Didactic Transposition in Mathematics Textbooks

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Didactic transposition: an epistemological model

Almost no one today believes that knowing is passively receiving everything ready-made from one's environment. The rejection of this belief is a fundamental assumption of the constructivism that is a major influence on the thinking of educators and cognitive scientists. Constructivists are not in accord, however, on the old quarrel between the ontological view that accepts the objectivity of knowledge and the phenomenological view that denies it. That is, radical constructivists take the drastic step of replacing objectivity with the intersubjectivity of knowledge, and thus set themselves free from the traditional epistemological paradox posed by the skeptics, whereas trivial constructivists consider only the process of cognitive construction and do not take the paradox seriously [von Glasersfeld, 1985].

In fact, the paradox itself is already a trap to both ontology and phenomenology. There is no way out of the dilemma without some cost. The solution given by radical constructivism is one way out at the cost of confining knowledge to a specific kind. That is, the negation of the objectivity of knowledge by the radical constructivists can be persuasive only when one says about knowledge that, apart from faith and dogma, it fits observations [von Glasersfeld, 1987, p. 5]. That negation is a consequence of considering the binary relation between knowledge and the knower.

The ordinary usage of the word *teach,* however, assumes the ternary relation of the teacher, the student, and the knowledge taught. Furthermore, knowledge to be taught in school has many aspects other than that it fits one's observations. We may need an alternative epistemology if we admit that most of the knowledge in school mathematics is a compound of knowledge that fits observations together with our values, instructional purposes, mathematical skills, and so on. "Constructivists need to clarify and develop their ontological commitments.... We need an epistemology that takes ontology into account" [Kilpatrick, 1987, p. 19]. Can we construct an epistemology that allows us to treat knowledge at least "as if" it existed independently outside of the knower without violating much of the constructivist position? One epistemological model that gives a positive answer can be found in the didactic transposition theory of Chevallard [1985, 1988; see also Balacheff, 1990].

Didactic transposition theory is based on an assertion that bodies of knowledge are, with a few exceptions, designed not to be taught but to be used. The didactic transposition of knowledge is the transposition from knowledge regarded as a tool to be put to use to knowledge as something to be taught and learned [Chevallard, 1988, p. 5]. Thus any modification of knowledge under instructional purposes can be called a didactic transposition. The difference between knowledge used and knowledge taught is that as long as you only use knowledge in doing something, you need not justify nor even take note of that knowledge to endow your activity with social meaning. In contrast, teaching requires the social acknowledgement and legitimation of the knowledge taught [Chevallard, op, cit., p. 9].

The first step in establishing some body of knowledge as teachable knowledge consists in making it into an organized and more or less integrated whole [Chevallard, op, cit., p. 8]. As a way of achieving this goal, taught bodies of knowledge have been derived from corresponding scholarly bodies of knowledge. A scholarly body of knowledge is nothing other than knowledge used both to produce new knowledge and to organize the knowledge newly produced into a coherent theoretical assemblage [Chevallard, op. cit., p. 10]. In teaching, however, the mechanism of knowledge production and assembly is hidden. It is the task of didactic transposition theory to illuminate that mechanism so that we can produce didactic transpositions that will be to our society's best advantage [Chevallard, op. cit., p. 12].

Two modes of knowledge

As an epistemological model, didactic transposition theory can be reinforced by reviewing knowledge from a philosophical perspective. If one wants to restore human attributes to knowledge, then the theory can be re-examined according to Martin Buber's philosophy of "I and Thou."

According to Buber, I may interact with people or things in two modes, the I-You and the I-It. In the I-You mode, I and You meet in genuine appreciation. I-You is the primary phrase of relationship, whereas I-It is that of experiencing and using. In the I-It mode, I use It for my purpose. Even when I am retrospecting with a strong emotion, whatever remains in my memory is the It in an I-It mode. The You never stays in a form. In Buber's [1970] words, "The It is the chrysalis, the You the butterfly" [p. 69].

Knowing has the two modes: I may know something as either You or It. And I may pass between these two modes. Let us say, for example, I come to know (discover, invent, or construct) something: It may be a geometrical fact about a triangle, an algebraic pattern in a group of numbers, or a solution path for a given answer. Now I know it, but I did not intend to find it because I was not even conscious of its existence: It might exist in another world but not in my world just before I know it. Thus, to know in this sense is very different from simply to remember. Knowing at this moment is an interaction in the I-You mode.

Knowledge in the I-You mode is always inclined to become knowledge in the I-It mode because I have to use it to do other things: I have to present it to others, I have to

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write it down, or I have to remember it. To retain it, I endow it with a form. The form may be a simple narration, an ideographic expression, or a rigorous numerical formula. Sometimes I may give the form the flavor of my personal taste. A mnemonic system may be used. Or more effectively, it can be related to and coordinated with other mathematical facts. Now I know it, because I retain it: I know it, because I use it. Knowing at this stage is an interaction in the I-It mode.

What is once a knowledge in the I-You mode can take various representations in the I-It mode. Also, knowledge that was once represented in the I-It mode may or may not be revived as knowledge in the I-You mode. Converting knowledge from the I-It mode to the I-You mode, however, is not the inverse process of going from I-You to I-It. It is rather the activation of the relationship between me and the knowledge as You. For knowledge to be meaningful, the activation of an I-You mode is essential. The I-It mode may or may not be helpful for this activation of an I-You mode. It may help me change the mode to I-You by recalling the situation in which I met the knowledge in a previous I-You mode. But it may also disturb me by giving me a mental set.

When we teach, the knowledge to be taught is presented as the It in an I-It mode. During learning, however, the knowledge at the moment the student comes to know is the You in and I-You mode. The teacher has the special task of converting the mode of knowledge from the It in a stable form, which especially in the case of mathematical knowledge has been accumulated throughout a long history and various cultures, to the You in the student. This task can be characterized as a communication of knowledge or a sharing of knowledge between the teacher and the student.

Fragility of knowledge

A process of communicating knowledge can be abstracted as follows. In his or her own personal context, an original thinker comes to know something mathematical. The thinker understands it in a particular way in a specific context. We can say that at this stage his or her mathematical knowledge is personalized and contextualized. Although both modes of knowledge are working, the I-You mode is essential in this process or personalization and contextualization because the knowing is always initiated through the I-You mode.

To be communicated, however, the knowledge needs to be organized and given a form. In most cases, the original thinker may have to hide something, such as the personal conditions that governed his or her success. The I-You mode begins to fade out. This is a process of depersonalization and decontextualization in which the I-It mode of knowledge becomes dominant.

The cycling of these epistemological processes implies that the I-It mode of knowledge comes to take various forms or representations within a community as well as within a person's mind. That is, knowledge is transformed while it is communicated. Some transformations are so effective for communicating knowledge that they are fixed and remain in a relatively permanent state. Those types of transformation can be classified according to the intention of the transformer, who may be a community as a whole or an individual person. The typical transformation of knowledge is done for a scholarly purpose. Mathematics educators are most interested in transformations having instructional purposes, that is, the didactic transposition of mathematical knowledge.

Just as the I-It mode of knowledge can simultaneously help and disturb the activation of the I-You mode of knowledge in a person, a transformation of knowledge is both necessary and deficient. It can help one to retain and convey one's thought in a relatively stable form, but it may also destroy original thoughts. In this sense, knowledge is very fragile. That is, as communication is repeated, it becomes harder for any item of knowledge to be retained with its original representation and meaning. Some knowledge may disappear with wrong and meaningless representations. Some may be expanded with rich meaning and better representations. Some may be transformed into knowledge that has a very different meaning from that of the original.

The fragility of knowledge has a paradoxical aspect in that a constant form cannot keep constant meaning. The more we adhere to a specific form, the more we lose the original meaning in the form. This paradox is well illustrated by Brousseau's [1986, p. 13] claim that a classroom lesson is irreproducible. To keep original meanings as long as possible requires not the reproduction of explicit forms of knowledge but epistemological vigilance [Brousseau, 1986, p. 32] with respect to the knowledge. Such vigilance is essential for protecting the fragile knowledge in the didactic transposition. Recording knowledge in a book is often seen as the most efficient way to preserve and transmit knowledge, but that simply increases the need for the teacher to exercise vigilance.

Didactic transpositions and mathematics textbooks

A mathematics textbook is a typical way of preserving mathematical knowledge. Thus, it is a very essential point along the route of didactic transpositions in school mathematics. It provides a source in which some aspects of didactic transpositions can be investigated. For example, many geometry textbooks in the United States introduce mathematical proof with two columns, for statements and reasons, a form seldom found in other mathematics textbooks. Such a presentation of proof is a didactic transposition from abstract knowledge about mathematical proof. It is also a form that can be discarded after one masters the technique of mathematical proof. The two-column proof as an outcome of a didactic transposition suggests a conjecture that, through a didactic transposition, knowledge in school mathematics tends to be presented within a format that may be transitory. By inspecting such conjectures, we can increase our understanding of didactic transpositions. Such understanding may help us improve our way of handling knowledge in school mathematics.

Although a mathematics textbook is a source in which one can observe some aspects of didactic transpositions, a didactic transposition in a textbook may have certain limitations. When Kilpatrick [1980] asked the question, "Is problem solving bookable?" at the 58th annual meeting of the National Council of Teachers of Mathematics in Seattle, he was questioning whether anyone could put into the linear, static form of a textbook the multidimensional, dynamic process of solving a problem. The most obvious difficulty is that if the text contains a solution to the problem, there is nothing to stop the puzzled student from looking ahead to the solution. But there are other difficulties as well. How do you "book" blind alleys, incorrect solutions, reformulations of the problem, hints, and so forth? Because mathematical problem solving can be located at the core of mathematical activity, Kilpatrick's question can be extended as "Is mathematical knowledge bookable?" The extended question requires us to clarify the special characteristics, including the limitations, of didactic transpositions in mathematics textbooks.

Some characteristics of didactic transpositions in U.S algebra textbooks were examined by Kang [1990]. For example, he observed that the textbooks were written under the assumption that mathematical knowledge is taught and learned through a procedure of explanation followed by practice. He categorized didactic transpositions in three algebra textbooks into four groups: localization of mathematical concepts, real-world models for mathematical concepts, word problem types, and bodies of extra-mathematical knowledge. He also found that the form of the knowledge involved in the didactic transposition in a textbook is unstable. That is, the knowledge in a mathematics textbook is about mathematical facts that are unsteady and being confirmed dynamically, whereas school mathematics as a declared body of knowledge has a static quality.

Declaration in didactic transposition

Chevallard [1988] explained the difference between knowledge used and knowledge taught in the light of the social aspect of knowledge. The most decisive attribute of knowledge used is its revelance. Knowledge that is not useful is likely to be discarded. Relevance, however, is not so important for knowledge taught. For a body of knowledge to be taught, whether it is useful in practice or not, it should be first acknowledged socially. That is, what is important for the knowledge taught is social acknowledgement and legitimation. In going from knowledge used to knowledge taught, relevance gives way to legitimacy. A didactic transposition is a process to bestow social acknowledgement and legitimacy on knowledge used. To receive social acknowledgement, knowledge must be declared beforehand. Being declared is part of teachability [Chevallard, p. 9].

Above all, school mathematics as a whole is a declared body of knowledge to be taught in schools. The declaration, however, must then be broken into parts. For example, in secondary schools in the United States, school mathematics consists of many courses, such as Algebra I, Algebra II, Geometry, and General Mathematics, that are used as norms for sorting and arranging the mathematical content to be taught. Although those courses are traditional in school mathematics, mathematical knowledge in general cannot be partitioned in such a way.

Declaration can be detected even in small bodies of knowledge. Consider the following example. At one time, and still today in some schools, the following expansion formulas were taught with considerable emphasis:

$$(a + b)^2 = a^2 + 2ab + b^2;$$
$$(a - b)^2 = a^2 - 2ab + b^2;$$
$$(a + b)(a - b) = a^2 - b^2.$$

Later, the following factorization formulas were taught:

$$a^2 + 2ab + b^2 = (a + b)^2;$$
$$a^2 - 2ab + b^2 = (a - b)^2;$$
$$a^2 - 2b = (a + b)(a - b).$$

The expansion and factorization formulas express the same mathematical fact. They are just special cases of the distributive law,

$$m(a + b) = ma + mb.$$

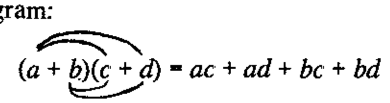
Although knowing this law might be enough for one to understand or explain all the expansions and factorizations of finite polynomials, the above expansion and factorization formulas were presented to students as important formulas to be familiar with. Obviously, the formulas were didactic devices. That is, the knowledge from the distributive law was didactically transposed to and declared as the expansion and factorization formulas.

Declarations may go further. Consider the example of the distributive law again. Instead of presenting many expansion formulas, just one formula,

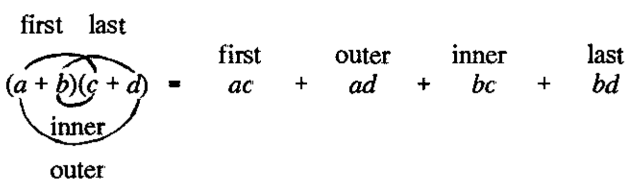
$$(a + b)(c + d) = ac + ad + bc + bd$$

could be presented. This formula is another didactic transposition from the distributive law.

Didactic transpositions can occur again at a higher level. That is, we can observe one or more didactic devices for this formula. One may teach the formula with the following diagram:



Or one may use more sophisticated device such as the mnemonic FOIL, where F, O, I, and L are the initial letters of the first, outer, inner, and last terms, respectively, with the following diagram:



The declaration of knowledge in a didactic transposition may be explicit, as in the FOIL method, or implicit, as in the above rule for multiplying binomials.

The changing environment in didactic transpositions

Besides the declaration, there is another attribute that distinguishes the didactic transposition from other transformations of knowledge. Usually, the knowledge to be used arises in a specific environment within which various combinations of situations are possible. In a given environment, appropriate knowledge arises to be used in a situation: knowledge depends on the environment. But in a didactic transposition, the emergence of the knowledge to be taught depends on the didactic intent. Thus, the didactic environment that enshrouds the knowledge to be taught has to be changed or rebuilt properly from the start.

The basic principle of changing the environment of a didactic transposition is to facilitate the learner's contextualization and personalization of the knowledge to be learned. Thus, the teacher's main job is to recontextualize and repersonalize the knowledge taught to fit the student's situation. This recontextualization and repersonalization, however, is temporary and hypothetical. Especially in textbooks, it assumes hypothetical students, teachers, and classrooms that seem representative. In this sense, didactic transpositions in textbooks are consequences or processes of pseudo-contextualizations and pseudo-personalization.

The context and meaning of an item of knowledge change along the route of its transformation. For example, although an item of knowledge about the distributive law may mean an axiom that sustains a particular number system for mathematicians, students are usually taught it as a law that produces specific expansion or factorizations formulas. The didactic intent reflects the judgement that the distributive law can be better understood not as an axiom but as a law. The didactic intent is not always obvious, however, especially to learners. Even teachers may not recognize the didactic intent that has changed the context of the knowledge to be taught. It is only because of our cultural conditioning that we usually take teachability for granted [Chevallard, 1988, p. 7]. When mathematical knowledge is given a pseudo-context, its range of application, the relation of mathematics to the real world, the purpose of posing problems, and the emphasis on connections between mathematics and other fields are changed [Kang, 1990].

Textbook authors usually regard the student as the main reader. They want teachers to encourage students to read the book. Because they cannot intervene directly in the communication between teacher and students, the authors usually write the textbook from the teacher's position. They give a pseudo-personalization to the mathematical knowledge by providing the teacher with an explanation that can be used in class and by providing exercises the teacher can give to the students.

In explaining mathematical concepts, textbook authors sometimes devise auxiliary concepts that are not necessarily required in forming the principal concepts. The auxiliary concepts may then present difficulties to students because they are incompletely explained [Kang, 1990]. Textbooks that present an area model for the multiplication of binomials, for example, may overlook the auxiliary lines or rearrangements needed when the model is used for the product of the sum and difference of two variables; they may also ignore the restriction to positive numbers. The activities provided for students tend to be grouped as routine exercises or as nonroutine problems following didactic imperatives rather than providing a variety of learning tasks.

Didactic phenomena

As one might expect, a practical problem of didactic transpositions is whether each transposition can be devised efficiently. A difficulty with such efforts is how to control the fragility of mathematical knowledge through a balanced facilitation of both pairs of processes—the student's personalization and contextualization and the student's depersonalization and and decontextualization. Four extreme phenomena should be seriously considered when any didactic transposition is being devised or observed.

For the process of contextualization and personalization, we consider two extreme phenomena: the *metacognitive shift* and the *formal abidance*. The metacognitive shift is a consequence of overemphasizing the process of a hypothetical student's personalization and contextualization, whereas the formal abidance is that of de-emphasizing or ignoring it.

For the second cycle of the epistemological rejection and investment, that is, for the student's decontextualization and depersonalization, there are also two extreme phenomena we have to consider: the *Topaze effect* and the *Jourdain effect*. The Topaze effect is a consequence of the expedient failure to consider the process of student's depersonalization and decontextualization, and the Jourdain effect is that of overestimating it.

Metacognitive shift

The metacognitive shift is a change of focus in a teacher's (or educator's) didactical effort from the mathematical knowledge to his or her own didactical devices. As Brousseau [1984, p. 117] showed, a typical example of this phenomenon is the use of graphs, such as an oval to represent a set, in the 1960s to teach structures, a method connected with the name of G. Papy. Another example is the use of "magic peanuts" to represent negative integers in the Comprehensive School Mathematics Program [see Howson, Keitel, & Kilpatrick, 1981, pp. 154-159]. In the use of the area model to explain the factoring of the polynomial $a^2 - b^2$, a metacognitive shift occurs when the geometric manipulation is emphasized over the algebraic procedure [Kang, 1990]. The metacognitive shift has the advantage of facilitating the student's personalization and contextualization, but it may lead the student's mathematics to take a quite different form from that of the mathematician.

Formal abidance

Formal abidance is a logical presentation of formulation knowledge, minimizing the metaphorical use of knowledge and neglecting a metacognitive strategy. It has been called a *metamathematical slide* by Brousseau [1986]: It "consists in substituting for a mathematical problem a logical debate about its solution and attributing to it all the sources of errors" [p. 32].

A typical example is the deductive presentation of mathematical knowledge in Euclid's *Elements* and its equivalents. In Olney's 19th century textbook, general rules were presented first and then examples were given. Given the binomial formula for $(a + b)^m$, for example, expansions of specific cases such as $(x + y)^5$ and $(x + y)^{-4}$ were explained thereafter [Rash, 1975, pp. 434-442]. Understanding the binomial formula meant applying a general rule to specific cases. In such a textbook, the author's effort is to detail the logical process of applying a mathematical rule. In some algebra textbooks today, formal abidance can be seen in the way substitution is presented as an axiom or principle [Kang, 1990] Formal abidance may not help students understand the inductive side of mathematics, but it may make depersonalization and decontextualization easier for them.

Topaze effect
The Topaze effect is a typical effect under the constraints of the so-called *didactical contract* (Brousseau, 1984) in which a teacher is led to empty the learning situation of all cognitive content when manipulating the meaning of a student's behavior.

The Topaze effect was named by Brousseau after Marcel Pagnol's famous play "Topaze" In the first scene of the play, the teacher Topaze attempts to get his 12-year-old pupil, with whom he practices French dictation, to write the silent plural "s" correctly in the phrase "*des moutons étaient en sûreté dans un parc*" (the sheep were safe in a park). While looking over the pupil's shoulder, he successively dictates "des moutons," "des moutonss," and finally "des moutonsss" He eventually gives, after several failures, an obvious hint of the plural, begging for some sign of consent [Brousseau, 1984, p 110]

The "funnel pattern of interaction," which is illustrated by Bauersfeld [1988, pp 33-36], is an example of this effect. In the funnel pattern, the teacher guides students toward a desired result by a series of questions. Another example is the giving of a solution to a problem in a textbook, as noted by Kilpatrick [1980] and discussed earlier. In algebra textbooks stressing the use of a calculator, the introduction of the power key, y^x, may cause a Topaze effect when lessons emphasize keystroke sequences rather than the meanings of exponents [Kang, 1990].

Jourdain effect
The Jourdain effect is a nontrivial degeneration of the Topaze effect: to avoid both a debate with students about a particular item of knowledge and eventually a confession of defeat, the teacher acts as if he or she recognized evidence of scientific knowledge in the students' behavior or answers, even though these responses were actually motivated by trivial causes and meanings [Brousseau, 1984, p. 115].

The Jourdain effect was named after M. Jourdain, one of the characters in Molière's "Le bourgeois gentilhomme." M. Jourdain asked a philosopher to teach him to spell and received an explanation of the distinction between prose and verse. Eventually he exclaimed, "Well, my goodness! Here I've been talking prose for forty years and never known it, and mighty grateful I am to you for telling me!" [Brousseau, 1984, p. 116].

In a mathematics classroom, for instance, a child who has been made to carry out somewhat curious manipulations with small yogurt cups or colored pictures is suddenly told: "You have just discovered a Kleinian group" [Brousseau, 1984, p. 116] Given an algebra textbook that treats substitution as an axiom, a teacher may interpret a student's use of substitution in an expression as evidence of axiomatic reasoning [Kang, 1990]. That is, the Jourdain effect occurs when the teacher takes student's prosaic responses as representations of mathematical knowledge.

One may argue that these four phenomena are extremes we cannot avoid. In fact, not only are they extremes, but it is also hard to define a clear boundary between normal and pathological cases of each phenomenon. The phenomena may represent a teacher's or textbook author's difficulty in considering both the form and the meaning of an item of knowledge. Kang [1990] found that authors of algebra textbooks did not show a pathological use of didactic transpositions. They maintained moderate attitudes regarding didactic phenomena. Such attitudes, however, suggest the limitations of using a textbook in a classroom. For example, a teacher can explain the factoring of the difference of squares using diagrams of rectangles more freely, and thus sometimes more effectively, than a textbook can, although the teacher may also pathologically transpose that body of knowledge. The limitations of a textbook suggest that a study of didactic transpositions of mathematical knowledge in classrooms should follow the study of those in textbooks.

Conclusion

Didactic transposition theory provides an epistemological model that we can use to avoid extreme phenomenological and ontological views on the nature of knowledge. When it is reinforced with Martin Buber's philosophy of the two modes of knowledge, the theory can also support meaningful interpretations of many phenomena in the teaching and learning of school mathematics. Buber's view that our knowledge is either of You or of It illuminates the vacillating quality of knowing during instruction: knowledge is depersonalized and decontextualized when represented for communication, personalized and contextualized when first encountered, depersonalized and decontextualized again as it becomes part of the learner's codified knowledge. Didactic transpositions must contend with this oscillation. Knowledge is fragile; as it is repeatedly communicated, it changes both its form and its meaning. Recording a body of knowledge in a book does not control its fragility by keeping its meaning constant. Although school mathematics has a static aspect in that it must be declared beforehand, the form of the knowledge being presented in a textbook is not stable. The processes of didactic transposition used in a textbook can be termed pseudo-contextualization and pseudo-personalization. The four processes of contextualization, decontextualization, personalization, and depersonalization are involved in the effective transposition of knowledge in teaching mathematics or in writing a mathematics textbook. Although textbook authors seem to employ moderate forms of didactic transpositions, teachers who follow textbooks uncritically may end up employing extreme forms. Thus, the effective use of mathematics textbooks in a classroom depends on the mathematics teacher's epistemological vigilance.

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Recognition of the indeterminacies present even within mathematics, for centuries the paradigmatically rational discipline, should not entail abandonment of ideals of reason, of ideals of coherence and consistency. The illusion of closure is a product of failing to distinguish the ideal from the real a failure to recognize the purely regulative function of the ideal But once this is recognized the indeterminacies in mathematical representations the undecidabilities in any formal system signalling the discipline s lack of closure can be seen to be its source of problem solving and creative power. not a sign of weakness or of imperfection [...] Mathematical constructions are both concrete and symbolic; they do not provide an escape from representation to reality but serve as a reminder that representations are part of human reality just as human reality is constructed by representations.

Mary Tiles
