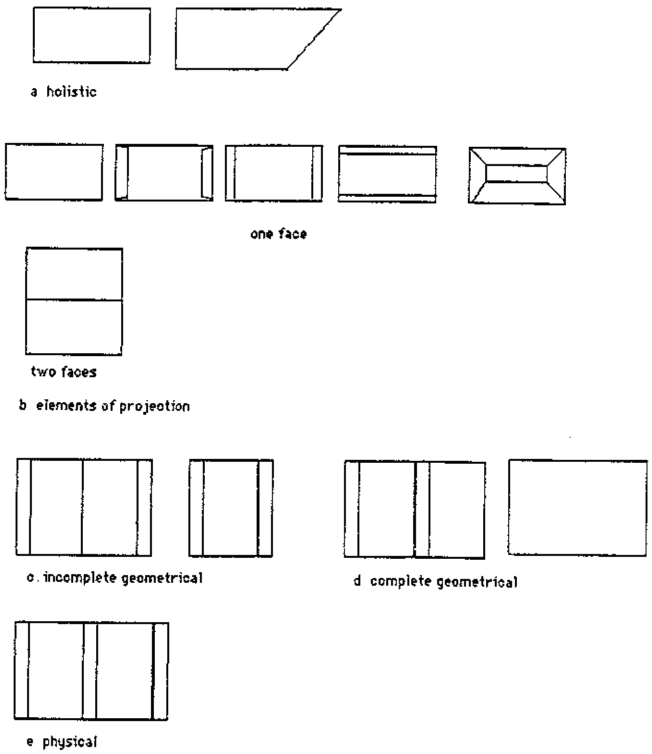

Figure 2:
Children's figures of the external matchbox

	9 year olds N=25 (%)	11 year olds N=42 (%)
holistic	20	5
elements of projection	44	24
one face	(40)	(21)
two faces	(4)	(3)
incomplete geometrical	16	19
complete geometrical	12	43
physical	8	9

Table 2:
Children's models of the external matchbox
(Percent in each age group)

Few nine year old children drew complete models. Most gave models with elements of projection or holistic models. On the contrary, older children mainly drew complete geometrical models, while a significant number of them drew models with elements of projection.

It is worth mentioning the second drawing in the holistic category (Figure 2). In the child's drawing, the slant side of the quadrilateral possibly represents the height of the box. She says, "As we open it, it becomes more slanted"

Children with the "elements of projection" model do not show the height of the box. One of the large faces is drawn in the first five examples, while both large faces are stamped in the last one. The pieces attached to the larger sides represent the open ends of the box, while those attached to the smaller sides, internally to the base, seem to be projections of the narrow faces.

Nets with all the faces of the box placed in the wrong position are also included in incomplete geometrical models.

In the fourth category, the correct net, but with no distinction between the faces, is also met.

Children used similar approaches to drawing the complete nets to those described in the previous task. It was interesting that some children pressed the box flat and copied it twice. Perikles used this approach, but he was not able to complete it. The following extract points it out.

Int: Perikles, tell me about the external box. How did you make it?
Per: I held the box and I put it there. (He flattened the box and drew its net placing it on the paper).
Int: You flattened the box, didn't you?
Per: Yes, like this.
Int: Are you sure that your figure is correct?
Per: I don't know, miss.
Int: Will it be like this when you open the box?
Per: I don't know.
Int. What do you think it will be?
Per: I think that it will be a rectangle
Int: What you have drawn is not a rectangle?
Per: It is, but it should be a little bigger than this.
Int: How could you find out?
Per: How could I find out? If I open it.
Int: How could you do it without opening it?
Per: Do I have to measure it?... No I cannot do it

Perikles could not reach the level of abstraction required for completing his net by the strategy which he had adopted.

Toilet roll

Children's drawings are shown in Figure 3. The percentage of children at each age level allocated to each model is given in Table 3

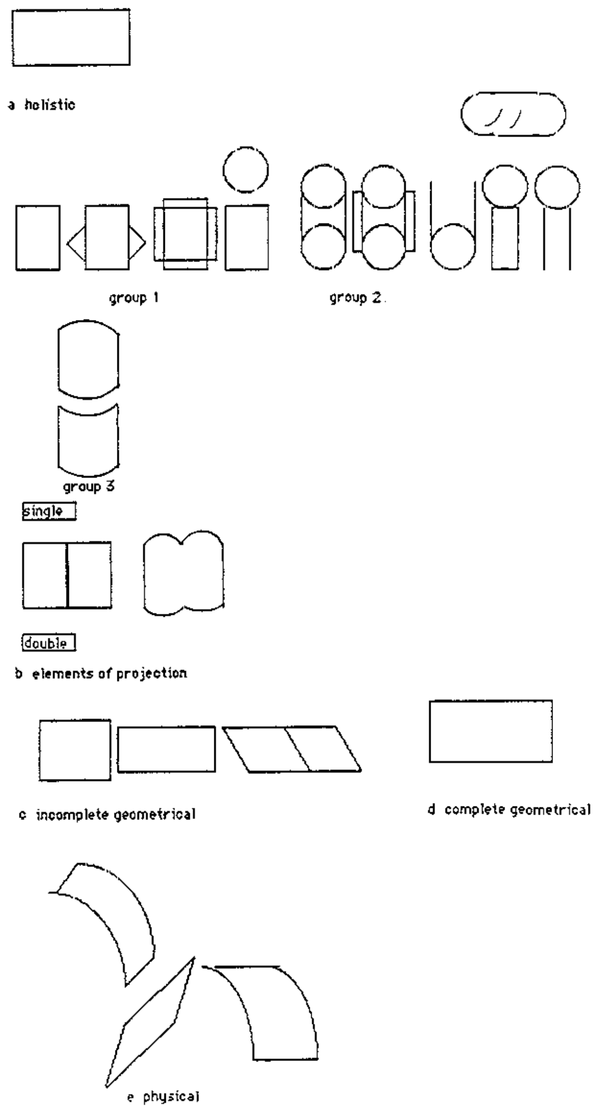

Figure 3:
Children's figures of the toilet roll

	9 year olds N=25 (%)	11 year olds N=42 (%)
holistic	12	–
elements of projection	40	31
single	(24)	(29)
double	(16)	(2)
incomplete geometrical	20	38
complete geometrical	12	29
physical	16	2

Table 3:
Children's models of the toilet roll
(Percentage in each age group)

Most of the youngest children and a lot of the older ones drew models with elements of projection. The complete and incomplete geometrical models are mainly met in older children's drawings. Whereas a considerable number of the younger children demonstrated a physical or holistic model, this was not the case with the older ones.

The figure (Figure 3a) of the holistic model is not clearly distinguished form those allocated to the incomplete geometrical. However the interviews provided evidence that children in the first category did not take into account any element of the solid or its net.

In the subcategory of single projection three groups of drawings can be identified. The first group consists of drawings with an orthogonal projection of the roll. In the second, children have attempted to represent the solid in a perspective view, while in the third group they have begun to show the existence of the back part of the roll. The children who did the last two drawings of the second group did not draw the bottom circle. As Yasiliki admitted, this was a way of expressing the openness of the solid: "I did not draw the bottom so it looks open".

In the subcategory of double projection children tried to show both front and back parts of the roll projected orthogonally.

A square of side equal to the height of the roll was often met in the incomplete models. Another net in this category was a rectangle having one dimension the height of the roll. A parallelogram resulting from a slant cut of the roll was an interesting case.

In the physical models children appeared to use their previous personal experience of unfolding a real toilet roll in order to draw the net Sofia states that she had unfolded the rolls in the past.

Children used a variety of methods to produce a complete net. The most common was the rotation of the roll as described by Eleni (11 years old). "I placed the roll like this and I drew it (a segment of length equal to height). Afterwards I put a mark on the roll and I rotated it until I found the mark again". Georgia (11 years old) attempted to find the length of the circle by using the ruler. She said that she needed a measuring tape for better accuracy. Ares (11 years old) pressed the roll flat, using an approach similar to that described for the external matchbox. An extract from his interview follows:

Ares: I folded it in the middle. I measured one side and multiplied by two and found the length that.
Int: The line around
Other child: The perimeter.
Int: Please measure it to see how long it is.
Ares: It is 7 5, about 7. I said seven by two is 14. I also measure this (the height) and it is 10 5. I draw the line and I put 10.5.

Ares is very confident of his method. He easily measures the dimensions and also has the ability to approximate. The development of mathematical language is also apparent through the activity.

Other children drew the length of the rectangle "by eye", as they admitted. The models produced by this method have been considered as complete.

Sardine tin

Most nine year old children drew figures with elements of projection while this was not the case for the eleven year olds. The majority of the older children gave incomplete geometrical models similar to a net of a box. In both age groups few children gave complete models. The holistic model was only produced by few 9 year olds.

In the "elements of projection" category children have copied one or both bases of the tin. In some cases they have drawn inside the base either the lateral surface or the tin's edge. The height is not shown.

As appears in the drawings, the curved parts of the lateral surface were represented in various ways. These will be discussed later in the article.

Almost all the 11 year old children who drew complete models kept the same dimensions as the sardine tin. We

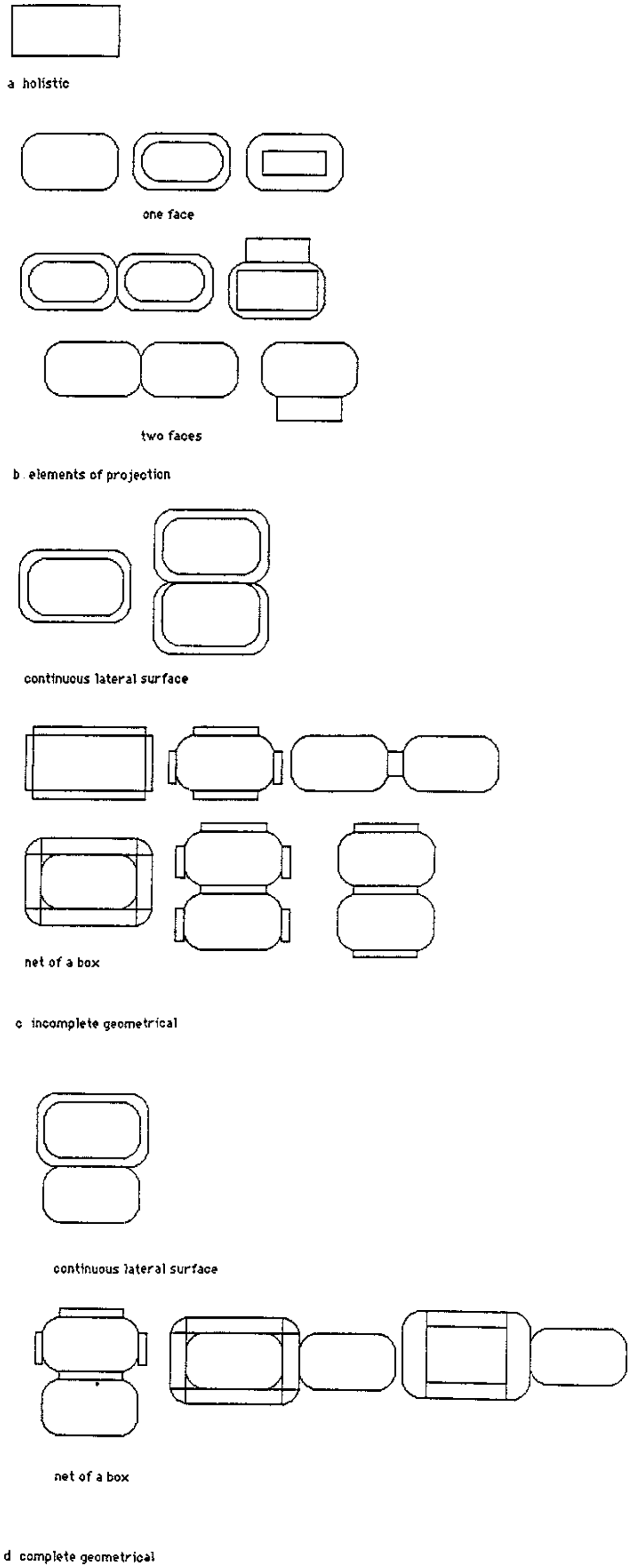

Figure 4:
Children's figures of the sardine tin

	9 year olds N=25 (%)	11 year olds N=42 (%)
holistic	8	–
elements of projection	56	17
one face	(32)	(14)
both faces	(24)	(3)
incomplete geometrical	12	66
continuous lateral surface	–	(7)
net of a box	(12)	(59)
complete geometrical	24	17
continuous lateral surface	(8)	(10)
net of a box	(16)	(7)

Table 4:
Children's models of the sardine tin
(Percentage in each age group)

also met complete nets with proportional dimensions in younger children's drawing

Children usually used the tin itself to imprint the bases and parts of the lateral surface. However they used their imagination to design the curved parts of the tin. Some children used a combination of copying the bases and measuring the height. Georgia (11 years old) drew a net with a continuous lateral surface. "I placed the tin and I drew a line around. Afterwards I measured this small side with the ruler. I found it to be 2cm and I drew lines 2cm apart, so I made it". She means that she drew the outside line 2cms from the inside.

Another distinctive characteristic of the tin is that its lateral surface has plane and curved parts. Some interesting representations of the curvature appeared in the children's drawings. Some children have not retained the curvature in the net, as is demonstrated in Figure 5a. On the other hand a number of children used several ways to display it. These appear in the subcategories of Figures 5d-g for the net of the box subcategory, and in Figures 5b-c for the continuous lateral surface one. An interesting case is shown in Figure 5g where the sides that represent the curvature are complementary.

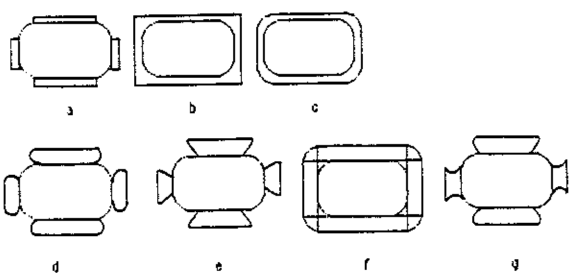

Figure 5:
Children's representations of the curvature in the sardine tin.

Results from the second phase of the experiment

Whole class discussion on the models

Children's involvement in the discussion was continuous and spontaneous. They enjoyed investigating not only their own models but their classmates' as well. This process helped them to reflect on their methods and find out their own mistakes. For example when an incomplete geometrical model of the sardine tin was displayed to the children, Perikles exclaimed "This is my drawing, but I have made a mistake! Although I've opened the tin, I drew the top surface over the bottom".

A brief description of the children's comments on the external matchbox models follows.

Children accepted the holistic model as correct but incomplete. Eleni stated, "Children who have made a distinction between the faces gave better answers". Ares agreed, adding "It is not complete because the faces have not been drawn". Perikles argued that it was not only the distinction between the faces which makes the model complete. "It is necessary to measure the dimensions", he claimed.

Two models with elements of projection were presented. First the copy of the large face was discussed (Figure 6a). On being questioned about their opinion of this model some children said that the height had not been considered while others supposed that the other faces had not been seen. Eleni showed another appreciation of this model: "these children have seen it without it being open". Second, the drawing with the two small faces internally added was discussed (Figure 6b). "Have they represented the gaps in the box?", one child wondered. "Have they drawn the two small faces of the box?" Perikles asked.

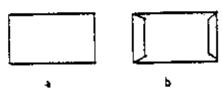

Figure 6

At the next stage the incomplete geometrical models were discussed. The model shown in Figure 7 was chosen for discussion. Children pointed out the incompleteness of this model. In their effort to interpret it some imagined the solid which could be constructed from this net. Archondoula stated "We cannot get this solid from this net. One of the smallest sides should be in the middle". Ares and Akes thought that the solid made from this net would be an open triangular prism. Ares said this solid was "sloping" while Akes said "It will be an isosceles triangle"

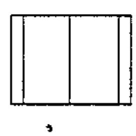

Figure 7

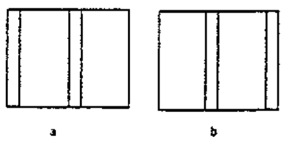

Figure 8

Children accepted the complete geometrical model with a distinction between the faces as correct. They also recognised that figures 8a and 8b give the same solid.

Different interpretations were expressed for the physical model. This appears explicitly in the following extract

Girl 1	The two small faces will be joined together
Girl 2	We don't have five faces in the box.
Girl 3	They cannot be joined together.
Boy	It seems that the matchbox has one more face.. Or they may have cut one face (the small one) in the middle.

On being asked how they had constructed the toilet roll net children recalled some of their own methods.

Georgia	I would use a measuring tape to measure the line around (complete geometrical).
Eleni	I would make a strip. I imagined that I opened the roll along the line (physical model)
Interviewer	Is it right?
Yiannis	Yes, because if she had folded it she would have made the initial solid

The interviewer displays two nets: a rectangle of arbitrary dimensions and a square. Children were asked to evaluate the nets.

Makes:	Both are correct because if I folded them I would make the roll
Ares:	The cylinder constructed on the square would be taller or ... narrower

Finally, children recognised in some examples of the elements of projections models that the solids had been represented "closed and not developed".

In the case of the sardine tin children admitted that they had been puzzled by the curvature of the lateral surface. The geometrical model of the open box was considered correct. Moreover, it appeared from the discussion that the children accepted the curved edges of the lateral faces as a way of representing the curvature

The "Sprite tin" case

From the analysis of children's drawings of the nets of the Sprite tin the following models emerged: models with elements of projection, incomplete and complete geometrical models. These models were similar to those described in the previous tasks. The holistic model was not met in this activity. Children's drawings for each category are demonstrated in Figure 9. Table 5 summarized the percentages of children that constructed these drawings.

In the first model, "elements of projection", three subcategories appeared. The first two were also encountered in the nets of the toilet roll. The third can be considered as more advanced since it involves an initial appreciation of the tin's development. This is expressed in the first drawing of Figure 9a by the rotation of the cylindrical surface and in the second by the drawing of the two bases of the tin.

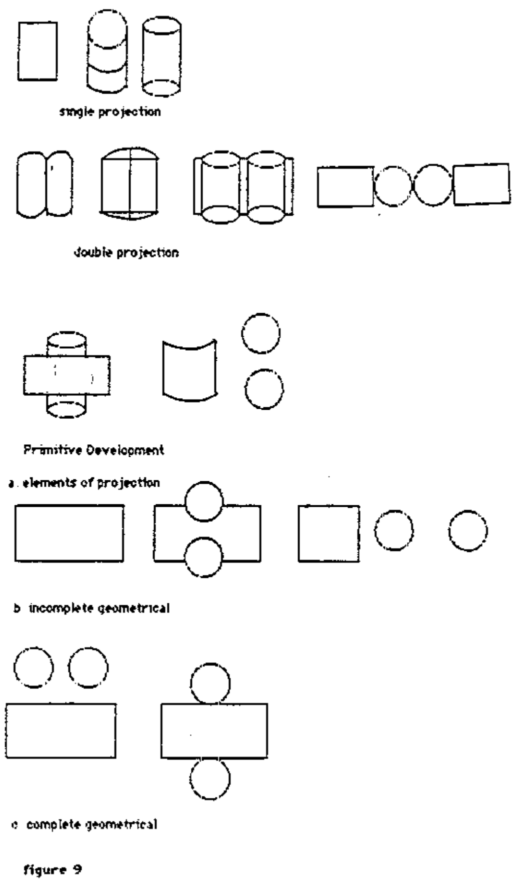

Figure 9:
Children's figures of the Sprite tin

Three types of net are included in the incomplete geometrical model (Figure 9b): the development of the cylindrical surface only, the appearance of the circles in the previous net cutting the lateral surface, and the net of a cylinder with arbitrary dimensions

In the third category children not only recognised all the elements of the solid but they were also able to represent them in the development

Some children did not see the net of the Sprite tin as the conventional one of a cylinder. They tended to represent some particular characteristics of the solid. Some examples are given in Figure 10

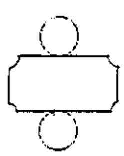

Figure 10

A remarkable net was Anna's (9 years old). She imagined the lateral surface of the solid cut in thin strips (Figure 11). Her intuitive approach bears a close resemblance to mathematical methods of approximation.

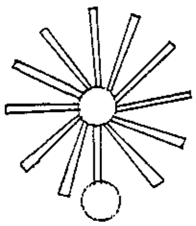

Figure 11

Half of the children, as indicated in Table 5, demonstrated models with elements of projection. The remainder performed complete and incomplete geometrical models.

	9 year olds N=25 (%)
elements of projection	20
single	44
double	(40)
primitive development	(4)
incomplete geometric	16
complete geometrical	12

Table 5
Children's models of Sprite tin
(in percentages)

Comparing individual's models
From an analysis of individual's models for each task, we saw evidence in many cases that the models were not only identifiable but also stable. This consistency was more apparent in older children. In younger children the stability mostly existed in the more advanced and the more primitive models (complete geometrical, holistic).

Children's models for the sardine tin and the Sprite tin in one class (class A) and for the toilet roll and Sprite tin in the other class (class B) were examined more systematically. These solids were chosen because of their commonalities. We attempted to focus on the effect of the whole class discussion on individual's models. The shifts in their models are indicated in Figure 12.

A similar method of representation has been used by R. Russell *et al* [1989].

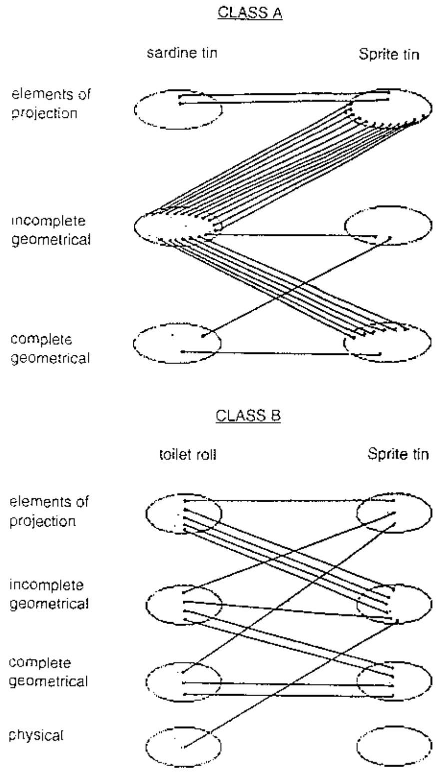

Figure 12:
Shifts in individuals' models after whole class discussion

In class A a significant change occurred in the children who had constructed an incomplete geometrical model of the sardine tin. Most of their models of the Sprite tin were now allocated to the "elements of projection" category. A third of them improved their models, giving the Sprite tin's complete net. This suggests that children found it more difficult to imagine the Sprite tin's development. This may be due to the dominance of the lateral surface of the Sprite tin. This is also expressed by the different type of projection used in each case. Children tended to project the lateral surface of the Sprite tin and the bases of the sardine tin.

In class B an improvement of children's models was noticed. The discussion may have helped the children to conceive the development. Almost all children shifted from "elements of projection" models to incomplete geometrical ones.

So far we have found that the discussion did not cause a dramatic change in children's performance. In spite of this fact, such reflective approaches, through the conflict situations that occurred, have helped some children to improve their models. More significant changes were not expected from such a short term intervention. Such intervention needs to be used more systematically in the classroom for its influence to be more effective.

Discussion
Although we did not intend to classify children's models in terms of levels of development, we identified a progression from more global and holistic views of solids' nets to more

quantitative and analytic consideration of them. As Bell *et al* [1983] mention, a number of studies have reported a similar transition. In Michelmore's study [1980] where children's representations of solids were explored, five categories emerged. Their evolution could be paralleled by the one in the present study.

Most of the studies in this area have considered geometrical models This study is about the development of physical objects. This results in the appearance of the physical model. The physical model cannot be regarded as a more advanced model in all tasks For example, for the matchbox parts, this category can be accepted as more developed. This is not the case for the toilet roll where some children's drawings show a lack of appreciation of the solids' properties.

As was expected, older children drew nets allocated to more advanced models. Although younger children had more difficulties, a number of them gave correct nets. This is contrary to what Piaget states, in Holloway [1967], that correct solutions are arrived at after the age of 9 years old and for some children only after 11 years.

It is worth mentioning that younger children drew nets with dimensions smaller than those of the solids. This was not the case for the older children who used more systematic ways. These children either rotated the solids and copied their parts or measured their dimensions This suggests that 11 year old children take the metric properties of the solids more into account.

This study explored the ways that children imagined and drew the nets of different physical objects, which probed children's understanding of properties of space. A number of reasons and factors influence the individual's performance across the different tasks. Some of them are the methods of construction and characteristics of the physical objects. Although children's interpretation of a task was different depending on their previous experience and their ability to visualise space, we detected a consistency in the models met for all the physical objects. The stability or generality of these models may lead to different educational strategies being advocated [West and Sutton, 1982].

This generality was also reinforced in the second phase of the study where were met the same models applied to the case of the Sprite tin. However a small change in the children's models was indicated after the whole class discussion in the second phase. Our own belief is that this kind of meta-learning needs to be emphasized at all levels of education so that children reach the level of conscious control of their learning. This is also regarded by Driver [1989] as of great importance.

Ben-Chaim *et al* [1989] found from their research that suitable intervention had great success. They suggest that spatial visualization training in particular concrete experiences should be a part of the middle school curriculum. This visualization was also needed for our tasks.

As Marrioti [1989] states, "Constructing the correct net of the solid implies coordination of a comprehensive mental representation of the object with the analysis of the single components (faces, vertices and edges)". The usefulness of such learning approaches with three-dimensional figures has also been proposed by Biggs [1984]. In the Greek primary mathematics curriculum very limited emphasis has been given to activities concerning the development of solids.

Further research in this area should provide information about different age groups' performance with other physical solids. Another interesting aspect could be the further investigation of the effect of the physical characteristics of the solids on childrens' models. The development and evaluation of specific interventions deserves wider research attention.

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Language lends itself to myth in another way: it is very rare that it imposes at the outset a full meaning which it is impossible to distort This comes from the abstractness of its concepts: the concept of *tree* is vague it lends itself to multiple contingencies [. .] One could say that a language offers to myth an open-working meaning. Myth can easily insinuate itself into it, and swell there: it is a robbery by colonization. [.] When the meaning is too full for myth to be able to invade it, myth goes around it and carries it away bodily This is what happens to mathematical language. In itself it cannot be distorted, it has taken all possible precautions against *interpretation*: no parasitical signification can worm itself into it. And that is why, precisely, myth takes it away en bloc; it takes a certain mathematical formula ($E = mc^2$) and makes of this unalterable meaning the pure signifier of mathematicity We can see that what is here robbed by myth is something which resists something pure. Myth can reach everything, corrupt everything and even the very act of refusing oneself to it So that the more the language-object resists at first the greater its final prostitution; whoever here resists completely yields completely: Einstein on one side *Paris-Match* on the other. One can give a temporal image of this conflict: mathematical language is a finished language, which derives its very perfection from this acceptance of death. Myth, on the contrary is a language which does not want to die: it wrests from the meanings which give it its sustenance an insidious, degraded survival, it provokes in them an artificial reprieve in which it settles comfortably, it turns them into speaking corpses.

Roland Barthes