On the Methodology of Analysing Historical Textbooks: Lacroix as Textbook Author

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New approaches to the history of mathematics education

During the last few years one can note a growing interest in the history of mathematics education — an interest which is instigated by issues of social history, by questions about the beliefs and intentions of the persons actively concerned with education (teachers, administrators, parents). Traditional historiography is not well adapted to answering these questions — it concentrates on administrative policy and its operationalisation by decrees, time-tables and the weekly share of mathematics instruction, but it does not bother much about real school life, about the problems of everyday practice. Evidently one needs more and different sources in order to investigate these issues.

If one starts from the fact established by school research in the 1970s that *teaching practice* is not so much determined by ministerial decrees and official syllabuses as by the textbooks used for teaching, one is led to study schoolbook *authors*. Actually, analyzing old schoolbooks can be regarded as a quite traditional approach to the history of mathematics instruction. But we still seem to be far from reliable methods of analysis since the usual methodology of studying an isolated text or of formally comparing several books tends to neglect the social and cultural context of their utilisation as well as the national specificities of the respective school systems and the respective professional status of the teachers who used them. It seems therefore reasonable to choose a basic "unit" for such studies where at least some of the relevant dimensions can already be seen in interaction. In fact, investigating the personality of a schoolbook author and the totality of his *oeuvre* can enable us to get insight into the social meaning of school knowledge and of the relations between the author and his "clients": the teachers. [1] As schoolbooks are usually reedited several times and during this time undergo substantial changes, one can put in relation changes in the textbooks and changes in the school structure, changes in the social appreciation of school knowledge and the active or passive reactions of the schoolbook author to these changes. Thus, choosing the work of one schoolbook author as a basic unit may be *one* approach by which one can better analyze the reality of school life in former days.

If one takes the perspective of investigating schoolbooks in order to study the establishment of "school mathematics" as a particular and separate *corpus of knowledge and methods*, then clearly the 19th century will be a natural focus for the research — and particularly the first half of this century, since that was the period of the establishment of a general, state-controlled school-system. [2] However, in order to put the proposed approach in operation it is not such a good idea to begin with Germany since the schoolbook production there was so enormously large and the number of schoolbooks authors so great that it would be extremely difficult to obtain reliable results. [3] Therefore I propose to begin these studies with France where we find during the same period a system with a reduced degree of complexity:

> — from 1803 onwards (up to the early 1860s) the utilisation of schoolbooks was administered by central state authorities: only schoolbooks examined and approved by a central commission could be used in primary and secondary (state) schools. As a consequence the number of schoolbooks — at least for mathematics and the sciences — remained "bounded"; a comparative analysis, thus, is not impossible. [4]
>
> — France constituted in the first half of the century a relatively closed system of communication so that — as a first approximation — foreign influences on school subjects and their methodology can be neglected

Having thus defined the frame, one has to identify an author whose *oeuvre* is sufficiently extensive and influential to function as a basic unit enabling one to make comparative analyses of the contemporaneous productions.

Life and work of Lacroix

In fact, there is such a key French author: *Sylvestre-François Lacroix* (1765-1843). His textbooks can be regarded over a period of about fifty years — 1795 to 1845 — as highly influential: they were all published in a substantial number of editions and were obviously widely used in schools. Moreover, Lacroix was one of the new French authors who composed textbooks for practically all branches of school mathematics and for all degrees of mathematics instruction, from secondary to higher and technical education (excluding only primary education). There are just two contemporaneous French authors who can at all compete with Lacroix with regard to the breadth of range of their textbook subjects (but not with regard to the multiplicity of addressed school types and levels): A. A. Reynaud (1774-1844), who reedited and transformed the textbooks of Bézout (the most successful author from the new technical and military schools of the second half of the 18th century), and P. L. M. Bourdon (1779-1854). [5]

One can therefore regard Lacroix as an author whose oeuvre contributed most decisively to the constitution of school mathematics in France. Lacroix and his work have not yet been studied extensively, but there are several biographical articles by R. Taton [1953a, 1953b, 1959] which can serve as a starting point. Furthermore, Lacroix's *Nachlaß* held at the Bibliothèque de l'Institut (Paris) yields information on his activity as a textbook author.

Lacroix — promoted by Condorcet — had already been a teacher of mathematics during the last years of the *Ancien Régime,* particularly in military schools. After the Revolution, he was among the most actively engaged campaigners for the establishment of a general system of instruction. Particularly noteworthy was his engagement to establish mathematics instruction as an integral part of general instruction. Though Lacroix pursued a substantial part of these activities within the state administration for public instruction, his administrational policies and their impact on mathematics instruction have not been investigated hitherto.

As I have shown elsewhere [6] the elaboration of good textbooks was of prime importance for the pursuit of the educational reforms aimed at after the French Revolution from 1792 onwards. Not that the education of teachers was seen as a first priority since the "corporatistic" spirit of the educating institutions was heavily distrusted and since it was left to the industriousness of aspirants for teachers' positions to acquire the necessary knowledge. Rather, centrally prescribed textbooks (only one per school discipline) would ensure loyalty to the republican ethos and the uniform application of "the good method" (which was thought to be incorporated into the good textbook). All subsequent French governments — despite their highly-opposed political orientations — pursued this policy of centrally chosen and prescribed textbooks. The first attempt in 1794 to produce the best textbook per discipline within five months by means of a public *concours* failed dismally [7], and it was later decided to set up a permanent commission to choose between texts which would be produced by private initiative for the market. The concours of 1794 had only had in mind textbooks for primary schools, and the teachers at the *Ecoles Centrales* — established in 1795 as upper secondary schools — had free choice in their textbooks. However, the majority chose the traditional textbooks of Bézout, and when Napoleon in 1803 suppressed these republican schools and replaced them by the *Lycées,* the teachers there were now forced, too, to use only those textbooks which were approved by the central commission.

Lacroix was a member of the jury for the 1794 concours so that he could not participate in it. Yet in 1795 he produced his first textbooks, the "Traité de géométrie descriptive", based on his experience as an assistant to Monge's lectures at the short-lived *Ecole Normale* of 1795. Since he taught mathematics at a Parisian *Ecole Centrale* he began to publish more textbooks, first for the use of his own school and then for a wider public. In 1797 he published the "Traité élémentaire d'arithmétique" [8], an adaptation of Clairaut's "Elémens d'Algèbre", and the first volume of his important "Traité de calcul différentiel et intégral" Urged, as he himself declared [Lacroix, 1799, p. 1.], by the necessity to provide the new schools with textbooks, Lacroix published within the next four years the bulk of his textbook *oeuvre* for all subdisciplines of school mathematics and for all degrees of schools. In fact, Lacroix taught mathematics at almost all the important Parisian institutions, sometimes several at the same time: year III of Ecole Normale, the Ecole Centrale (des Quatre Nations), the Lycée, the Ecole Polytechnique, the Faculté des Sciences, the Collége de France. His efforts were crowned by a singular success: the commission which in 1803 had to choose the textbooks for the *Lycées* adopted exclusively books by Lacroix for mathematics [9] Even in later years Lacroix's books figure prominently.

One can note as Lacroix's principal objective the development of a coherent corpus of school mathematics, from secondary to higher education. Lacroix expressed this "universal" approach by the main title which he adopted in order to imply the integral character of his *oeuvre*: "Cours complet de mathématiques" His enumeration in 1819 of the different parts of this "complete" textbook series (with ten elements), and of the number of new editions, shows the broad range of topics as well as the remarkable success of this textbook series over a period of twenty years: [10]

COURS COMPLET DE MATHEMATIQUES à l'usage de l'Ecole centrale des Quatre-Nations; Ouvrage adopté par le Gouvernement pour les Lycées, Ecoles secondaires Colléges, etc., Par S. F. LACROIX, Membre de l'Institut et de la Légion-d'Honneur Professeur au Collège royal de France etc 9 vol. in- 8
Prix pour Paris 38 fr 50 c
Chaque volume se vend séparément, savoir:
Traité élémentaire d'Arithmétique, 14e édition, 1818 2 fr
Elémens d Algèbre, 12e édition, 1818 4 fr
Elémens de Géométrie, 11e édition, 1819 4 fr
Traité élémentaire de Trigonométrie rectiligne et sphérique, et d'Application de l'Algèbre à la Géométrie, 6e édition 1813 4 fr
Complément des Elémens d'Algèbre 4e édition, 1817 4 fr
Complément des Elémens de Géométrie Elémens de Géométrie descriptive, 4e édition, 1812 3 fr
Traité élémentaire de Calcul différentiel et de Calcul intégral, 2e édition, 1806 7 fr 50 c
Essais sur l Enseignement en général, et sur celui des Mathématiques en particulier, ou Manière d étudier et d'enseigner les Mathématiques, 1 vol in- 8, 2e édition. 1816 5 fr
Traité élémentaire de Calcul des Probabilités, in- 8, 1816 5 fr
Traité de Calcul différentiel et de Calcul intégral, 2e édition, revue et considérablement augmentée, 3 gros vol in- 4, avec planches Prix pour Paris, 66 fr

Mathematics history and textbooks

Lacroix's universal approach is the more important in that he undertook it in an epoch when for the first time a general and public system of education became established — in France, and soon after in Prussia. This new

educational system gave mathematics a considerable position. Evidently this large diffusion of mathematics throughout society was an historically unprecedented event. The structure of the presentation of mathematical knowledge was not prepared and adapted for the demands of this sort of teaching. A restructuration and redefinition of mathematical knowledge became necessary. It was exactly this objective which Lacroix realized very early on and to which he became profoundly committed. And it is his historical merit to have substantially contributed to the restructuring of a poorly-organized and scattered corpus of mathematical knowledge, guided by educational objectives.

Lacroix also seems to have been the first who explicitly reflected these demands — from the very moment when he began to work at his "Traité de Calcul différentiel et de Calcul intégral". He intended in this textbook not only to assemble the original results of the various researchers, dispersed in the publications of the multitude of European academies, but also to *structure* and to *elementarize* them, i.e. to analyze the *elements* of the calculus, regarded as a conceptual field, [11] and to present the calculus as an ordered and well-defined sequence starting from these basic elements. Already in a letter of November 9, 1789, to Legendre, in which he asked for certain research material on the calculus needed for his book, Lacroix discussed this objective:

> Even the most complete elementary textbooks — the Integral calculus by Euler and that by Cousin [12] — need additions, and in order to increase the coherence of its parts one has maybe to change the manner of their presentation. [13]

There is a wide spread conviction that textbooks authors cannot be discussed in connection with the progress of mathematics. For example, Taton alludes to the general opinion about Lacroix "as an author of minor rank, without lasting influence" [Taton, 1953 a, p. 593]. However the contemporaries of the first systematic effort around 1800 to produce *livres élémentaires*, basic textbooks for the general education system, judged its impact on science differently. A telling example is the review of the second revised edition of J. A. J. Cousin's calculus textbooks, published in 1796 in the *Decade*, the journal of the then leading philosophical group — the *Idéologues* — which was at the same time the most influential pressure group behind the educational reforms [14] The review's anonymous author — though sharply distinguishing between the audience for *livres élémentaires*, the addressees of general education, and those who intend to "approfondir une science", who are advised to read the *inventors* — accords to the textbook author under certain conditions the rank of an inventor:

> The author of an elementary book attains the rank of an inventor if he can present the elements, first, in the best order, in the most simple and the most clear manner; if he removes from the science all its technical wrapping and if he illustrates after each step the space traversed in such a manner that the pupil always knows well where he is [15]

And one year later, in 1797, the French Academy (Institut de France), in its expert report on Lacroix's project for a new textbook on the differential and integral calculus, emphasized the intrinsic relation between progress in research and clarity in the fundamentals(while no longer ascribing a privileged role to the inventors):

> To present difficult theories with clarity, to connect them with other known theories, to dismantle some of the systematic or erroneous parts which might have obscured them at the time of their emergence, to spread an equal degree of enlightenment and precision over the whole; or, put shortly: to produce a book which is at the same time elementary and up to the mark in science, this is the objective which Citizen Lacroix has taken to himself and which he could not have attained without engaging himself in profound researches and by progressing often at the same level as the inventors. [16]

Given the considerable impact of textbook production on the progress of science, we have yet to state that the impact is not always a "positive" one, i.e. a contribution in the traditional way to an increase in differentiation and theorization. One example of the other type of impact is provided by Prussian school mathematics: the ever more profound and aspiring attempts there to restructure and elementarize mathematical knowledge led numerous mathematics teachers to a *souci de rigueur* which would ensure clear and stable foundations but which eventually ended in the static vision of a science reduced to pure formalisms. Typical of such "fundamentalism" is a schoolbook of 1872 by Robert Grassmann (1815-1901), a brother and close cooperator of Hermann Grassmann. In this book, "Doctrine of forms of mathematics", the author reproaches all the existing mathematical "systems", and in particular the arithmetical ones, for being neither rigorous nor coherent and based on vicious circles. In the editions of 1891 and 1900 the reproaches are reinforced — we quote from the third edition:

> The present treatise on the doctrine of numbers or on arithmetic claims to be the first rigorously scientific but at the same time entirely elementary presentation of the doctrine of numbers. Except for the works of the brothers Grassmann at Stettin and of Professor Schroeder at Karlsruhe (. . . Hermann Grassmann, 1861, Robert Grassmann, 1872, Schroeder, 1872), all the other presentations of this doctrine contain in their basic chapters the most dubious vicious circles and fallacies which prove nothing and are only capable of accustoming the reader to unscientific reasoning and confusing his thinking. [17]

Another possible impact comes from the attempt to counter certain disciplinary differentiations and to emphasize certain "integrationist" tendencies. A telling example of this tendency is associated with the history of

negative numbers in France: affected by L. Carnot's fundamentalist criticisms in 1803 against the admission of algebraical concepts which could not be directly interpreted in "real world" terms, i.e. in geometrical terms, the strong movement in French mathematics towards algebraization stopped quite abruptly and was replaced by a return to geometric foundations [cf Schubring, 1986a]. As a consequence, "quantity" was maintained as a basic *common* notion, embracing the entire domain of geometry, arithmetic and algebra ("continuous" and "discrete" quantities), a move which did not advance conceptual diffentiation. And Lacroix was quick to adapt his textbooks to this epistemological rupture and to use them to spread the new integrationist conception.

Methodology of textbook analysis

Looking at textbooks from this perspective we see that they do contribute to the history of mathematics so that one may study textbooks in the context of this history. The crucial question, however, resides in the problem: how can one analyse textbooks? The first, seemingly obvious, answer is to apply the same (or an adapted) methodology as used for other historical texts. This traditional methodology, well known as the "history of ideas", however, suffers particular shortcomings which are increased to a higher and more significant degree by textbook-type texts. Due to the scarcity of reflections on the historiography of mathematical methodology, one is forced to judge it by its practice. The general practice, then, in the historiography of modern mathematics seems to be to interpret some selected scientific texts in an almost internal manner, often comparing them with the publications of other famous authors.

The sporadic and isolated character of these interpretations is emphasized by J. Dhombres when he underlines the comprehensive approach of Judith Grabiner in her analysis of Cauchy's book:

> In other words, to expose the originality of a mathematician, we not only have to consider the definitions introduced and specific innovations made in terminology, but we have to see what practical uses are made of them in proofs. This is a decent and straightforward attitude which has not always been respected by historians of mathematics, particularly when they are confronted with the task of describing new paths in the rigorization of some mathematical theories. [Dhombres, 1985b, p. 87].

> But even if one can attain a systematic internal analysis of the structure of a text, the mere description of it will not be sufficient for the historian: he wants to place the author and his work within the development of mathematics and he wants to evaluate the originality of the author's contributions to this development. Such an evaluation might seem to be not so difficult for scientific mathematical texts since the overall development of mathematics is well-known and since one might check the effect of a particular work by, say, citations analysis in later works. Yet there are not only works which have remained unknown for generation, but this view also presupposes a continuous and linear development of mathematics which may unduly underestimate forgotten approaches and traditions in mathematics.

That this basis for textual interpretation is not sufficient is shown more clearly by textbooks: the corresponding measuring standard, the "étalon", would be the corpus of school mathematics and its development. However, the far greater number of contributors makes it even more difficult to evaluate the originality of a contribution than in research mathematics *sensu stricto*. [18] Moreover, one has to admit frankly that one knows very little about the constitution and the development of school mathematics [19] There is therefore no direct access to an immediate internal interpretation of a textbook.

As a consequence, it is necessary to enlarge the interpretation of a text in order to reconstruct its meaning: a first basic rule for such an endeavour is that a text can only be interpreted adequately together with its *context*. And, as an approximation to a reconstruction in its proper conceptual field, one should analyze its *contemporaneous context*. This enlarged methodology of textual interpretation very much resembles the "hermeneutic" methodology established by the eminent German philologists F. A. Wolf and A. Boeckh in the late 18th and the early 19th century for the interpretation of classical literary texts. Remarkably enough, they insisted on a thorough study of Greek economy and politics in order to understand Greek poetry. [cf Schubring, 1986b] In fact, for the genuine understanding of a text, it is not sufficient to undertake "summits": a certain few ingenious authors separated from each other by epochs and countries. One has to reconstitute the whole context of the debates and the conceptions of the contemporaneous authors together with their embeddings in the cultural structures of the time.

Such an enlarged approach to the interpretation of texts seems to be not very far from the usual understanding of "social history" methodology (so that much of the externalism/internalism dispute would become obsolete). Indeed, school textbooks almost inevitably invite one to study the social context in particular — one might even say: the social pressure which is exerted on school knowledge and on the functioning of the system for *transmitting* knowledge. The production of school textbooks is considerably more subject to social and institutional constraints than that of university textbooks or research publications.

To give an example of a particular social dimension to the context: school textbooks are, unlike academic presentations of mathematical theories, often reedited. The number of editions of a schoolbook reflects, together with the modifications of the text, the degree of social acceptance of the transmitted knowledge as well as the social pressures tending to impose changes (in methodology, "metaphysics", content, etc.). Consequently, the number of editions neither expresses the success of the book with its

users nor the intellectual progress of the author. Stated another way, the traditional approach to mathematical historiography, which already shows shortcomings in connection with research texts, is largely insufficient for the study of schoolbook texts.

To draw some conclusion from this methodological discussion, a more *holistic* approach seems to be necessary. To approximate such a holistic approach, I propose a "three-dimensional" scheme for the analysis of an *oeuvre* of historical textbooks:

> — the first dimension consists in analysing the changes within the various editions of one textbook chosen as starting-point, say an algebra textbook or an arithmetic one;
>
> — the next dimension consists in finding corresponding changes in other textbooks belonging to the same *oeuvre,* by studying those parts dealing with related conceptual fields, say geometrical algebra, trigonometry, etc ,
>
> — the third dimension relates the changes in the textbooks to changes in the context: changes in the syllabus, ministerial decrees, didactical debates, evolution of mathematics, changes in epistemology, etc

A first example of such a three-dimensional investigation is a study of the history of negative numbers in France and Germany between 1750 and 1850 [Schubring, 1986a]. Remarkable changes in the conception and presentation of negative numbers can be put in evidence by analyzing textbooks of arithmetic, algebra, trigonometry, geometrical algebra, etc., and relating them to mathematical and epistemological changes. [20] A manifest and persistent change was brought about by a decisive epistemological rupture: a return to an anti-abstract, "substantialist" view of mathematics, most effectively expressed by L. Carnot in his seminal *Géométrie de position"* of 1803. [21]

Patterns of textbook analysis

In this last section, I want to elaborate on several patterns of textbook production which are pertinent to a systematic analysis within the three-dimensional scheme, notably in the context-dimension.

The textbook author

The first pattern concerns the complex *role of the textbook author*. One can say that the name(s) on the front pages of a textbook almost never represent(s) the only author(s), but that such names stand for a "collectivity" of authors. This collectivity is a consequence of the fact that, at least since the end of the 18th century, textbooks are tied to an institutional context and are thus molded by the constraints of and the social demands of the respective institution, through its syllabus, its typology of knowledge, and its tradition. One should therefore consider the institution as part of the collectivity of authors [22] The weight of these collective and institutional factors is also indicated by the numerous textbooks which have been published *without* indications of authorship

This leads to another differentiation of the role of textbook authors The collectivity of authorship is also shown in the fact that a textbook is in general molded in contents and structure by the already existing textbooks for the particular institution, and by frequent "borrowings" from other books, or even by direct copying.

The common and the private

This is an expression of a particularly remarkable pattern: school knowledge is regarded, unlike research knowledge, as a sort of "*common" property*. Copyright regulations and respect for the rights of authors find practically no application in this field of publication. The overall acceptance of the community-character of textbooks is underlined by the rare exception. One such exception has been provided by Lacroix who frankly avowed his borrowings In the first edition [1797] of his *Traité élémentaire d'arithmétique* he explained clearly that the book is "to a considerable degree the work of Citizen Biot, mathematics teacher at the *Ecole Centrale* of the *département de l'Oise*" [Lacroix, 1797, v. I. xj]. He did the same in the second edition of 1800 (but no more, however, in later editions). Analogously, Lacroix avowed in the first edition of his algebra textbook:

> Urged by the shortness of time which does not allow me to completely compose a treatise of algebra in the time that remains before I need it, . . . I have completed the notes and additions which I had inserted into the fifth edition [of Clairaut's algebra, G. S] by new articles or by pieces selected from Bézout's algebra, and I have done this in such a manner that a coherent whole has emerged. . . . All that has been taken from Bézout's algebra has been put between brackets. [23]

Indeed, the parts copied from Bézout have been enclosed in angular brackets. Summing up the parts (borrowed only for "completion") marked by brackets I arrived at a total of more than three-quarters of the entire book! The second edition of 1800 underlines the "common property" pattern in an intriguing formulation:

> [In the first edition] I borrowed from the third part of Bézout's textbook series some articles which deal only with details of operations and which are common to all books and all methods. I did so in order to fill the gaps which were left between the notes and the complements by which I had accompanied my edition of Clairaut and in which the most subtle aspects of algebra were discussed. [24]

Lacroix mentions that he had revised these almost trivial parts less thoroughly than the subtle ones and adds that he will from now on omit the distinguishing brackets.

This common-property pattern inherent in textbook knowledge leads to the serious difficulty of clearly identifying what has been the original contribution of the textbook author to his product Personal declarations of originality are not at all trustworthy, and the same holds true for the references which an author may quote (if any) To disclose the originality of a particular textbook one has to compare it with traditional and with contemporaneous productions

— evidently not an easy but a terribly large-scale and laborious task.

Market strategies

The common-property character leads to a further important pattern: the appropriation of "common knowledge" is determined by an interest in private profit and in securing markets and spheres of influence. It is evident that the *market strategies* of authors and their effect on textbook compositions have to be considered as an essential, though hitherto neglected, element of context analysis.

The historical ways in which market mechanisms have functioned need special attention, particularly since the period just after the turn of the 19th century was in many countries the time when a free market for schoolbook production first emerged. One can already identify two dimensions by which the functioning of market mechanisms can be investigated. The first dimension concerns the competitive behavior of different authors and the second one concerns the relation between the author and his editor on the one side and the respective Ministry of Instruction or the state administration.

With regard to the first dimension, *direct competition,* it is clear that is produces detectable conflicts more explicitly in central administered states than in decentralised ones. Indeed, an author in one of the great number of different German states could hope much more realistically to find a place on the market than an author could do in France. I can illustrate the intensity of personal and direct competition by a remarkable conflict between Legendre and Lacroix which can be reconstructed from their correspondence. Both competed as textbook authors for the same market, the *Ecoles Centrales,* in 1799, when the teachers of these schools were still entitled to a free choice of their textbooks (until 1803).

Legendre had learned that Lacroix was going to publish a geometry textbook. He was seriously concerned that his own geometry book, until then the only modern French text, would be threatened by a dangerous competitor. Legendre therefore asked Lacroix for a meeting and urged him to resign from the publication of the projected geometry book. Lacroix ceded and promised to resign. However, this was not to be his last word in this affair. Three days later, Lacroix's editor Duprat (also the editor of many other important mathematical texts) went to see Legendre: Lacroix had not only withdrawn the geometry text but also his books on arithmetic and algebra. Duprat therefore complained to Legendre of the economic losses that would result from the dropping of three works. Legendre reacted in a letter of 16.2.1799 to Lacroix and again developed the plan which was destined to sweeten Lacroix's resignation: Lacroix should continue, as a teacher at an *Ecole Centrale,* to use Legendre's geometry textbook, but he should be free to "supplement orally" this text and "to follow up the text by other works composed by you" As examples of such follow-up textbooks, Legendre mentions trigonometry, arithmetic and algebra "on which I have never intended to write". [25] But since "the sacrifice which you have made in my favor" (namely to retain Legendre's geometry as textbook at the *Ecole Centrale* now causes too many problems, due to pressure from Duprat, Legendre now desists from solving problems of competition by monopolization:

> But since this sacrifice costs you too much, and since it is too expensive for Citizen Duprat, and since it is better that among three concerned persons only one is sacrificed instead of two, I gladly agree that I am the one to be sacrificed. . . . I therefore regard the promise which you gave me three days ago as not having occurred and give you liberty to publish your own geometry. [26]

Lacroix would soon afterwards have the satisfaction of knowing that his freshly published geometry was immediately preferred by some teachers to Legendre's geometry In the 9 Germinal (30.3.1800), Biot informed him, directly after receiving the new book and working it through, that he replaced by this, for the rest of the course, the hitherto used Legendre text, explaining his choice by a comparative evaluation:

> I avow that (before reading your book) I knew nothing better than Legendre's geometry though I knew that many things are missing and in particular that analytical way which prepares the intellect for research into the truth and which gives him the instrument unceasingly needed for discoveries. Your book seemed to me to unify those two subjects which one must demand of each elementary textbook, i.e. the rigour of demonstrations and the indication of the way which one has to follow for the solution of problems. This last point was missing in Legendre's geometry. [27]

With regard to the second dimension of market strategy, the *relation between author, editor and state administration,* one can suppose that there is in general a tendency in each instructional administration, given by "natural logic", to control all activities inside its range of competence, and that this tendency will converge with the desire of many an author to attain a monopoly for his textbook. This convergence might even coincide with the desire of an editor to dominate the market, but it will be in conflict with the interests of the totality of editors who want free access to the market In fact, I note that in several German states, pressure from groups of editors has effected a withdrawal of the state from monopolizing or regulating the schoolbook market (for instance, in Bavaria)

For the purpose of our analysis, it is particularly intriguing to investigate the convergence between authors and state administrations since one notices again and again the too simplistic strategy of "innovation": the implementation of reforms or improvements (that textbook authors claim to have attained) by means of administrative decrees. An example of this behavior is Bavaria where the famous philologist F. Thiersch succeeded in 1831 in establishing a monopoly of one schoolbook per discipline) to ensure the widest dissemination for his book) by using an institution for publishing the books which had been founded by the Jesuits in 1614 for disseminating anti-rationalist pam-

phlets. Another example is provided by the mathematician A. L. Crelle in Prussia who in 1829 tried to impose a single textbook, intended to be the best, on all *Gymnasien* as a means of solving every problem of mathematics education. [28]

Textbook and teacher

One of the most important patterns in the functioning of textbooks is the *relation between textbook and teacher* in the respective educational system. Yet though educational theory says that the teacher plays the key role in instructional and methodological reforms in schools, it seems that historical studies of textbooks tend to neglect the interaction between teacher and textbook. This neglect is the more regrettable since the mutual functions of teacher and textbook in the instructional process constitute almost the nerve center of educational policy. Obviously the interaction of these two central forces touches on a zone of conflict which would merit better understanding.

For instance, if one studies the establishment of the first systems of general and public instruction, in France and in Germany, one notices an opposition between an emphasis on teacher education and an emphasis on textbook production. Whereas French policy is characterized, since 1763, by the dominance of textbooks — first as a means of supplying (missing) good teachers, and later for assuring control and imposing uniformity — Prussian policy consisted in following the ideas of neohumanism, conceiving of the teacher (in secondary schools) as a scholar and attributing to him autonomy in methods and in the choice of textbook. [29]

Elementarization and method

Another highly important pattern in the investigation of textbooks is the *process of elementarization,* or, to put it another way, the transposition of knowledge into teachable knowledge and a related method [cf. also Chevallard, 1985] This process was the subject of a unique national effort in the years after the French Revolution, the program to compose *livres élémentaires* It seems that Descartes was the first to use the term "livres élémentaires". On realizing that the learned corporations to whom he had first addressed his activities (in Latin) were not capable of changing themselves, he later addressed a broader audience in French D'Alembert also asked for *livres élémentaires* as a necessary basis for education. The Enlightenment philosophers, however, understood elementarisation as a problem exclusively within the internal logic of a science: in the article "Eléments" of the famous *Encyclopédie, element* was defined as the first link in a logical, deductive chain.

> All would be reduced to this first proposition which one could regard as the *element* of the respective science since this science would be contained entirely in it.

This conception entailed no didactical problem of its own; only practical reasons spoke against such a logical reduction to basic elements.

> Followed strictly, this rule would reduce textbooks to almost nothing, which would render their use and application too difficult. [30]

The first *concours* of 1794 for the composition of *livres élémentaires* followed this undifferentiated view of methodology: the allowed time of only five months shows a conviction that it would be relatively easy task. Moreover, eminent scientists were in mind as the likely authors

Among the older French elementary textbooks only those by A. C. Clairaut presented a methodological discussion and a reflective organisation and were not just sedimentations of various traditions of knowledge. Lacroix, who was the person responsible in the adminstration of instruction for the evaluation of the texts submitted for the *concours,* repeatedly expressed his regret that Clairaut's texts on geometry and algebra could not win. Lacroix was basically sympathetic to Clairaut's claim that a textbook should follow the path of the discoverers This claim concorded neatly with Condillac's [31] proposition:

> The best method to teach others is to lead them along the same way which one had to follow in order to teach oneself. [Condillac, 1977, p 308; my transl.: G.S.]

On the other hand, Lacroix had to criticize the methodological approach because of his rich teaching experience: regarding the algebra book, Lacroix remarked that Clairaut had shed new light on the principles of algebra but that he had extended the path of discovery beyond its natural limits. Lacroix accepted this path as necessary for the encouragement of those who began the study of algebra but regarded the approach as too pedantic and too overcharged with details if pursued beyond its first elements. The latter parts of Clairaut's algebra were not in general liked since the basic rules were not clearly presented but only developed through examples. Learners had therefore not been able to apply the rules on their own Lacroix contrasted his own approach to that of Clairaut as follows:

> I [Lacroix] have therefore not hesitated [in changing from my edition of Clairaut to the first edition of my own algebra] to convince myself that it is necessary to restrict the path of discovery considerably and that then — when the student has overcome the first obstacles, when he has understood the objective of his science, when applications have convinced him of the utility of his effort — one must in order to encourage the student to continue, present the subjects to him in just that sequence by which one logically emerges from the other I have therefore thought that I ought to follow the way of discovery only when introducing the *elements*. [32]

And Lacroix remarked of Clairaut's geometry book that the lack of rigor in its demonstrations and the small range of its treated subjects had prevented its appreciation in the *concours* of 1794/95.

In fact, the problems of establishing satisfying elementary treatises led to new and thorough reflections on the process of elementarisation. One stepped back from Clai-

raut's purely pedagogizing approach as well as from the deductive conception of elements. The criticism of the submitted texts, explained by a spokesmen of the *concours* jury in 1795, shows its awareness of a problematic in its own right:

> [The submitted texts] have in general confounded two entirely different objectives, namely: elementary books and abridged treatises. To shorten and condense a voluminous work is to write an abridged version. However, presenting the first germs of a science and exposing its matrix in a certain manner, that means elementarizing. In other words, an abridged version is the exact opposite of an elementary book. [33]

From these experiences it followed, too, that the composition of textbooks is entirely different from research work and that both types cannot, at least in general, be successfully realized by the same persons. Lacroix therefore emphasized, in 1798, what is now generally acknowledged, that the composition of good elementary books is one of the most complicated tasks. And Destutt de Tracy, a then leading philosopher, influential in educational policy, underlined in 1801 that the work of elementarization can contribute to the progress of science since gaps and missing connections in the foundations are frequently discovered. [cf. Schubring, 1982, p. 114].

Textbook forms

The *form of textbooks* is a further pattern which has up to now not been studied historically. One knows that at least the following different forms exist: the long-form "Kompendium" (aiming at exhaustive presentation), the short-form "Leitfaden" (a concise guide), collections of examples, separate editions for teachers and for students. However, very little is known about the emergence of these different forms, about the drastic reduction in books addressing self-educated learners (widespread throughout the 18th century), and about the differentiation between books for teachers and books for students. Evidently, it would lead to a much deeper understanding of classroom practice if we knew more about the typical situations in which they were used and how these situations differed according to the types of textbooks and to the differing functions of teachers and textbooks.

As far as I know, the first differentiation between teacher's and students' editions occurred in the wake of the French Revolution: the first explicit mention is made in the famous education plan by Condorcet of 1792 which recommends separate methodological books for teachers. In fact, the *concours* of 1794 for the composition of *livres élémentaires* suggested the task of writing methodological advice for teachers. The only mathematical text judged to merit a distinction in this *concours* was a textbok of arithmetic: it was the first to be published in two volumes, one book for students and an accompanying methodological commentary for teachers. [34] Lacroix did not publish any methodological commentaries on his various textbooks; however he did publish an entire volume of general methodological discussions on school mathematics [Lacroix, 1805]. This book had wide dissemination and went into new editions throughout the first half of the 19th century [1816, 1828, 1838].

Effects of textbooks

A last, but indispensable pattern of textbook studies, is the investigation of the *effect of a textbook oeuvre*. A study of historical textbooks would remain quite unsatisfying if one could make no statements about their impact on school mathematics, on method, on classroom practice, on the teacher's use of them. Here, however, little research has been done and it is not at all clear how one can operationalize (or "measure") the effect of textbooks. In a central administered state like France the situation is somewhat less complex since one can say (at least up to the 1860s) which books have been approved and one can find out from the files of the central commission which books were refused. As a further "measure" of effect, Dhombres has proposed a comparison of the respective circulation numbers per edition. Yet there remains the problem of how to evaluate the impact of textbooks on the methodological thinking of teachers.

In the case of Lacroix we are lucky enough to be able to dispose of yet another indicator of the exceptional success and impact of his textbook *oeuvre*: translations into other languages. For instance, the great majority of his works have been translated into German; of some books there are even several translations (of the different editions). Many of his books have also been translated (and even adapted) into English [35] though the relatively small community of those in Britain who were actively interested in mathematics somewhat limited their impact. Charles Babbage, for instance, who with his friends Peacock and Herschel had published in 1816 a translation of Lacroix's abridged version of the differential and integral calculus, regretted in 1820 in a letter to Lacroix that the conditions in Britain did not allow a translation of the complete three-volume edition:

> I very much wish we had a good translation of your large work [. . .] it is indeed invaluable, and I am convinced that nothing would so much increase the progress of mathematical knowledge on this side of the water as such a work in our language. [36]

Notes

[1] The student is only apparently the main addressee of a schoolbook. Actually, at least since the end of the 18th century — i.e. since the emergence of a general school system— the teacher is addressed as well as the student or is even the main addressee. This new triple relationship between schoolbook, teacher and student will be touched on below.

[2] The development of school mathematics in England was somewhat delayed relative to the Continent, cf. Howson, 1984.

[3] Cf. Schubring, 1986c. An evaluation of a relatively complete bibliography has yielded a number of at least 1,534 schoolbooks published in the various German states between 1775 and 1829 (omitting new editions)! A bibliography of German mathematical schoolbooks is in preparation by P. Damerow and G. Hentschke. A highly valuable investigation into French textbook production

between 1775 and 1825 has been published in an article by J Dhombres [1985a]. The *Service d'Histoire d'Education* (Paris) is establishing a databank (Emmanuelle) with the titles of all French schoolbooks from 1789 until today [cf. Choppin, 1980, 1982] A good impression of schoolbook production in the USA between 1775 and 1900 is given by the Catalog of titles collected by federal agencies since the 1860s [Svobodny, 1985] This Catalog should be compared with the Plimpton Collection of old American schoolbooks (Butler Library, Columbia University).

[4] Up to about the 1830s this restrictive system of admission resulted in a remarkably low production of new schoolbooks — as one can see from the list of books forwarded to the Commission or the Ministry (AN, F^{17} Nr. 1559 and 1560), whereas from the early 1830s onwards the production increased enormously — particularly for languages, but also for mathematics — so that a considerable clash arose between the number of schoolbooks produced and approved (See the list of schoolbooks approved in France between 1802 and 1850: *Liste chronologique* , 1851 This list has to be used with care since it omits to say if the books were approved for use in schools or for deposit in school libraries or for award as school prizes.)

[5] In subsequent generations of French schoolbook authors such univerality no longer occurs. Authors rather confine themselves to two or at most three branches or subdisciplines of mathematics. Textbooks addressing themselves to the entire range of say. secondary education only reappear at the end of the 19th century — but now in the form of a series of titles by individual authors, united by one (or more) principal authors: "*sous la direction de M.* . . " (for preference a school inspector or a university professor)
Indeed, this change seems to be due to a process of specialization, and the universalistic *oeuvre* of Lacroix, Reynaud, Bourdon seems therefore to be a transposition of the German encyclopedic models of the eighteenth century: the textbook series by Christian Wolff, A. G Kästner, et al., embracing all mathematical disciplines and addressing themselves to secondary and to higher education In fact, in the Germany of the first half of the 19th century, I know of only one author with a comparably broad *oeuvre* Martin Ohm (1792-1872) who can in several respects be seen as following in the tradition of the 18th century [cf Schubring, 1981]. It would be interesting, therefore, to compare Lacroix and M Ohm as textbook authors Though a study on M. Ohm as textbook author has been undertaken recently [Bekemeier, 1984], it is to regretted that it can hardly be used for such a comparison: the study restricts itself to an internal analysis of just two books out of the entire series. And though M. Ohm published several adaptations of his system for different audiences, no analysis has been undertaken to relate the differences in content to social views of knowledge.

[6] See Schubring, 1981, 1982, 1984

[7] The evaluation process took more than one year In mathematics, for instance, none of the submitted texts was found to be entirely suitable An arithmetic text received a prize as the best Legendre's geometry book got an honorable mention

[8] No trace of this first edition is found in the traditional bibliographies. However, an inspection of the second edition (1800) reveals that the first edition constituted a part of the fifth edition of Clairaut's algebra, published by Lacroix in 1797 in two volumes

[9] *Recueil de lois* . . , tome 2, 1814, p 390

[10] This list was published by the editor of the second edition of the third volume of Lacroix's *Traité de calcul différentiel et de calcul intégral* (1819) on the page opposite the title page.

[11] Cf. the notion of "conceptual field," see Vergnaud [1981].

[12] Lacroix alludes here to the books by L. Euler, 1768-70, J. A. J Cousin, 1777.

[13] "Les livres élémentaires les plus complets, le Calcul Integral d'Euler, celui de M. Cousin ont besoin d'addition et peut être même pour mettre plus d'ensemble dans les parties, faudrait il un changement dans la manière de les présenter" Letter of Lacroix to Legendre, November 9, 1789. Bibliothèque de l'Institut (Paris), Papiers de Lacroix, mss 23296.

[14] Cf the excellent study on the Idéologues by Moravia [1968]; also Gusdorf [1978]

[15] "L'auteur d un livre élémentaire se met au range des inventeurs, lorsqu'il dispose, le premier, les élémens dans l'ordre le plus convenable, le plus simple, le plus clair; lorsqu'il débarasse la science de tout échafaudage technique; lorsqu'après chaque pas il montre l'espace parcouru, de manière que l'élève sache toujours bien où il est". In: *La Décade Philosophique, littéraire et politique*, an 4, Vol 9: No. 78, 30 Prairial (18 6 1796) p. 517.

[16] "Présenter avec clarté des théories difficiles, les lier avec d autres théories connues, dépouiller quelques unes de la partie systématique ou erronée, dont elles ont pu être obscurcies à leur naissance, répandre sur le tout un égal degré de lumière et de précision, en un mot, faire un ouvrage qui soit à la fois élémentaire et à la hauteur de la science, tel est le but que s'est proposé le Cn [citoyen] Lacroix, et qu'il n a pu remplir sans s'engager dans de profondes recherches et marcher souvent de front avec les inventeurs" In : Séance du 11 nivôse an V (31 décembre 1796), *Procès-verbaux des Séances de l'Académie*, tenues depuis la fondation de l'Institut jusqu'au mois d'août 1835 Tome I, Hendaye, 1910, p 155

[17] "Die vorliegende Darstellung der Zahlenlehre oder Arithmetik macht den Anspruch, die erste streng wissenschaftliche und zugleich ganz elementare Darstellung der Zahlenlehre zu sein Sehen wir von den Arbeiten der Gebrüer Grassmann in Stettin und des Prof. Schroeder in Karlsruhe ab (. Hermann Grassmann, 1861, Robert Grassmann, 1872, Schroeder, 1872), so bieten sämmtliche andere Darstellungen der Zahlenlehre in ihren grundlegenden Abschnitten bei ihren sogenannten Beweisen die bedenklichsten Zirkelschlüsse und Trugschlüsse, welche nichts beweisen und nur geeignet sind, die Leser an unwissenschaftliches Denken zu gewöhnen und zu verwirren" (R Grassmann, 1900, p. iii)

[18] The underlying assumption is that each relevant social group constitutes its own corpus of knowledge, associating it with specific mental characteristics and epistemologies. In our case, the group of mathematics teachers has become — following its emergence as a particular professional group — the support system for the corpus of school mathematics (differentiated within itself according to the different social groups of mathematics teachers: say, primary, secondary, and vocational school mathematics) Since school mathematics functions as a kind of mediator between research mathematics and everyday knowledge in society, it can serve as an analytical tool for studying the relation between mathematics and society at large

[19] There is the highly valuable analysis of the development of elementary mathematics by Tropfke [1903, 1980] However, given the enormous dimensions of this field it cannot really go into details, particularly not of the different national developments of school mathematics And even for, say, German school mathematics, there is no reliable account of its structure and development

[20] For lack of space, the related changes in the organisation of mathematics instruction — which were particularly numerous and drastic in France between 1795 and 1845 — could not be discussed explicitly in that article

[21] By an analogous analysis, one can relate the revision of the first two volumes of M. Ohm's *System* [1822] for the second edition [1829] to a Prussian ministerial decree of 1826. M Ohm's second edition is no longer so radically axiomatic and formal and is written in much more popular terms The decree of 1826 banned arithmetic from the lower grades of the *Gymnasium* and restricted proper mathematics instruction to the middle and upper grades, so the incentive to construct an integrated mathematics course, developing "logically" from arithmetical principles, had vanished.
Generally speaking, textbook authors have to be somewhat "opportunistic" and follow the "laws of the market", i e obey curricular and methodological changes. Lacroix, however, proved to be remarkably resistant to the temptations of opportunism — particularly on a political level: he never abjured the philosophy of the Enlightenment and of Condillac, though these had fallen under a shadow since the time of Napolean and especially after the restoration of 1815. Lacroix even explained in an additional note to the third edition of his *Essais* . . his defence of this philosophy. [Lacroix, 1828, p 346 sqq. and 351 sqq]. And he did not hesitate to criticize Laplace publicly for his political opportunism in changing certain philosophical statements in his textbooks. [Taton, 1953b, p 357-360]

[22] Even with Cauchy's famous textbook *Cours d'Analyse* one can

show that the Ecole Polytechnique functioned as institutional author, cf. Schubring, 1986b, 84-85

[23] "Pressé par le temps qui ne me permet pas d'écrire en entier un Traité d'Algèbre d'ici à l'époque où j'en aurai besoin, . . ., j'ai complété, soit par des articles nouveaux soit par des morceaux extraits de l'Algèbre de Bézout et de manière de former un seul tout, les notes et les additions que j'avais insérées dans la 5e édition du premier de ces ouvrages [Clairaut] . . . On a renfermé entre des crochets tout ce qui est tiré de l'Algèbre de Bézout". [Lacroix, 1799, p. 1-2]

[24] Dans la premiére édition, "j empruntai, pour remplir des lacunes qui laissoient les notes et les additions que j'avois faites à cet ouvrage Clairaut, et dans lesquelles se trouvoient dévelopés et discutés les points les plus délicats de l'Algèbre, quelques articles de la troisième partie du Cours de Bézout, qui n'avoient pour objet que des détails d'opérations communs à tous les livres et à toutes les méthodes " [Lacroix, 1800, p. XV.]

[25] In the first edition of his geometry, in 1794, Legendre had excluded trigonometry in order to preserve the purity of method. In later editions, however, he added a section of trigonometry

[26] "Mais puisque ce sacrifice vous coute trop, qu'il est trop onéreux au Citoyen Duprat; puisque de trois victimes il vaut mieux qu'il y en ait une de sacrifiée que deux, je consens volontiers à être cette victime; . Je regarde donc la parole que vous m'avez donnée il y a trois jours, comme non avenue, et je vous laisse toute liberté de publier votre géométrie" In: Bibliothèque de l'Institut (Paris), Papiers de Lacroix, mss. 2396: Letter by Legendre to Lacroix, 27 pluviôse an VII

[27] "J'avoue (qu'avant de vous avoir lu) que je ne connais rien de mieux que la Géométrie de Legendre, quoique je savois bien qu'il y manquait beaucoup de çhoses, et sur tout un peu de cette marche analytique qui prépare l'esprit à la recherche de la vérité et qui lui donne l'instrument de découverte dont il a sans cesse besoin. Votre livre m'a semblé réunir les deux objets que l'on doit se proposer dans tout livre élémentaire, savoir la rigueur [de] démonstrations, et l'indication de la marche à suivre dans la résolution des problèmes Ce dernier point manquait dans la Géométrie de Le Gendre." Letter of J. B. Biot to S. F. Lacroix, 9. Germinal, In: D. E. Smith Historical Collection, Columbia University Libraries: Butler Library.

[28] Both examples are discussed in more detail in Schubring [1985] and [1988].

[29] The historical opposition between teacher education and textbooks is exposed in more detail in Schubring [1984] (for France), and Schubring [1983] (for Prussia)

[30] "tout se réduiroit par conséquent à cette premiere proposition, qu'on pourroit regarder comme l'*élément* de la science dont il s'agit, puisque cette science y seroit entièrement renfermée . Mais en s'attachant scrupuleusement à cette règle, non-seulement on réduiroit les *éléments* à presque rien, on en rendroit l'usage et l'application trop difficiles " [quoted from Schubring, 1982, p. 113]

[31] Condillac was accepted as a philosophical authority by Lacroix and, in general, by those who were active in educational policy after the French Revolution (up to about 1800).

[32] "Je ne tardais pas à me convaincre par moi-même qu'il étoit nécessaire de resserrer beaucoup la marche d'invention, et que lorsque l'élève a passé des premières difficultés, qu'il a aperçu le but de la science, que des applications l'ont convaincu de l'utilité de son travail, il ne faut plus, pour l'engager à continuer, que lui présenter les matières dans l'ordre où elles naissent les unes des autres
Je crus donc devoir ne m'astreindre à la marche d'invention que pour en faire l'introduction des Elémens " [Lacroix, 1800, ix]

[33] "avaient généralement confondu deux objets trés différents, des *élémentaires* avec des *abrégées*. Resserrer, contracter un long ouvrage c'est l'abréger; présenter les premiers germes et en quelque sorte la matrice d'une science, c'est l'élémenter: ainsi, l'abrègè c'est précisément l'opposé de l'élémentaire." [quoted from Schubring, 1982 p 112]

[34] The author was J. B. Sarret; see Schubring [1984, p 364] Both volumes were published in 1798

[35] There are also Italian, Spanish and Dutch translations.

[36] Letter of Charles Babbage to Lacroix, 28 11.1820 Bibliothèque de l'Institut (Paris), Papiers de Lacroix, mss. 2396.

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