RE-THINKING PLACE VALUE: FROM METAPHOR TO METONYM

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Incident 1

Rebecca	How many tens do we have?
Children	Nine!
Rebecca	What will happen if we put all them together?
Dolores	Two tens together makes twenty!
Rebecca	Interesting. What will three tens together make?
Armando	Thirty.
Rebecca	What will nine tens make together?
Dolores	Ninety.
Rebecca	Make another ten.
	Armando makes two herds, of sizes nine and one, and pinches them together.
Rebecca	What will you get if you put the ninety and the ten together?
Armando	Tenty?

Incident 2

Teacher	The kids need to be able to see what a hundred really is.
Nathalie	They can get a different sense of a hundred by seeing how long it takes to get to a hundred by counting though.
Teacher	Yes, but that doesn't help them estimate what a hundred really looks like, like when you have to solve a problem involving a hundred things.
Nathalie	Yes, for estimation, I can see why you say that. But I think there are other situations in which you might not need to know what a hundred really looks like. Imagine, for example, you were asked what comes after a hundred twenty-four.
Teacher	But the kids need to be able to know that a hundred twenty-four is one hundred and twenty and four.

We juxtapose these incidents to contrast different ways of thinking about place value. The first grade children in Incident 1 are counting with their teacher, Rebecca; not counting objects, just counting, playfully perhaps and in this case counting in tens on an iPad app (one that will be described later in the article). There is an aspect to counting that involves noticing patterns in language. It appears Armando has followed a pattern of word endings: thir-ty, nine-ty, ten-ty. He is holding the second syllable constant changing only the first, which is a language-based way of expressing that the units digit is not changing, but the tens digit is.

Earlier in this episode, Rebecca asked Kerstin, a classmate of Armando and Dolores's, how she knew that seventy and ten make eighty, where both the numerals 70 and 10 were visible on the screen, and she responded, "I know that eight is after seven", showing attention to place value. We are not surprised that young children can attend to place value, even though it is not explicit in the curriculum until grade 2 (in Canada and the US) or Year 2 (in the UK). What we find significant is the way in which the children are attending to place value seemingly without concern for the actual size of the numbers involved.

The second incident is a partial transcript of a discussion with a teacher, following a seminar that included introducing the iPad app the children in Incident 1 were using. It seems to reflect a concern that playing with language risks losing the sense of what a number 'really is', especially as it relates to place value and perhaps imagined base-ten blocks. If base-ten blocks are associated with what 100 or 10 or 1 *really mean*, then number work is necessarily limited in the early years to what can be manipulated and seen, a presumption reflected in curriculum documents. We wonder whether physical representations that have become so common in primary school classrooms in Canada and the UK gain part of their appeal through the visual and through the sense that 'seeing' is 'knowing'. The work the children are doing in Incident 1 is oral and, hence, temporal and therefore perhaps seemingly fleeting—we acknowledge the concern this can provoke in an era of accountability.

Nonetheless, we want to consider in this article what can be gained from working with children on counting, not in the context of one-to-one correspondence with objects, but instead drawing on the kind of counting seen in Incident 1, an intransitive or ordinal counting. In doing so, we question what it means to know about place value. Understanding place value is given high prominence in national curriculum documents around the world and is linked to the achievement of number sense. Many authors view place value as the foundation of our numeration system and it is also a commonplace that students can find understanding place value hard. It is the aim of this article to suggest the current prominence given to place value, in the sense of interpreting numbers solely as quantities, may actually get in the way of the learning of many students. Over the course of this writing, we propose an alternative approach to developing awareness of number, focusing on its linguistic and temporal aspects.

Some background on number

Based on research in neuroscience, Dehaene and colleagues (2003) introduced the idea of a triple-code model, where number is seen to comprise:

1. a visual Arabic code in which numbers are represented as a string of digits;
2. an analogical quantity or magnitude code;
3. a verbal code in which numbers are represented as a sequence of spoken words in a natural language. (p. 488)

They suggest understanding number involves two major processes of transcoding, from (1) directly to (3) and back again (which they call 'asemantic') and a semantic route from (1) to (2) to (3). The label 'semantic' (indicating 'meaning', presumably) is reserved only for the sense of number that incorporates magnitude, code (2). The assumption here seems to be that meaning (semantics) only comes from magnitude and we see in the work of Dehaene and colleagues an emphasis on quantity that is common in the field. In the exchange at the start of this article (Incident 1), Armando's process for inferring what ninety plus ten will be, based on hearing and seeing what three tens are called and then nine tens, would therefore be judged as asemantic.

We associate what Armando does with an ordinal sense of number. Ordinality refers to the aspect of number that is linked to the sequence of counting numbers; the ordinal aspect of '4' is that it is the '4th' one and comes after '3' and before '5', it is ten before 14, and so on. The semantic, magnitude-based route to number, we associate with cardinality.

In the early years of schooling in the UK and Canada, as well as in teacher education in these countries, the focus of work is on the second, magnitude code. As one UK teacher we have worked with put it, the orthodoxy is that young children must work with numbers that are 'graspable', *e.g.*, working with beads, counters and one-to-one correspondence. In terms of place value, this approach evolves into working with similarly 'graspable' metaphors for number such as base-ten blocks.

Indeed, Ross (2002) argues for the use of digit-correspondence tasks, in which "students are asked to construct meaning for the individual digits in a multidigit numeral by matching the digits to quantities in a collection of objects" (p. 420). We want to ask, how else might it be possible to develop a sense of place value? We do not want to argue that the quantity code is unimportant. However, we suggest there is much to be gained from exploring an increased focus on the verbal code and, with it, the temporal.

Seidenberg's (1962) detailed account of the ritual origins of counting suggests the ordered recitation of the list of number words long precedes, historically speaking, the more cardinal counting of things (animals, people, money, *etc.*). He argues that acts of ordinal counting are principally about calling forth the next or another, making the new or next appear, not just about ordering that which is already visible. In other words, the very *meaning* of a number comes more through its relation to the previous and following numbers, rather than through a connection to a specific magnitude. And, according to Seidenberg, it is this meaning that eventually enabled the cardinal uses of number that are so prevalent today. On this hypothesis ordinality (which is temporal) was the fundamental aspect of number from which cardinal uses developed. There is some logical plausibility to this suggestion: without a pre-existing, fixed number sequence, how would it have been possible to give meaning to a verbal count of things?

While ordinality is typically mentioned in mathematics curricula for the first years of schooling, it is usually a small part of the set of concepts associated with developing number sense. Not so long ago, however, debate over the primacy of the one over the other was lively amongst mathematicians, philosophers of mathematics and psychologists. In mathematics, both Peano and Dedekind argued for the primacy of ordinals, whereas Russell advocated for cardinality. Brainerd (1979) asserts that Piaget ignored the logical distinctions underlying these two alternatives and used his experimental evidence as a basis for combining the ordinal and cardinal. After Piaget, who may well have espoused a balanced approach to number, the work of Gelman and colleagues (*e.g.*, Gelman & Meck, 1983) elaborated an almost exclusively cardinal conception of number, which has gained widespread acceptance in the field, continuing to this day.

It is not clear to us why ordinality lost out, but it may have to do in part with the influence of set theory, in which the numbers are taken to be sets, and are therefore foundationally cardinal. It may also be related to Piaget's theories around abstraction, linked to the assumption that counting 'how many' things there are (and therefore focusing on cardinality) is more 'concrete' than working with symbols (Tahta, 1991).

We have outlined reasons for considering the possibility of pursuing a more ordinal approach to number. Elsewhere, we have drawn attention to the work of other mathematics education researchers who proposed a more ordinal approach to number, such as Gattegno and Davydov (see Coles, 2014). For Davydov (and Gattegno), number arises out of considering relationships between measures (Coles, 2017), so that, in terms of Dehaene and colleagues' triple-code model, there is no magnitude code in an absolute sense and size is always in relation to a unit or another magnitude.

Before turning to our main interest, which is in place value, we would like also to bring into play a distinction that Tahta (1991, 1998) made that is relevant to the issue of meaning and what it is to be 'graspable'. In a discussion about the challenges involved in teaching and learning arithmetic, he distinguishes between 'metaphoric' and 'metonymic' ways of accessing number. A metaphor replaces one thing with another to help make sense of the original (*e.g.* the number '2' becomes a rod of length 2cm or two unit cubes). Metaphoric ways of approaching number, therefore, might involve an abacus, a ten-frame, rods, blocks or any other direct re-presentations. Lakoff and Núñez (2000) suggest the very doing of mathematics involves thinking with metaphors (such as 'container–contained') and mappings between the metaphor and the mathematics. Metaphorical models of number are seen to shed light on what number 'is'.

By contrast, metonymy is about 'part–whole' relations, where one aspect of a thing can serve as a substitute for the whole (*e.g.*, the number '2' can be associated with the act of matching two of the same rod against a single rod of equivalent length). Metonymic relations between words might be based, for example, on their sound or look ('look', 'book', 'hook', 'spook'). In Incident 1, we see Armando invoking a metonymic aspect of number ('thir-ty', 'nine-ty', 'ten-ty'). Presmeg (1992) used the term *metonym* to refer to reasoning from a particular mathematical object to a conclusion about a general class of objects (part to whole). Armando's use of ten-ty is also a different kind of metonym; the numeral (symbol) or the number word is another *name* for the number, not its meaning, and yet work with number names and the verbal code can still be both meaning-*ful* and highly useful.

Tahta (1998) argued that there is too much metaphoric work in early number, which we note seems to focus almost exclusively on cardinality, and not enough metonymic work, which seems more ordinal in nature. The question of meaning relates to the assumption that a metaphor carries more meaning, in that it is another thing that is physical and familiar. It is about identity and this seems to be the concern of the teacher in Incident 2. We see this assumption carried through to the work of Dehaene and colleagues (2003) and the apparent claim that the only 'semantic' (or meaning-based) route to number involves quantity. A metonym does not establish an identity, but rather resides in relations between things, and relations are ways of marking distinctions (a relation includes some things and excludes others) or differences. We are interested in exploring the meanings that children can make through metonymy, in particular in regard to the meaning of place value.

'Understanding' place value

We have briefly mentioned some of the ways place value is typically addressed and described in the literature. Here, we look more carefully at what researchers have seen as the requirements of 'understanding place value'. Ross (2002) suggests understanding place value involves a combination of four properties:

1. Additive property. The quantity represented by the whole numeral is the sum of the values represented by the individual digits.
2. Positional property. The quantities represented by the individual digits are determined by the positions that they hold in the whole numeral.
3. Base-ten property. The values of the positions increase in powers of ten from right to left.
4. Multiplicative property. The value of an individual digit is found by multiplying the face value of the digit by the value assigned to its position. (p. 419)

The emphasis on the quantity code is striking here. We might formulate alternative descriptions of these properties, based on the verbal code:

1. Additive property. The name of the whole numeral is made by saying the names of the individual digits in order from left to right (with the exception of numbers 11-19 in English) with the inclusion of the occasional 'and' (e.g. 632 is read aloud as "six hundred and thirty two" in British English).
2. Positional property. The positions of the individual digits in written form are assigned consistent names.
3. Base-ten property. The names associated with the positions increase in powers of ten from right to left (-'ty', 'hundred', 'thousand', etc.).
4. 'Multiplicative' property. The name of an individual digit is produced by saying the face value of the digit followed by the name assigned to its position (*e.g.,* 'six' followed by 'hundred').

We agree with Pimm (2017) that, verbally, 'place' value is arbitrary (Hewitt, 1999) in the sense that we could just as easily say "six-ty and one hundred" as "one hundred and sixty"; it is not the place in the verbal sequence that makes a difference, but the label. On the reading above, the concept of place value can be interpreted without referring to cardinality at all, through conventions of naming. Place value reduces to knowing the order of positional names and the order in which they are customarily read.

Tipping our hats to Piaget's reliance on empirical evidence, we would like to offer two examples in which an ordinal conception of place value can be elaborated with young children alongside the more typical cardinal one. We are interested in the possibility of a more balanced view of place value, one in which ordinal approaches become an alternate and effective strategy for solving problems such as what comes after 200 or what the '2' in 254 'means'. The examples below are taken from joint work that we are doing on using *TouchCounts* (the iPad app mentioned earlier) and 'Gattegno charts' (described below) in working with children in grade 1, 2 and 3 in both Canada and the UK. We explain these resources and introduce the examples in turn.

Working with TouchCounts

TouchCounts (www.touchcounts.ca) is a free multitouch app. It has two worlds, Enumerating and Operating. In the Operating world, tapping on the screen creates autonomous numbered sets, which we refer to as *herds*. One starts by placing one or several fingers on the screen, which immediately creates a large disc that encompasses all the fingers and includes a numeral corresponding to the total number of fingers touching the screen. At the same time, every one of the fingers in contact with the screen creates its own much smaller (and unnumbered) disc, centred on each fingertip. When the fingers are lifted off the screen, the numeral is spoken aloud and the smaller discs are lassoed into a herd and arranged regularly around the inner circumference of the big disc (Figure 1a shows two herds, of size four and ten).

Figure 1. *(a) The herds;* *(b) Pinching two herds together;* *(c) The sum of two herds.*

After two or more such arrangements have been produced (as in Figure 1a) they can be pinched together (addition, as in Figure 1b) or separated (subtraction). Dynamically, when pinched, they become one herd that contains the 'digital' counters from each previous herd, thus adding them together. The new herd is labelled with the associated numeral of the sum (Figure 1c), which *TouchCounts* announces aloud. Moreover, the new herd keeps a trace of the previous herds, which can be seen by means of the differentiated colours of the individual small discs within the combined herd.

In the opening episode (Incident 1), the children had created herds of size ten on the screen, which Rebecca had invited them to do, in part to help them make 100, which they had excitedly declared they wanted to do. When they pinched two herds of tens together, *TouchCounts* would label a new herd 20 and say "twenty". The children had pinched many herds of size ten together and so had heard/seen the multiples of ten quite frequently. In asserting that combining herds of size ninety and ten would make "tenty", Armando appears to follow a pattern in language ("eight, nine, ten") that is not linked to cardinality.

Similarly, Kerstin does not infer that 70 + 10 = 80 because she has counted on from 70 or because she has used a strip of unifix cubes or a base-ten block to represent 8 tens. In a way, she is ignoring quantity altogether, possibly even dismissive about whether she is talking about tens, hundreds or thousands, because her focus is on the relation between 7 and 8. In other words, for Kerstin, computing 70 + 10 would be no harder than 7 + 1 and no easier than 700 + 100.

In the Enumerating world, each time children touch the screen with one finger, they create a numbered object (a yellow disc) on the screen and, as a result, one number name is said aloud. The number names are said in order (one, two, three, four, *etc.*) and the symbols appear in order as well (1, 2, 3, 4, *etc.*). In this way *TouchCounts* creates a one-to-one-to-one correspondence between *touch, number-name, object* and *numeral.*

When gravity is turned "on", a "shelf" (horizontal line) appears on the screen. When a finger is placed on the screen below the shelf, the yellow disc falls away under the "force of gravity"; when a finger is placed above the shelf, the yellow disc is "caught" and stays on the shelf. For example, in Figure 2, there have been four taps below the shelf and the fifth one above the shelf.

In another grade 1 classroom children were putting multiples of five on the shelf, by tapping four times below the shelf (this started sequentially, but eventually became simultaneous), and once above, after a while creating an auditory pattern "four, five, nine, ten, fourteen, fifteen, *etc.*" with the following numerals visible on the screen: 5, 10, 15, etc. Having reached 100, the children were eager to keep going. After a few more multiples of 5 were added to the shelf, a boy exclaimed "Oh! It's not two hundred".

Teacher	What do you say?
Boy	It's not two hundred.
Teacher	Why do you say that?
Boy	I thought that two hundred was right after one hundred, but it's not.
Teacher	No, how far away is it from one hundred?
Boy	It's, it's, it's one more hundred away from one hundred.

The boy's use of the words "right after" evokes a sense of the sequence of numbers, the temporality of the unfolding of numbers more than their relative size. When the teacher says "how far away", one gets the sense of the number unfolding along a path, even though the children had not worked with large numbers on a number line. So when the boy repeats the word "away" and says that two hundred is one more hundred away, the focus seems not on the size of 200, but instead on the time that it might take to get to 200,

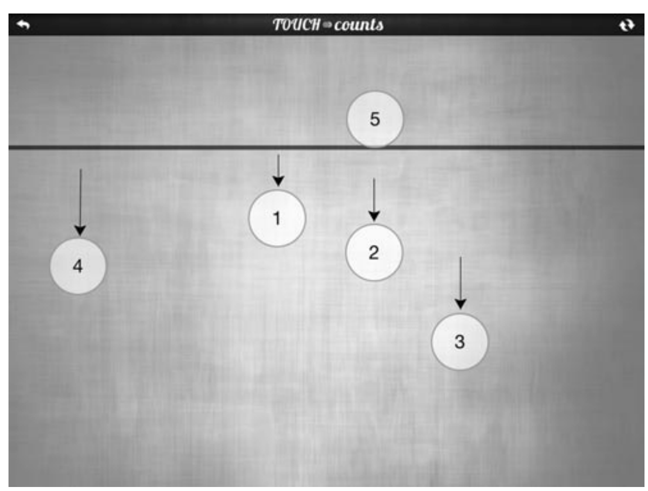

Figure 2. *Counting to 5.*

relative to how long it took to get to 100. Unlike Incident 1 then, where the focus is on the language and the symbols, here time is used in the ordinal counting. We suggest there is a sense here of the 'size' of 200, but not linked to quantity so much as number name structure.

Working with a Gattegno chart

The 'Gattegno tens chart' is a visual structuring of our numeration system (Figure 3). The chart can be seen as a layering of number lines at different levels of magnification and we believe work on the chart supports students' visuo-spatial patterning of number, which is suggested to be significant for learning (Zorki *et al.,* 2002), although this is not something we have researched. Different rows can be displayed, including decimal ones, but the units row is always included (alternative versions have the biggest row at the top).

One way of working, as a teacher, with the Tens chart is by pointing to numbers and asking a class to chant back in unison. After tapping successively on two numerals (*e.g.,* 50 then 7), the class chant back "fifty seven", the number is 'made' by two taps from different rows, in decreasing order. Students can do the pointing for others to say. The teacher can point to a number and others chant back the number one more, one less, ten times bigger, ten times smaller, *etc.,* than it. A further challenge might be to skip count using the chart (forward, backward, by any number, starting from any number).

Alf worked on the chart with a student, Aaron (a pseudonym), who had been judged as the lowest attaining in mathematics for his age (7) in his school. His teacher had asked Alf to help Aaron understand place value. At the point the transcript begins, Aaron had just asked to do some counting in tens. [1]

Alf	How's it going to go if we count in tens on this? *[points to Gattegno chart]*
Aaron	Ten *[5]* just thinking.
Alf	Do you know which number you point to on here? *[points to Gattegno chart]* if we're counting in tens?
Aaron	I think, cos you've got ten *[points at 10]* miss out one *[points at 1]* two three *[points to the right of 1]* and you go and you miss out 9 *[looks towards 9 but no pointing]* and then you go there! *[points at 10]*
Alf	You miss out all of those and then you go to that one. *[points at 20]*
Aaron	Twenty-two.
Alf	Just twenty. So: ten
Aaron	Twenty *[points at 20]* thirty *[points at 30]* forty *[points at 40]* forty-five *[points at 50]* fifty.
Alf	Just fifty.
Aaron	Sixty, seventy, eighty, nineteen.
Alf	Nine-ty.
Aaron	Ninety, hundred *[points at 100]* hundred and two.
Alf	To do that we'd do a hundred and ten *[directs the pointer, with Adam still holding it, to tap on 100 then 10]* and then a hundred and *[2]* twenty.
Aaron	Hundred and twenty [...] *[continues counting in 10s up to 190, pointing as he goes, tapping on 100 then 30, etc.]* [...] hundred and ninety, hundred and three.
Alf	A hundred and ninety and then *[points to 200]* two *[gestures hand forward]*
Aaron	Two *[1]* ty, hundred.
Alf	Two hundred.
Aaron	Two hundred, two hundred and one.
Alf	Two hundred and ten.
Aaron	Two hundred and ten, two hundred and twenty [...] [*continues in 10s up to 290, pointing as he goes*] [...] two hundred and ninety.
Alf	Amazing. I don't think we've ever been up this high counting. Keep going.
Aaron	Three hundred, I might get there *[points to 10,000]*
Alf	Yeah, you might do! Can you keep going from three hundred?

1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000

Figure 3. An example of a Tens chart.

Aaron initially struggled to get going on his suggestion of counting in tens. He seems to be aware that counting in tens from ten, he will miss out 11, 12, 13 … 19. However, he is seemingly not able to name where he will get to. From the first time Alf worked with Aaron, in September, he had struggled to name '20'. It is also perhaps significant to note that Aaron had been working on counting in ones on the chart and so would have done some 'tapping' on the chart moving from 10 to 11 to 12 (the numbers he is aware he will miss out when counting in tens). After the end of the transcript, he fluently moves through from 300 to 490, making one error, by moving from 470 to 490 to 480. The confusion of the ordering of 8 and 9 is another issue that had arisen in several earlier sessions with Alf. Later in the session, Alf asked Aaron to write down 490, which he did accurately.

We see in what Aaron is doing the beginnings of a metonymic fluency with place value. He can name and write three-digit numbers, while still having some confusion about the names for '8' and '9'. With inconsistency in his naming of 8, 9 and difficulties in recognising the verbal code for '20', we suspect in many contexts Aaron would not be offered work on higher numbers. The inconsistencies he demonstrates in the naming are, in Hewitt's (1999) terms, difficulties with 'arbitrary' aspects of counting. There is no reason why 8 should not be called 'nine'. Confusing these numbers indicates as little about a child's mathematical awareness as does calling a 'fork' a 'knife'. What Aaron does show is awareness of number-naming structure.

Physical action (of pointing) on a visual structuring of the number system seems to support Aaron's use of the verbal and temporal pattern of counting in tens, that we see as a demonstration of awareness of place value. Aaron uses the additive, positional, base-ten and 'multiplicative' properties of number (indicative of an 'understanding' of place value in the verbal sense) every time he points to, for example, 300 followed by 40 and says 'three hundred and forty'. There is a rhythm to Aaron's counting that is impossible to convey in written form. At certain points in his counting the numbers come quickly (-twenty, -thirty, -forty, -fifty) as though Aaron is being carried along a (metonymic) chain of signifiers by their own force of association.

Over-attachment to metaphor

In the previous sections, we have recounted some of our experiences of working with children, in which we see the possibilities for developing fluency around place value in an ordinal and metonymic way. The children's enchantment with big numbers seems to make this kind of work motivating and engaging for them. We suggest that both *TouchCounts* and the Gattegno chart can be used powerfully precisely because they are *not* offering metaphorical representations of number. Instead, they offer spaces for children and teachers to work on linking symbols, sounds, names, touch and gestures to each other.

In *TouchCounts*, children summon numbers into existence through touch and engage in learning the 'rules of the game' of our number-naming system. With the Gattegno chart, the verbal, number-naming code is linked to physical pointing and/or tapping. In our work with children, we have been surprised at how well *TouchCounts* and the Gattegno chart complement each other. Similar tasks seem possible in both environments and offer different available connections and awarenesses, for example, creating the five times table above the shelf in the Enumerating World and then counting in fives on the chart. We suspect that one reason for the strong connections between such different-looking resources is that they both offer metonymic ways of working with number.

The number name serves as a label, an associated part standing in for the 'whole', a metonym for number. What we are suggesting is that we can begin working with children on the ways we want them to operate with number, from the beginning, rather than approach number in a metaphorical manner that then has to be left behind if they are to achieve sophisticated fluency with counting, a fluency which is strongly linked to arithmetic fluency, one of the so-called 'basics'.

We see it as an open question, what might be an adequate balance of ordinal and cardinal approaches to learning early number, in any given context. There is an urgent need to explore this question, given the persistent difficulties that so many students have with number, which we believe in part derive from early learning that does not lay the ground for later developments in mathematics.

Notes

[1] Transcript notation: *[3]* indicates a pause of three seconds, *[text]* description of gestures, and [...] some dialogue skipped. Otherwise standard punctuation is used.

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