# CIRCLES, MATERIALITY AND MOVEMENT

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> The cut-outs still retain a sense of movement, both through the trace of Matisse's scissors and the ways in which he continually shifted and adjusted the shapes themselves. (Tate Modern, Room 1 wall note)

Please draw a circle. I would prefer you use some object, such as the base of a cup, a paper-clip or a compass, as opposed simply to drawing one freehand. Try to draw a 'good' circle. Once you have finished (and, in passing, how did you decide when you were done?), look at what you have drawn. What do you see? Is your drawing a condensation of traces of a lived experience? Or did you impose a form (from some Platonic realm, perhaps) upon your diagram? Or something else? I would probably agree that your circle looks like a circle, but, of course, it is not a perfect circle (neither is mine). At best, it provides a close approximation.

In the spirit of Matisse's cut-outs (images he made from coloured paper near the end of his life when he could no longer hold a paint brush), imagine you want to use your drawn circle to cut out a circular disc. In keeping with good elementary school practice, hold the round-nosed scissors stationary in your right hand (or in your left, if you are left-handed), elbow tucked in by your side, and turn the paper with the other hand to meet the blades' advancing maw. How is your paper-holding hand moving?

What about the diameter? Diameters can be brought into existence by the human act of folding the circle onto itself, and doing so twice brings the centre into being (I will come to the question of definitions shortly). These elements are produced by human action, not defined into existence. And there is a different confusion with regard to the 'circumference' (etymologically, Latin, from *circum+ferre* "to lead around") of a circle, when this term does not refer to a measure, but rather to the thing itself: in this sense, the circumference of the circle *is* the circle.

The circle is among the most recognizable mathematical objects and yet simply referring to it gives rise to a question of ontology. A few moments ago, your constructed circle might have been intricately tied to your movements, but now if you look at it, what remains is only a static trace, unmoving and fixed, relatively unchanging over time. Is it the movement or the trace that *is* the circle? Or perhaps both? This distinction between the act of constructing and subsequent fixed traces seems to create a profound tension. For students in mathematics class, especially for those just becoming familiar with circles (and other mathematical objects), the contrast between creating and reifying produces a profound divide.

As a teacher, educator and researcher, I am concerned that the method of drawing out structure and form while disregarding particular processes of becoming loses sight of how mathematics emerges and what mathematics is. Instead of valuing the human activity and process which brought the circle to life, a teacher may simply see the students recreate a 'form', distinct, separate and devoid either of personal or of material meaning.

Perhaps the circle, as with any mathematical object, ought not be seen as a reproduction of an ideal form, but rather as a meshwork of materials and forces. The term 'meshwork' is used by Tim Ingold (2007, 2011), a contemporary anthropologist, who offers it as the binding together of multiple "entangled lines of life, growth and movement" (2011, p. 63). His conception of the line as an organic flow of continuity conveys a process-oriented approach to activity, one which he contrasts with approaches that view the world as a set of distinct, interacting things.

Life activity is seen as movement along lines (not necessarily straight lines), as opposed to interactions among discrete entities. In this meshwork, we continuously become, as we act, entangled in multiple lines. But the lines that we move with are not predetermined like those on a map. They are constantly becoming and are transformed by us—as we are by them. Knowledge is created through this process. Ingold wishes to undo the tendency to view life through the concept of 'network', where nodes are seen as primary and connections secondary, and instead make 'lines' (in their full metaphoric sense) central: "not a network of connected points, but a meshwork of interwoven lines" (p. 63).

In this article, I adopt Ingold's approach in order to think about circle-making, and, in the bigger picture, about practicing mathematics. Ingold does not write specifically about mathematics, but his approach to life and engaging with physical materials aligns with my sense of mathematics teaching and learning. Here, I explore mathematical activity using Ingold's meshwork metaphor, where lines (processes) bring nodes (entities) into existence and not working from a starting point of pre-existing rigid boundaries and distinct 'states' of things.

There is also an echo of a mostly lost ancient Greek approach to geometry, one that views lines (and curves) as primary and singular, while points are seen as secondary. A point needs to be brought into existence by means of lines or curves intersecting: they are not already there, as in the current set-theoretic specification of a curve as a collection of points satisfying a relation. Hence, we have the historic use of curves (*e.g.,* certain conic sections) as tools to bring into being points satisfying certain properties or relationships (a reincarnation of this perception can be seen in contemporary dynamic geometry software). One of Ingold's provocations resides in his claim that "things do not exist, they occur" (2011, p. 160). This article explores what insights emerge when we shift from thinking about extant objects to examining processes occurring, and asks both what mathematics and what mathematics education might learn from Ingold's meshwork metaphor.

## Circle as representation to circle as process

In mathematics, the circle stands as a dominant form, so even in the common practice of constructing a circle, the form seemingly makes its presence known. To construct a circle, then, is to appeal to the existence of an eternal entity before construction begins. The mathematician/logician Alfred North Whitehead, upon whose work Ingold bases much of his, describes this as an example of how an "indetermination is referent to determinate data" (1929/1978, p. 45); that is, the drawing of a circle is dependent on the circle as a pre-existing image. Whitehead finds this notion problematic, because there is an implication that the indeterminate—which is to say, the constructing of a circle—is only meaningful in its reference to the abstraction of a circle. The drawing becomes meaningless in and of itself, because it will always 'fall short' of the referent image.

For Whitehead, processes constitute the ontological realities of our world. In such a process-oriented ontology, the activity or movement takes precedence over the image identified "in the mind". He outlines two perspectives—the subjectivist and the sensationalist—to clarify the difference between activity and image. From a subjectivist point of view, "the datum in the act of experience can be adequately analyzed purely in terms of universals", while from a sensationalist viewpoint, "the primary activity in the act of experience is the bare subjective entertainment of the datum, devoid of any subjective form of reception" (p. 157). In other words, the subjectivist seeks an image in the construction, while the sensationalist experiences the construction itself. With this distinction in mind, one may question, as an educator, what is mathematical in requesting a geometric construction from our students.

To separate the circle from the circler is a choice of distinction; it is to assert that the subject and the object manifest as separate entities rather than intertwined relations. But to outline *a posteriori* distinctions between image and image-maker is a boundary-making decision that assumes "a transitive relation between image and object" (Ingold, 2011, p. 6). Ingold argues that, if this were the case, then "[in] theory, the time it takes [to draw a circle] could be compressed into an instant, and history itself would merely be a succession of such instants" (p. 13). To distinguish what is "already there, or has already come to pass" stands opposed to the idea of "joining together [as] a practice of […] the intimate coupling of the movement of the observer's attention with currents of activity" (p. 223). Thus, a geometrical drawing is not an image that can be separated from lived experience.

## How to specify a circle: by property or genesis

If we look at an instance of a definition of a circle from a contemporary school textbook, we might find the following: "Every point on the circumference of a circle is the same distance from the centre of the circle" (Kirkpatrick et al., 2010, p. 91). This definition has erased any reference to the circle's genesis, the process of its construction: in Balacheff's (1988) words, it has been detemporalised, depersonalised and decontextualised. This particular definition is brief, efficient and static. The circle is described abstractly by referring only to its defining quality. Using the distinction made by mathematical historian George Molland (1976), it is a specification *by property*, which he contrasts with a specification *by genesis*, involving declaring what needs to be done to create the object, to bring it into being.

For an example of a specification by genesis, consider Hero's definition: "a circle is the figure described when a straight line [segment], always remaining in one plane, moves about one extremity as a fixed point until it returns to its first position" (given in Bartolini Bussi & Boni, 2003, p. 17). In this definition, we find a circle is something that is constructed by movement. Although there is still no clear creator, it is based on a dynamic, mechanical process.

These two definitions are in stark contrast to one another. While the first is disembodied and apparently describes a distinct, unchanging object, Hero's circle involves movement, an animating life force. The first can be thought of as a form, while the latter can be thought of as, literally, a line of becoming. Ingold identifies a common experience of drawing a circle, when one loses "the trace of the twirling movement that went into its formation" (2011, p. 13). Even more significantly, perhaps, the first definition is of a circle as a set of points (each one independently satisfying a single given condition), rather than as a coherent gestalt or singular object in its own right: it arrives pre-atomised, stuffed full of points. The latter is 'described' (*i.e.,* drawn), a trace generated in time, emergent from and dependent upon another 'curve', a straight line segment.

The circle is a snapshot. It is a particular stoppage on the line of becoming. Ingold would not agree that the 'circle' as an object is part of the mesh work, since mesh works are made of continuous lines rather than points. This is a boundary-making activity, a cut. It does not represent reality but is one way of creating a reality. To create a noun from a verb is a boundary-making activity as well. According to Ingold's example of the wind—whose nominalization [1] now requires speakers to link it to the verb 'to blow'—to construct a noun from a process is a misrepresentation. The wind is not a noun; it is a verb. Similarly, the sky is not something you see, it is something you see *with*; the act of seeing involves the sky.

For Ingold, a trace is any mark remaining on a solid surface that has arisen from continuous movement. The continuity unfolds in the movement. In the context of mathematics, the mark left by the tip of a moving writing instrument is particularly relevant. The trace begins at a certain point, but does not outline a pre-existing form; rather, it is a mark of entanglement and movement. In the case of a drawn line, the movement of the human hand is an integral part of the genesis of the trace, but, ultimately, the trace results from the culmination of an assemblage [2], including unpredictable influences.

Although the examples above highlight differences between state and movement, the theoretical nature of these definitions mistakenly depicts a smooth process of existence. That is, both circles appear without consideration of an actual real-life process that includes physical materials and life forces. Mathematician Brian Rotman (2008) appeals to the process of moving from speech to writing in order to identify a loss of phenomenon in moving from one medium to another. He argues (on p. 25) that the "beating, here-and-now" speech of the speaker is replaced by the "abstract, invisible author", as well as highlighting the "unique event" unfolding over time, as being replaced by fixed, repeatable, actual alphabetic inscriptions.

Moving from the materials and forces of a circle's construction to the mathematical form is an analogous process, in that circle-making loses its in-the-moment living experience. Instead, the act of circling can be seen as a manifestation of the arcing hand, the arching of the arm and wrist, the moving of the body so as to position the head to see behind the hand, the anchoring of the paper, the slippage of the compass because it is the now of the actual occurrence, where the rhythm and movement of circling is instantiated.

## Transport and wayfaring

Recent scholarly criticism of schemata (often used in theories of learning) or networks (*e.g.,* as used in Bruno Latour's Actor-Network Theory) is that they focus on nodes as opposed to connections. Although connections between these nodes are often represented with arrows going back and forth, they actually presuppose end-points. To counter this perspective, Ingold (2007) uses the term *wayfaring* as an alternative to speaking of a network and as a means of moving toward his mesh work metaphor.

To illuminate his idea of wayfaring, Ingold, borrowing from the Canadian author Rudy Wiebe, contrasts the conceptions of movement across water and across land based on practices implemented by the British Royal Navy (and its search for the North-West passage) and the Canadian Inuit. While the Royal Navy quantifies waters by means of latitude and longitude, the Inuit observe the movement of humans and animals, valuing their paths of motion, looking at the land as a "mesh of lines rather than a continuous surface" (p. 149) [3]. Ingold terms the Inuit act of following lines of travel 'wayfaring', which he contrasts with 'transport', where movement across a surface is conceived as going from point to point. A wayfarer threads her way through the world; she moves and creates a path as she goes. She does not follow a path already laid out (see also Varela, 1987). The unfolding of the mover is along paths not between places; paths are instantiated in the world as a line of travel. In contrast, the traveller is not changed through movement because the traveller is encased in a vessel (which might be their own body) and is carried from one place to another.

Ingold distinguishes a wayfarer as a line while transport is a connection between points. We might say that lines are about movement (or verbs), while points are about states of things (or nouns). 'To circle' is a transitive verb in English. And mathematics is notorious for turning actions into objects, turning verbs into nouns (see, for instance, Barton, Fairhall & Trinick, 1998; Lunney Borden, 2011). Ingold describes movement as *knowing*. In movement, a person elicits a knowing which is not to be taken as a "property of knowing", but as a "practice of knowing" (p. 159) [4].

## Student as wayfarer

In a traditional educational framing, the control of the circle's drawing is (literally) in the hands of the student. In this framing, the child is granted the privileged position of being the main subject while the tools are made invisible; yet the paper, pencil, compass and desk all bring the circle into being as much as, if not more than, the child. This framing is a strong statement of what our educational practices value: it focuses on the finished state and attributes the product to the student, hence rendering it—and him or her—available for evaluation.

Where does this framing leave us when the child alone does not draw a circle (or what a teacher might call a circle)? When there is a slippage of the tools and the circle morphs into something other than a circle, who or what is the cause? Or when the edge of the paper declares its presence and trumps the completion of the whole circle, where do our judgements lie? Where is attribution placed, where lies any responsibility or blame? The student-compass-paper-pencil-desk assemblage moved. The kinks with the circle construct, instead of being seen as a misrepresentation or a mistake, might instead be seen as the trailing off of a life force of the sliding hand or a gestural trace within a mathematical performance of circling. Perhaps it is even such a life force that produced a spiral.

Valerie Walkerdine (1988) criticizes the way in which the teaching of mathematics is often viewed as imparting to the individual a sense of control and mastery over the physical world. She observes that in "examining practices it is unnecessary to invest children with capacities or abilities which unfold" (p. 13), arguing instead for a focus on the "practices themselves" (p. 14). Reified, human-centric views may thus grant the individual power, but when the individual appears powerless in relation to the mathematics, it is seen as a deficiency on the part of the individual. Positioning mathematics as a separate entity from the human grants mathematics power over individuals (Davis & Sumara, 1997). This introduces a power struggle between the student and mathematics, which, I suggest, is neither a productive nor a supportive approach to mathematical learning.

Every organism has its own movement, and every 'object' that the organism encounters causes a disturbance or perturbation in its trajectory. Lines of becoming weave in and out of other such lines. From this weave emerges the metaphor of meshwork. Ingold draws attention away from the centrality of the person by arguing against interior attributes or characteristics. He describes a human not as entangled with things, but rather the combination as an entanglement itself, offering an alternative to conventional interpretation when he describes his field of anthropology as "the study of human becomings as they unfold within the weave of the world" (2011, p. 9). The person is her relations, seen as a manifestation of a process of becoming, of continuous creation or, simply, as being alive.

## Drawing back to the beginning: knowledge as movement

For me, Ingold's focus on movement and the forces of life is important in mathematics education because it draws attention to the *now*. Mistakes can be seen not as mis-directions or errors in achieving a preconceived form, but instead as arising from living forces. To appeal to a form, separate from activity, supports a representationalism that entails an imposing of form over a life force. As Whitehead states, this "does violence to that immediate experience which we express in our actions" (1929/1978, p. 49). Epistemology is seen in the Western world in terms of ideas and images, but meaning comes from our embodied experience. To disregard the actuality of the drawings and appeal to the 'form' makes, I argue, a travesty of the process of making, insulting both the thing made and the one-who-makes. To detach, impersonalize and shed the umbilical connection with its genesis generates a profound loss, because when an image has lost its history, vanished too are the forces that brought the circle to life.

The student is no longer a political figure attributed with agency and intention, but is instead a shared co-construction with materials and other living forces. It is the sensation-based perspective of Whitehead that draws back to the practice itself, offering students the opportunity to create with a 'compass' a storied knowledge resulting in questions such as "Which aspects of equidistance cannot be resolved?" or "What are the rhythms of circularity?"

This article is not merely about circles; the example of the circle has been used to highlight the differences between 'making' and 'image', 'wayfaring' and 'transport', 'lines' and 'connectors'. To extend beyond geometry and drawing I note how number and algebra can also be drawn into the conclusions of this paper. Similar to Rotman's observation of the loss of phenomena in moving from spoken to written form, the shift from the temporal, relational and often awkwardly phrased saying of numbers by students ('tenty', see Coles & Sinclair, 2017) to number as quantity (three pencils, ten chairs) highlights a challenge of valuing the embodied melodies and rhythms of experience, rather than the abrupt symbolic representation on paper. Or in a series of algebraic steps that a student has produced, a teacher may see a step-by-step trajectory aiming towards a correct and final expression. But what of the in-between? In between the written steps, that which is never seen (on paper at least), seems to be the place where the student is thinking, sketching and planning. Algebra takes place between the lines. If simplification of an algebraic expression is seen as an progression of steps to a final product, the wayfaring of actual practice is hidden (see also Proulx & Pimm, 2008).

It seems that mathematics may be something to be lived rather than understood. The sustainability of mathematical practice emerges from the merging and assembling of materials and through the coming together of multiple lines in a meshwork. Mathematics is emergent and alive through its doing. A student construction or expression invites the viewer to join with its creator as a fellow wayfarer to look "with" the process to see mathematics not as a fixed form, one separate and distinct from human activity, but rather as an occurrence of sound, touch, rhythm and materials. To enact mathematics, we need to draw back to our lived experience. Look back at your circle: what do you see now?

## Notes

[1] For more on nominalization in science and mathematics, see Halliday (2003).

[2] *Assemblage* is a notion introduced by Gilles Deleuze and Félix Guattari, and much used both by Bruno Latour and by Karen Barad. The article on Deleuze in the Stanford Library of Philosophy glosses it as follows: "'assemblages', that is to say, an emergent unity joining together heterogeneous bodies in a 'consistency'" (http://plato.stanford.edu/entries/deleuze/).

[3] This way of thinking aligns with the Marshall Islander stick charts used to support navigation, which were studied by Ascher (2002) as an example of ethnomathematics.

[4] This quotation aligns closely with the enactivist maxim, "All doing is knowing and all knowing is doing" (Maturana & Varela, 1992, p. 27), but I suggest Ingold is interested less in a cognizing agent and her internal, structural, biological changes and more concerned with the entanglement of multiple lines. Maheux and Roth (2011), for example, outline a criticism of enactivism, due to the basis of its framework resting on the cognizing agent's co-ordinating, evolving self developing, moving towards a more 'rich' oneness with the environment.


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