# In Fostering Communities Of Inquiry, Must It Matter That The Teacher Knows "The Answer"?*

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I would like to begin with a note of thanks to Sophie Haroutunian-Gordon for an ongoing conversation that has led me to compare and contrast the nature and implications of the teaching practices in the two most important aspects of my professional life, my undergraduate courses in mathematical problem solving and my graduate research group in mathematics education In a recent paper, Haroutunian-Gordon and Tartakoff [1996] discuss an approach to instruction called "interpretive discussion," in which students and instructors work together to develop and address questions to which neither knows the answers. Sharing in such explorations was clearly an enriching experience for the students and their instructors As Haroutunian-Gordon and Tartakoff note, one of my instructional aims is to have the students in my problem solving course function as a community of problem solvers, posing and working on problems in ways that can be considered similar to the ways mathematicians do. When we discussed our courses, Sophie raised the following question:

> Is there a fundamental difference in the character or results of instruction when the teacher is a co-explorer with students, covering new ground for him- or her-self, as opposed to an "expert" traversing very familiar territory?

As it happens, the instruction in my problem solving courses and in my research group provide a nice comparative base for addressing the question. Doing so is the focus of this paper

I first offered a course in mathematical problem solving about twenty years ago The course, which has long served as a laboratory for my thinking about mathematical problem solving, has been under development ever since In the beginning I had some ideas about competent mathematical practice, and I tried to design instruction that would, on the basis of those theoretical notions, help my students become better problem solvers. The attempt was half-baked, of course — some theoretical notions seemed reasonable, but just didn't feel right in practice. And, I found myself doing things as a teacher that hadn't yet found a place in my abstract conceptualization of the course. Hence each empirical experience provided grounds for reflection, for a new round of theory, and in consequence, for a new version of the course Over the past twenty years, the course and those theoretical notions have evolved in dialectic with each other. Since the course is highly interactive and depends on the students, it is different each time I teach it — but it is also as familiar as the back of my hand. Not only do I know most of the mathematics involved (the exception being when we work on new problems, or take off in new directions, which do happen), but I am so familiar with student work on the problems used in the course that I can often predict what my students will do before they do it My goals — to create a "community of inquiry" in which my students learn to *do* mathematics, and to have them emerge from the course with a different sense of the enterprise as well as improved problem-solving skills — are achieved through a carefully designed set of activities over which I have a great deal of control. Indeed, one would be within one's rights to raise an eyebrow at the phrase "community of inquiry" — after all, I'm on familiar ground most of the time.

My research group at Berkeley is slightly more than a decade old Its developmental path has been quite different, as is its modus operandi. In the research group we do "real work," whether for me or my (master's and doctoral) students. Broadly speaking, we are interested in understanding the nature of mathematical thinking, teaching, and learning. Our work is wide-ranging, and has included projects as diverse as "building a model of the teaching process," "trying to understand transfer," and "exploring the roles of cultural, school, and personal factors in the shaping of individuals' mathematical identities " Some of the work, for example the work on teacher modeling, is driven by my agenda Much of it, for example the work on transfer or mathematical identity (to mention just two) is driven by my students' interests Either way, I am frequently at the limits of my competence. In my own work, I tend to explore widely — I have the most fun when I grapple with problems that are really challenging and just perhaps doable As I explain below, the group joins me on those excursions — and the phrase "the blind leading the blind" would not be too inaccurate a description of what takes place Our forays into the problem space are largely unstructured; we go where it seems reasonable to go, and see what we can find. Nor am I usually on familiar turf when we explore the students' interests. Many of our group sessions are at the students' request, and consist of mutual explorations of an issue: the student is having trouble formulating a problem, has a snippet of data (say a segment of videotape) to share and discuss, or a tentative explanation or proposal to try out on us. While I have a fair amount of experience, the issues raised by these students just don't have routine answers — or if they do, I rarely know them I'm legitimately co-exploring with my students. In that sense, we *are* a community of inquiry. My goals, of course, include having students emerge from it with a particular sense of the enterprise and a set of finally tuned problem-solving (i.e., research) skills.

In sum, these two aspects of my life take place in radically different environments, and yet they have remarkably similar goals. What they share is that in both contexts I want my students to emerge as skilled practitioners of the discipline — as people whose epistemological stance is compatible with that of the field (or at least my view of it), and who engage in the discipline in ways that are compatible with that understanding. Yet there are fundamental differences (above and beyond the obvious difference in scale) between the two environments. There is essentially no artifice in my research group, in that my students and I are engaged in joint work, and their development has the character of an apprenticeship into a particular kind of intellectual community. In contrast, one might argue that all is artifice in my problem solving course. The environment is deliberately structured by me to have particular characteristics, and the students know it; I am clearly "in control" in a way that differs from, but is no less strong than in standard courses (and that differs substantially from my role in the research group); and even if the students don't know where they're going, I usually do. Modulo the difference in scale, could these two radically different experiences have outcomes that are similar in kind? And if so, why?

I shall argue that the answer to the first question is yes: despite the radical differences in environments, the outcomes can be similar in kind. The reason I believe, is that some of the felt experience is the same from the students' point of view; in a particular kind of environment, the acknowledged artifice can become irrelevant, as can the question of whether the instructor knows "the answer."

Given the constraints, the evidence I offer here will be largely anecdotal (although there is ample documentation to back it up) and the conclusions will be largely suggestive. My intention is to provide a few examples that characterize the flavor of the two environments and that point to similarities in outcomes, and then to speculate on why such similarities can exist.

## The research group

I begin with a cursory description of the research group, because it is, in some ways, simpler to understand — the model is well known. The bare bones were described above. We meet regularly, in two kinds of meetings: to pursue a particular thematic agenda (currently "building a model of teaching"), and to pursue lines of work in which my students are engaged (how student representations of mathematical concepts develop and evolve, the nature of teachers' "content knowledge," etc.) The group is a place for "real work" and reflection on it: at various times all of the major aspects of research (planning, data gathering, mulling over data, revising, reflecting, writing) get carried out within it.

There are, of course, cycles of personnel, corresponding to the academic year: new members join the group, and those who finish their degrees move on. At the beginning of the year there are introductions to what we do, and then we get down to business. The business is. ..whatever we happen to be working on.

I have a favorite story about the nature of our work, and one student's evolving sense of how we do it. When this student entered the group, we were just about to embark on a new series of analyses, in which we tried to understand the nature of tutorial interactions — how and why tutors did what they did when they were working with students. We had a series of videotapes of tutorial interactions, and I hoped to make sense of them. Simply put, I knew what I wanted to accomplish — to develop an analytic perspective that did equal justice to the tutor's interpersonal and subject-matter concerns — but I didn't know how. As I explain in [Schoenfeld, 1992], this is typical of my work: I gather rich data (typically videotapes), mull over them until I have some ideas about what might explain what's happening in them, and then try to construct analytic frameworks to pursue those ideas. The early work is often minimally structured, and many tentative explanations or frameworks wind up on the scrap heap before we come up with ones that seem solid enough to pursue.

The student asked what my research plan was, and I told her I didn't have one: I didn't know the data well enough to say what we were going to do. She insisted that I tell her — surely I knew. I said no, I honestly didn't know where the data were going to take us. She said OK, but grudgingly. Some time later she acknowledged that she thought I was merely withholding my plan, so she could discover it herself.

As it happens, this student became one of the major analysts of our data. She had numerous insights about them, which became part of our analytic framework. Over the two years or so we worked over those data, she saw a series of vague intentions coalesce into a (more or less) coherent research plan, and she participated in the codification of once-vague intuitive notions into a rigorous analytic model.

The next fall a new cohort entered the research group, and I gave my standard introduction to the way we worked. I talked about the way we often mulled over data and tried to make sense of them; that we'd start confused, but ultimately make progress. At that point the student, now a senior member of the group, said "and let me tell you: When Alan says he doesn't know what he's doing, you can really believe him!"

Enough said? Much more could be said, because it took me a decade to figure out how to make such a group truly functional, and to meet the needs of the students — but I hope this anecdote and the paper referenced above serve to convey the spirit of the enterprise. I shall say more about the problem solving course, because it is more non-standard.

## The problem solving course

I have written extensively about the course (see, e.g., my 1985 book *Mathematical problem solving,* and my 1994 article "Reflections on doing and teaching mathematics"), and it has been the object of a fair amount of analysis. In recent years four colleagues [Arcavi, Meira, Smith, and Kessel, 1994] have analyzed the opening "moves" of the course in great detail; aspects of the course have also been examined in my research group, as we have tried to build our model of the teaching process.

The key phrase in the description of my course is that I have tried to craft it to be "a microcosm of selected aspects of mathematical practice." That is, I have tried to create a classroom environment in which the students and I are behaving as though we are a mathematical community — with one member of it significantly more knowledgeable than the others, perhaps, but acting differently in degree rather than in kind The classroom community that results is artificial on the one hand — it is "manufactured," managed, and will disband at the end of the semester. On the other hand, it does function as a real community, in which a particular set of values and perspectives is developed and shared, and in which membership takes on a distinctive character — one's world view evolves as one becomes a member of that community I shall argue that, to the degree that the community is meaningful to the students, the effects of the course are real; also, the more that the community is meaningful to the students, the less the artificiality of its construction is relevant to its effects on them

Let me begin by highlighting the ways in which the course differs from my research group There are, of course, obvious differences of scale and context. To mimic Gertrude Stein, "A course is a course is a course is a course. . ." The problem solving course is a regularly scheduled offering in the mathematics department. It meets for three hours a week, and students receive three credits for it. I assign homework and give tests (though of a non-standard type), and I assign grades And, there is little expectation *a priori* that the course will have much effect on students' lives. Some students take it for fun (there is a "problem solving subculture" in mathematics), some because they want to learn to become better problem solvers, some out of curiosity.

As described above, the research group is often minimally structured; there is no *a priori* sequence of investigations, and we follow what leads seem promising as the research evolves. Importantly, that is the case for me as well as for my students: we have often wound up focusing (for months and years!) on things I could not have predicted And, as mentioned above, I don't know the answers to the problems we're confronting

From my point of view, the problem solving course could not be more different From the beginning of the course to the end, I know where we are going. Not only do I know the mathematics of the problems we are going to work (that is, the material is sufficiently familiar that I can produce solutions to the problems without difficulty), but I know what the students are likely to produce as they work on the problems, and I am prepared for those responses.

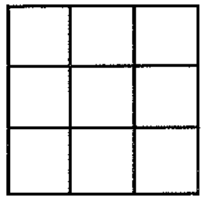
Let me describe the nature of that familiarity, in the context of the classroom structure. Typically I hand out a set of problems. The class breaks into groups of three or four, and the groups work on the problems As they do, I circulate through the classroom, getting a sense of the ways in which they are progressing, and dealing with whatever questions the students raise At some point we reconvene as a "community of the whole" to discuss the problems I ask what students tried, and why. Typically one or more students present their ideas at the board I may ask more questions of the class, prod them to think about the mathematics in different ways (can you solve it another way? are there extensions? generalizations?), or talk about the strategies they used to approach problems — what worked and what didn't If we haven't made sufficient progress on the problem, I suggest that they work on it some more, and that we'll get back to it. If we feel we've gotten closure, I do a wrap-up.

For some problems, especially those at the beginning of the course (where the shaping of the students' experience is especially critical), I can predict almost the complete substance of our interactions. To use the language of our teacher model group, my "lesson image" for a full half-hour discussion of the class is complete down to the level of the questions I am likely to ask, the responses students will produce, the ways in which I will react, and so on.

For example, one of the first problems I have the class work is the "magic square." Place the digits from 1 through 9 in a $3 \times 3$ grid so that the sum of each row, column, and diagonal is the same See [Schoenfeld, 1990] for an extended discussion of the problem.

### The Magic Square Problem

Can you place the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 in the box below, so that when you are all done, the sum of each row, each column, and each diagonal is the same? This is called a magic square.



If you think that the $3 \times 3$ magic square is too easy, here are two alternatives.

(1) Do the "$4 \times 4$" instead of the "$3 \times 3$"

(2) Try to find something interesting to ask about the $3 \times 3$ (This alternative is better. There are lots of things you can ask.)

I know from twenty years' experience of working on the problem that when the class works on it in small groups, all of the groups will arrive at solutions within five to fifteen minutes Almost all of the solutions will be found by trial-and-error A number of the groups will have made guesses about the "magic number," the number that is the sum of each row, column, and diagonal, and one or two groups may have proved it must be 15; a number of groups will have guessed (on the grounds of symmetry) that the number in the center square must be 5, and used that guess as the basis for generating their solution.

I have a standard script for discussing the problem when we convene as a group of the whole I ask for the solutions to the problem, and then discuss ways we might have obtained them Simple as it is, this problem serves as a vehicle for illustrating a range of problem solving strategies. We don't want to plug in numbers at random; that would take too long. Is there some question we could ask, whose answer would help narrow down the possibilities? (This is the formal strategy of "establishing subgoals.") Yes, they say: What's the sum? How could we figure that out? We talk about that, and come up with a way. What else would be useful? The center square, they say. Together we come up with a good guess, and then a proof that the answer is right. We then do a small bit of trial and error, and fill in the square

I then turn to the class and ask OK, are we done? The students' answer is always in the affirmative: yes, we've solved the problem.

"Oh no, we're not," I say. "You've only found one way of solving the problem. There might be others." I then mention the strategy of "working forwards." Since the magic square has three rows, three columns, and two diagonals, all of which must add to the "magic number" 15, we will need to find eight sets of three numbers that add to fifteen. Can they generate them?

Again, I know from long experience that the students will generate them randomly: (4, 5, 6), (1, 8, 6), (3, 5, 7), ... At some point there may be a duplicate, say (6, 8, 1); and they'll run out of steam with either 6, 7, or 8 triples generated. I then ask "Do we have them?" They're not sure, of course; I proceed to answer my own question with "How would you know? You've been completely unsystematic about this!" We then develop a system for generating the triples in order, and proceed from there. There are discussions of extensions and generalizations, and of related mathematics.

I could provide more detail, but I think the point is clear. The atmosphere is open; the students are generating ideas that are fresh to them; and I am working on the spot with whatever they generate. Yet, they know that I've chosen the problems and that I know the "answers." And, in fact, there is a very large likelihood that almost everything that transpires in that discussion will be familiar to me — in the sense that improvisational theater is different but the same every night

## The compare-and-contrast

In the preceding descriptions I have gone out of my way to highlight the differences between the two instructional experiences. Here I shall temper the description of the differences, point out some fundamental similarities, and argue that the import of the similarities far outweighs the import of the differences.

First, I have painted the pictures of "degree of control" and "degree of knowledge" in extremes. In both environments, I'm pretty knowledgeable and have a pretty good sense of where we're going; in both, I exert fairly strong leadership. Ofttimes in the problem solving course we go off into new territory — both mathematical and cognitive. A major difference, however, is that in the research group we all know that we're exploring new ground. In the problem solving class, my guess is that the students have little idea what's new and what's familiar for me. My guess is also that it doesn't matter very much. There are major reasons for this, which I shall explore below: issues of "authority" and the role of the community, and issues of trust

As before, I shall focus more extensively on the problem solving course; the norms for and properties of the research group are more familiar and better understood. In the research group,

a. There is a common understanding that we are all seeking a particular kind of knowledge, and that while some of us may know more than others, "answers" are not generally known in advance

b. The real "authority" is not the Professor — it's a communally accepted standard for the quality of explanations, and our sense of what's right. That is, we all play by the same ground rules, and if a particular explanation seems to make the best sense, it wins — no matter who generated it. Ofttimes that comes from someone other than me — indeed, I may object or have a competing explanation, but the best one takes precedence. (This is oversimplified, but its intention is clear.)

c. There is a feeling of trust, in that we must feel free to have our ideas (and not ourselves) compete. The workings of the intellectual community engender and support this, and support the notion that we are all trying to have our work (individually and collectively) meet the (internally imposed) communal standards described in (b).

The problem solving course is more complex, but I will argue that as the course develops there are very strong parallels. Here is where I proceed largely by telling stories. I have videotapes of the course, and ultimately I would like to offer a careful analytic substantiation.

I offer one prefatory comment. What "counts" are the students' perceptions of the experience, not my version of the reality. In particular, I described above just how much of the course is "choreographed" — by dint of experience I know what students are likely to do, and am prepared for it. But what matters is what the students experience. On the one hand, the students know I've taught the course many times, and expect me to be expert in the content. On the other, I can say with confidence that they have no idea that I could provide detailed descriptions of what they'll do, as I did above. (This semester some students in my research group were genuinely astonished when I told them about my rather complete "lesson image" for the problems, which they had experienced in a problem solving course the previous semester. "I had no idea," one said. "I thought you were just reacting to what we produced on the fly.") I am not duplicitous about this; it just doesn't come up in the problem solving course, and it doesn't seem to matter. What matters is that I'm reacting directly and honestly to what they produce. This relates to the issue of trust, which is pursued below.

[N.B. I can only give a cursory version of the story here. See "reflections on doing and teaching mathematics" [Schoenfeld, 1994] for more detail regarding authority, ownership, and invention.]

### *Authority*

The issue of authority is a fundamental one for mathematics classes. In most classes, students propose things; faculty evaluate and certify them. That is, the faculty, by virtue of their experience, are given the right to say what is right or not; the students suggest ideas in which they may have various degrees of confidence, but ultimately bow to the faculty's judgment. This is not how it works in mathematics. Of course, there are authorities in mathematics (experts in particular areas), but the ultimate authority in mathematics is the mathematics itself — and there are numerous examples in the history of mathematics where someone with a good idea prevailed, despite the suspicion or antagonism of the "experts." Indeed, no potential mathematician survives for very long unless he or she internalizes the standards for mathematical correctness:

you won't do very well unless you know what you have is right, by the appropriate standards. In short, it is the mathematics that speaks, and that says whether something is right or wrong At the level of doing mathematics professionally, the process of refereeing papers submitted to journals can be seen as a communal process of checking (the mathematics is complex, and one can make errors) and valuation (is this important enough to merit publication, even if it's right?), not one of "certification" by experts of work proposed by people who are unsure whether their work is correct. That is very different from the classroom process.

An explicit goal of my course is to deflect inappropriate professorial authority (that of certification) in order that my students learn to internalize the standards of mathematical judgment for themselves Early on, I refuse to tell the students whether something is right or not; I say "don't convince me; convince the class. Do you folks buy it?", and we discuss reasons for believing the claim (or not) During the course, I elaborate on John Mason's framework [see, e.g., Mason, Burton & Stacey, 1982] for developing mathematical arguments: First convince yourself Then convince a friend Finally, convince an enemy. We work on what is compelling and why — that is, what meets mathematical standards of justification. When an argument does, you know it's right — it doesn't have to be "certified."

While this may seem somewhat artificial, it is, in fact, a mirror of both the standards in the mathematical community and those in my research group (where, because our work is now in the social sciences, the standards are more nebulous). And, it pays off. Consider what happened when, toward the end of my problem solving course, I assigned the "concrete wheel" problem given below

**The concrete wheel problem**

You are sitting in a room at ground level, facing a floor-to-ceiling window which is twenty feet square A huge solid concrete wheel, 100 miles in diameter, is rolling down the street and is about to pass right in front of the window, from left to right The center of the wheel is moving to the right at 100 miles per hour. What does the view look like, from inside the room, as the wheel passes by? (See Figure 1 )

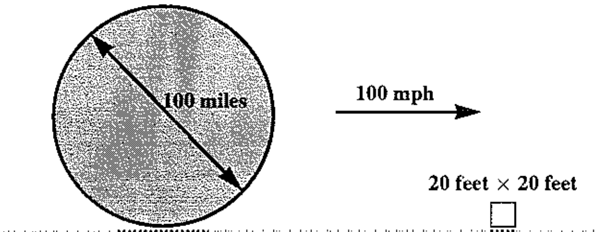



Figure 1
The situation described in the concrete wheel problem

This problem tends to provoke immediate and widely divergent intuitive reactions from students, among them:

a) The room will go (almost) instantaneously dark as the wheel first passes the window. It will stay dark for a short while and go (almost) instantaneously light as the wheel leaves.

b) Same as (a), but the room stays dark for a relatively long time.

e) The room darkens slowly, as though a large window shade was being pulled more or less:

i) horizontally from left to right, as follows;

ii) diagonally from the upper left corner as follows:

iii) or vertically downward as follows:

The room then stays dark for a short/long period of time, after which it lightens in a way complementary to the way it darkened

This time was no exception. The students got engaged in the problem and argued heatedly for some time about the answer There were two opposing camps, and one finally prevailed (on what struck me as good mathematical grounds). When things wound down and I offered to wrap up as I usually do, one student said "Don't bother. We got it " The class concurred.

That is exactly as it should be. The students had internalized the standards of correctness; the mathematics was clear; and there was no need for "certification" from me

***Who knows what, when: Ownership and invention (and trust)***

As indicated above, the course begins with students believing I am an "expert:" I am the faculty member in charge of the course, and they have whatever beliefs students typically have about the faculty member's competence when they enter the course. Indeed, at the onset of the course I deliberately produce various displays of competence to induce them to put themselves in my hands — the course "breaks the rules," and they'll only get something out of it if they believe I'm worth trusting. At the beginning of the course, then, the students may well conceive of me as very knowledgeable; they might, as my research student did, believe I "know all the answers" and that if I'm withholding them, it's as a training device. What happens over the course of the semester is that this issue simply becomes moot. There are times when I know what's going on, times when I'm on very thin ice. There are times when they know it, and times when they don't. And as far as I can tell, it simply doesn't matter — the community has evolved to a point where such issues are of little concern. Here are a few indications.

Once, when we were working on a conjecture related to the Pythagorean theorem (which I know to be false, but which was a quite reasonable conjecture given what the class knew) a student asked "If we can prove this, do we have a publishable theorem?" I temporized, and we continued working Eventually the students discovered that the conjecture — which had indeed been plausible — was false. In this case, the students believed they were "on the edge" of knowledge for the field, and I was a *consultant* rather than a *certifier* In other discussions, the students obtained some results that were new to me — e.g., that there are infinitely many Pythagorean triples (numbers, $A$, $B$, and $C$ with the property that $A^2 + B^2 = C^2$) that have the property, like the (3, 4, 5) and (5, 12, 13) Pythagorean triples, that the largest number (corresponding to the hypotenuse of a right triangle) is one larger than the second largest I told the students afterward that this was a discovery for me. I don't think they suspected that at all during our discussion, and I don't think it mattered — we were simply doing mathematics

One last case in point: one day my students got very excited about a particular topic that arose in the consideration of a problem we were working on, and they pursued it for the better part of an hour As usual, I played the role of facilitator. What the students didn't know is that I have a strong dislike for that area of mathematics — of my own free will, I'd never have explored it!

In sum, I think the community within my problem solving course had evolved to the point where the descriptors of my research group above, which I repeat here:

a. There is a common understanding that we are all seeking a particular kind of knowledge, and that while some of us may know more than others, "answers" are not generally known in advance

b The real "authority" is not the Professor — it's a communally accepted standard for the quality of explanations, and our sense of what's right That is, we all play by the same ground rules, and if a particular explanation seems to make the best sense, it wins — no matter who generated it Ofttimes that comes from someone other than me — indeed, I may object or have a competing explanation, but the best one takes precedence. (This is oversimplified, but its intention is clear.)

e. There is a feeling of trust, in that we must feel free to have our ideas (and not ourselves) compete. The workings of the intellectual community engender and support this, and support the notion that we are all trying to have our work (individually and collectively) meet the (internally imposed) communal standards described in (b)

were either true or moot much of the time.

On (a), we were indeed seekers of knowledge. At times my knowledge was essential, and relied upon — but then, I depend on colleagues' mathematical knowledge too. And at times it didn't matter what I knew: they and possibly I were at the edge of what we knew, and what counted was the mathematics

On (b), I could always exert my professional or individual authority if I wished — but as the concrete wheel problem shows, the power relations in the course changed to where what counted was the correctness of the mathematics, not the strength of my assertions.

On (c), I am convinced that trust is a central factor in what allows the problem solving course to succeed. The course has non-standard goals, and a non-standard modus operandi It will only work if the students are willing to take risks — to become members of a mathematical community where it is OK to make conjectures that turn out to be wrong, to reveal what you know and don't know, and to join in collaboratively in what is often seen as a competitive game.

In short, there are remarkably strong parallels between the two environments — despite the fact that one is "real" and one (by some measures) "artificial." What matters, I think, is that both are communities dedicated to exploration and sense-making, and that properties such as those described immediately above hold true Of course, it has also been my experience that when my students and I are engaged in legitimate explorations, we tend to wind up at times in novel territory — even if we start out on what is very familiar ground.

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