the process of writing the words. Try bearing it in mind when you invite students to solve physics problems requiring ratio and proportion as well as physical ideas. In many cases of mathematical paralysis, some component skill requires full attention, thus derailing the overall plan of attack. In other words, too many elements of the problem are symbolic not enactive. Being only vaguely aware of this, the student remains frozen. By bringing these matters to the student's attention, other options become available.

I end, as usual with a problem. This one seems to me to illustrate pretty fully the qualities of specializing and generalizing and E-I-S transitions.

Reflecting

Denote lines in the plane by lower case letters, $a, b, c, \ldots$, and points by upper case letters $P, Q, R, \ldots$. To each line, say a, there corresponds a transformation of the plane, reflection in that line, which it is convenient to denote by a also. To each point, say P, there corresponds reflection in that point, denoted by P also.

Transformations can be composed. Sometimes the result will be to leave all points of the phase fixed: denote this identity transformation by I. For example,

reflection in a followed by reflection in a again is denoted by aa or a^2 and is always I.

Similarly $ab = I$ if and only if $a = b$, and $PQ = I$ if and only if $P = Q$. Investigate the geometrical meaning of the following statements:

$(PQR)^2 = I$; $(ab)^2 = I$ but $ab = I$;
$(aP)^2 = I$; $(abc)^2 = I$;
$PQRS = I$; and so on.

Investigate how to express symbolically, geometrical statements of the form:

R is the mid point of PQ $(P \neq Q)$;
a and b are perpendicular lines meeting at P;
and so on.

Stuck?

Do you understand the question? Try some specific examples! Have you tried various interpretations for a, b, P and Q? Have you tried special cases when $a = b$, etc?

I wish to express my thanks to the EM235 Course Team: Leone Burton, Nick James, Ann Floyd, Jean Nunn and Tim O'Shea for constituting such a creative environment that some fuzzy notions became clearer and more useful.

Reference

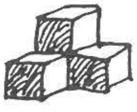
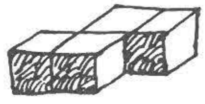
Bruner, J.S., *Towards a Theory of Instruction*. New York: Norton, 1968

Four-Cube Houses

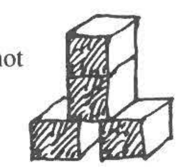
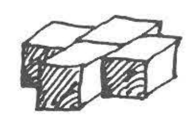
HANS FREUDENTHAL

The following is a summary of an experiment undertaken by E.J. Wijdeveld in a third grade class. It has been published in full*, with 38 colour pictures, taken in the classroom, didactical remarks, and critical comments by a teacher who repeated the experiment. Meanwhile the experiment has been repeated many times, even at PTA meetings. It has become a classic in our primary education.

Paulus the Forest Midget is a wellknown feature on Dutch children's TV. The teacher tells a story about Paulus and the midgets. There is restlessness in the midget town. Some houses are more beautiful than others. Paulus is called in as a troubleshooter. He proposes to rebuild the town. The midgets will live pairwise in houses, each consisting of a drawing room, a kitchen, and two bedrooms. All rooms are to be (congruent) cubes, and each house will be built from four cubes, which touch each other along complete faces, thus

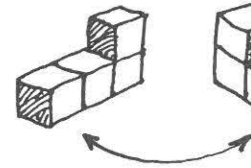
 or  and so on.

*Edu Wijdeveld, Vierkubers — een onderwijsleerpakket voor de basisschool. *Wiskobas-Bulletin* 6, nr 2 (1977). IOWO (Instituut Ontwikkeling Wiskunde Onderwijs). (Out of print)

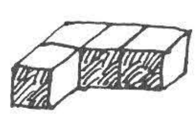
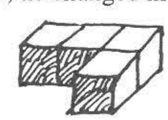
But not  or  and so on.

It would be a dull town if all the houses were the same shape. So it is understood that they should build as many different houses as possible. The children will help them to design such four-cube houses. They are sitting at tables in a circle around the teacher who acts as Paulus, each child with four cubes. Each has built one house.

But are they all different? No, quite a lot are the same. It is easy to see whether two of them are the same, but not so easy to tell *why* they are so. You can show it by turning them.

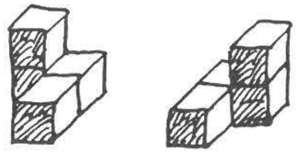


By chance Eloy and Guus, at the same table, had built the "same" house. When Eloy noticed it, he changed his house a bit.

Now they cannot any more be turned into each other, he said.

On a table in the center Paulus rebuilt each new house he noticed. Again and again it had to be discussed whether it was really a new one.



Aren't they the same? No, Guus said, climbing to the second floor you must turn left in the one and right in the other. They were "mirror" rather than "turn" houses. (But some mirror houses may be turn houses too.)

When Paulus had finished assembling, there were 13 houses on the central table. Isn't it possible there are more?

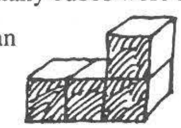
The children tried unsystematically. All the houses they constructed had already been found before. Then Marc discovered a new one. And by mirroring a house one is likely to create a new one. Finally they had got 15 houses, just as many as there were pairs of midgets.

But how should they describe them in order to get them built? "We can take them from the table and colour the place where they stood", a child said. Was it a good idea? No, it was not. There may be different houses standing on the same kind of ground floor.

Guus had a better idea: writing on each ground square a number that tells how many cubes were standing on it. Then

1	1	2

can only mean



Each child made a diagram of his or her own house, and finally all the diagrams were put on one big sheet.

The next day the children reconstructed their houses on the ground floor of the classroom, and finally they had again 15 different ones. But were there really no more than 15?

Eloy said: "Let us start with four rooms in a row." There is only one.

1	1	1	1

"Then all with three in a row."

1	2	1

2	1	1

1	1	1
	1	

1	1	1
1		

1	1	1
		1

They continued with two in a row, and finally the tower.

1	
1	1
	1

	1
1	1
1	

2	2

1	1
1	1

2	1
	1

1	2
1	

2	1
1	

3	1

4

Paulus advanced a new problem: the building lots have to be bought and the walls painted. A square of ground floor is worth 100 florins, and painting a side square costs 10 florins. Some houses were more expensive than others. 2 1 1 and 1 2 1 were of course equally expensive. But why did 3 1 and 4 differ by 90 florins?

Brigitte could explain it. If you lift the top cube from the tower and put it by the side of the ground cube, there are two faces put together that need no painting, which makes a difference of 20 florins, but then you have to paint a new one on the top, which is 10 florins, and 100 florins for the new ground square.

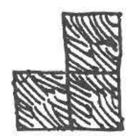
The midgets were disappointed. It was not a good idea to pay different prices. Paulus then proposed they build one big building of sixty cubes.

So the children started designing 60-cube houses by making diagrams. It yielded tremendous activity, a display of buildings, from trivial solutions to sophisticated arithmetical architecture.

The third lesson started with a recapitulation of the preceding ones. The children looked at a sequence of 30 slides that had been made while they worked. Then a new problem was posed.

The midgets had in fact built the original 15 four-cube houses. One night a burglar searched for one of the houses. In the twilight he could discern something that looked like three cubes in a row. Which house was it?

Anyway, the house the burglar meant was two stories high. When he went further, he noticed something like



Which one could it be? He went around to see it from the side and then it looked like

Paul made the house

	1
1	2

But

1	2
1	

was also right. Lisette added

1	2
1	

Everybody said she was wrong. The front was correct but seen from the right side it was wrong. Lisette laughed. "How can you tell whether the burglar turned right? I say he turned left to look again, and then it is correct."

The lesson finished with some more problems of the same kind.