WHAT IS OUR FIRST PHILOSOPHY IN MATHEMATICS EDUCATION?

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What theoretical bases underpin research and practice in mathematics education? For most of the late 20th century, the theoretical underpinnings of mathematics education were mathematics and psychology. But in the past two decades, other disciplines have grown in importance, including philosophy, sociology and linguistics, and have been used by a growing number of researchers to underpin their work. My own concern has been to draw on the insights and theories that philosophy offers. But this raises a question: which of the branches of philosophy demands priority in mathematics education: what is our "first philosophy", if such exists?

The quest for a first philosophy has been an important theme in the history of philosophy. It concerns the answer to the question "What is the most fundamental part of philosophy whose questioning and assumptions precede any subsequent reasoning or theorizing?" Aristotle (1998) argues for metaphysics as first philosophy, claiming it is the most fundamental part of philosophy and has the highest level of generality. Descartes (1641/1996) chose epistemology as first philosophy because it is the basis for all our knowledge and knowing. Heidegger (1926/1962) proposed ontology, within metaphysics, as his first philosophy because, he argues, without Being, as opposed to beings, there is nothing at all. Finally, Levinas (1972/2003) posits ethics as first philosophy, on the grounds that any philosophical questioning presupposes a human questioner and being human rests primarily on ethics. Thus, historical answers to the question of first philosophy offer several branches of philosophy as candidates. What does this mean for mathematics education?

Can mathematics education have a first philosophy? Is there a branch of philosophy that is a *sine qua non* for mathematics education research and possibly its practice as well? Are there philosophical assumptions that cannot be avoided in pursuing any inquiries whatsoever in our field? Can these assumptions be located in one branch of philosophy? In this article, I argue that much is presupposed when we embark on research in mathematics education, including philosophical assumptions. Identifying a first philosophy for mathematics education, if one exists, is a vital task, because any theories we use rest on assumptions, both overt and covert. These assumptions must be cognizant of and consistent with such a first philosophy. This condition is important because, as I argue in the conclusion, some recently popular theories, such as radical constructivism, fail such a test.

Three candidates for a first philosophy for mathematics education research were mentioned above: ontology (representing metaphysics), epistemology and ethics. In addition, two further branches of philosophy are relevant, the philosophy of mathematics, which inquires into the nature of mathematics including its objects and knowledge, and critical theory, which considers the role of scientific and mathematical knowledge in society, as well as issues of social justice and social critique. I consider, below, the claims of each of these five candidates to be the first philosophy of mathematics education.

Critical theory

The Frankfurt school of critical theory was founded during the crisis in Germany that led to the rise of Hitler and Nazism. Critical theorists include Adorno, Fromm, Habermas, Horkheimer, Marcuse and others. They developed their philosophy drawing on the ideas of Marx and Freud, based on a commitment to egalitarian social justice values. It is a utopian perspective presupposing the perfectibility of human society, and incorporating a critical view of the prevailing functionalist ideology. One of their main targets is instrumental reason, of which mathematics is a central part. Instrumental reason is the objective form of action or thought which treats its objects simply as means and not as ends in themselves. It focuses on the most efficient or most cost-effective means to achieve a specific end, without reflecting on the value of that end. According to the *Encyclopedia of Marxism* [2], the Frankfurt School saw instrumental reason as "the dominant form of reason within modern capitalist society [...] leading to the destruction of Nature, the rise of Fascism and bureaucratic capitalism, [and] the reduction of human beings to objects of manipulation" (see under *Instrumental reason and communicative reason*).

Through its attack on instrumental reason, critical theory offers a strong critique of the way mathematics is used in society, and this critique can be taken so far as to question the basic assumptions upon which mathematics education rests (Ernest, 2010). Several scholars have applied the insights of critical theory to critique mathematics education, including D'Ambrosio (1985) and Skovsmose (1985, 1994). D'Ambrosio (2003) offers a trenchant critique of the current state of the world and the roles of mathematics and education in it. He points to the contradictory roles of mathematics and mathematics education as both complicit in the problems faced by all and as contributors to the potential means of their solution. Thus from the perspective of critical theory, the most pressing issue for mathematics education is to contribute to the improvement of the human condition through addressing the universal problem facing humankind, namely "survival with dignity" (D'Ambrosio, 2003, p. 235).

Critical theory, as a philosophy, enters into mathematics education by insisting on the responsibility to offer values-based criticisms of society, mathematics and the social practices of mathematics education, notably the teaching and learning of mathematics. But in analyzing the strengths and weaknesses of mathematics, society, mathematics education and their complex inter-relationships, is critical theory serving as a first philosophy? Critical theory draws on a number of other philosophical areas and assumptions, including ethics and values (as the basis for the critique), epistemology and philosophy of mathematics (in the critique of mathematics and knowledge in general), and social philosophy and theory (for the social and educational critique). Thus critical theory serves rather as a "last philosophy" for mathematics education, if such were to be defined, bringing together and combining other areas of philosophy to analyze and underpin mathematics education, especially with regard to its role in society and its politics. Therefore, despite the vital importance of its critiques, critical theory cannot be said to serve as the first philosophy of mathematics education, its unique philosophical foundation, since it draws on other preceding branches of philosophy.

The philosophy of mathematics

Perhaps the most frequently considered area of philosophy in mathematics education is the philosophy of mathematics. It is argued that understanding the nature of the subject of mathematics and its philosophical underpinnings is necessary, both for teaching the subject thoughtfully and for research in mathematics education. Traditionally the philosophy of mathematics has been concerned with two main questions. How is mathematical knowledge justified, and what are the objects of mathematics? These questions are both relevant for mathematics education. However, a recent maverick tradition in the philosophy of mathematics (Kitcher & Aspray, 1988) has challenged not only the traditional answers to these questions, but also the assumption that these questions are its sole concerns. Elsewhere, I have argued that the philosophy of mathematics should account for more than epistemology and ontology in mathematics (Ernest 1998), including the following issues.

1. Mathematical knowledge: its character, genesis and justification, with special attention to proof.
2. Mathematical theories, both constructive and structural: their character and development, and issues of appraisal and evaluation.
3. The objects of mathematics: their character, origins and relationship with the language of mathematics, the issue of Platonism.
4. Methodology, mathematical practice, the activities of mathematicians, past and present.
5. Applications of mathematics: relations with science, technology, other areas of knowledge.
6. Values, the ethics of mathematics and its applications.
7. The learning of mathematics: its nature, role in the onward transmission of knowledge and creativity of individuals.

Reforming the philosophy of mathematics to meet these broadened objectives would, at the very least, provide an underpinning for the central focus of mathematics education, namely the teaching and learning of mathematics, especially through issue 7, above.

The maverick tradition in the philosophy of mathematics has also challenged the traditional absolutist accounts of mathematical knowledge as certain, absolute, superhuman and incorrigible. The alternative fallibilist (Lakatos, 1976), humanist and social constructivist accounts view mathematical knowledge as fallible and humanly created. These accounts resonate with many of the most controversial theoretical developments in mathematics education, including radical constructivism, social constructivism, socio-cultural theory, postmodernism and critical mathematics education. Even the problem-solving and investigations movements in mathematical pedagogy have drawn on these newer philosophies of mathematics and their challenge to mathematical absolutism.

However, it is not just these newer developments in the philosophy of mathematics that are claimed to underpin mathematics education. The educational relevance of the philosophy of mathematics as a whole has been argued more widely: "Whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (Thom, 1973, p. 204). Thus, as Hersh (1979) argues:

> The issue, then, is not, What is the best way to teach? but, What is mathematics really all about? [...] Controversies about [...] teaching cannot be resolved without confronting problems about the nature of mathematics. (p. 34)

In discussing such philosophies of mathematics embedded in the mathematics curriculum I have shifted from referring to formal academic philosophies as discussed by professional philosophers (*e.g.,* intuitionism, logicism, formalism), to discussing informal philosophies, perhaps better described as images of mathematics. There is an analogy here with Tall and Vinner's (1981) distinction between concept definition (formal, explicit, publicly justifiable description) and concept image (visual and other representations and associations). Concept images represent a deep level of meaning, partly implicit, and may influence their holder's dispositions and actions. Similarly, images of mathematics can include a wide range of representations and associations from sources including philosophy and accounts of the nature of mathematics, but also including representations from the media, classroom presentations and parent, peer and other narratives about mathematics. Personal images of mathematics can utilize mental pictures, including visual, verbal, and narrative representations, originating from past experiences, social talk, *etc.,* and include cognitive, affective and behavioural dimensions, including beliefs. Clearly, with all this variety of representations, a personal philosophy or image of mathematics cannot be the same as a fully articulated academic philosophy of mathematics.

A great deal of research has been conducted into individuals' personal images, beliefs and attitudes about mathematics over the past twenty-five years (Leder *et al.,* 2002; McLeod, 1992). This work makes it a major growth area in research in mathematics education. But while images of mathematics in some loose sense are the counterparts of philosophies of mathematics on the personal or social level, there is a large philosophical gap between the two. The significance of images of mathematics in mathematics education research does not establish the primacy of the philosophy of mathematics in underpinning mathematics education. I have argued elsewhere that there is no direct correspondence between philosophies of mathematics and the images of mathematics of schoolteachers, in the mathematics curriculum, in classroom representations of mathematics, or even in students' images of mathematics (Ernest, 1995, 2008). Whether a teacher has a traditional absolutist or fallibilist philosophy of mathematics may not correlate as well with their personal image of school mathematics as do their values, whether at the authoritarian or humanistic end of the spectrum. In addition, the enactment of any planned image of school mathematics in the classroom as a realized image is subject to the constraints and opportunities afforded by the social context of schooling. Lastly, the image of mathematics as held by the learner is only the result of this realized image after it has been filtered through the student's interpretations, pre-existing attitudes and beliefs. At each stage in this process there are swaying influences at work, such as a teacher's strategic compliance to social pressures on the presentation of mathematics, the influence of the hidden curriculum of schooling on the realized image, and the impact of students' preconceptions in interpreting their classroom experiences and developing their own images. These processes are all illustrated in Figure 1.

More specifically, Figure 1 illustrates how an absolutist philosophy of mathematics combined with humanistic values can give rise to a teacher's humanistic image of school mathematics via "crossing over". Here, a deep commitment to the ideals of progressive mathematics education co-exists with belief in the objectivity and neutrality of mathematics. Likewise crossing over the opposite way enables a fallibilist philosophy of mathematics to combine with authoritarian values resulting in an authoritarian image of school mathematics (Ernest, 1995, 2008).

My overall point is that there is no logical implication leading directly from a teacher's philosophy of mathematics to their intended classroom image of mathematics, let alone to what images are realized in the classroom or are experienced by students. This logical gap weakens the claims of the philosophy of mathematics to be a first philosophy. Philosophy of mathematics alone does not provide a foundation for the concerns of mathematics education, especially the current narrow, dominant form of the philosophy of mathematics. Consequently the philosophy of mathematics does not qualify, in my opinion, as first philosophy for mathematics education. The most relevant philosophies of mathematics do not stem directly from the philosophy of mathematics itself.

Epistemology

There are three main planks to the claims of epistemology to be the first philosophy of mathematics education. First, epistemology is central because the teaching and learning of mathematics and research are all about knowledge and knowing, and epistemology is the theory of knowledge. Second, theories of individual learning have an epistemological dimension since they are about how we acquire knowledge, and they are central both to the theory and practice of mathematics education. Third, the research methods and methodologies we use in our inquiries in mathematics education are all underpinned by epistemology.

Epistemology is the theory of knowledge and concerns both the structure and character of knowledge, as well as the means for its justification. Mathematics education is an interdisciplinary field situated at the confluence of mathematics, the social sciences, and the humanities. Mathematical knowledge is justified at the highest levels by being cast into explicit and precise theories, with clear, step-by-step proofs of all of its claims and theorems. Social science draws upon a

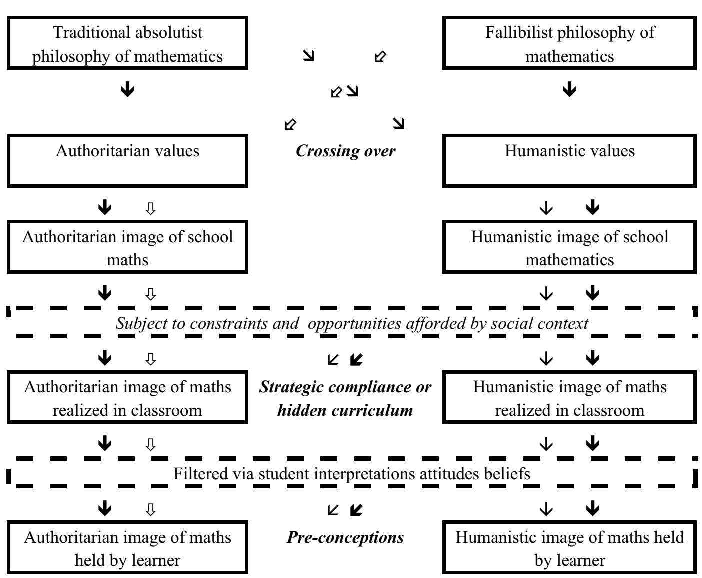

Figure 1. Philosophies of mathematics, values and images of mathematics.

number of research traditions and paradigms for its knowledge claims. Likewise, the humanities use a range of methodologies including text-based, conceptual and theoretical methods. All of these approaches are used to a greater or lesser extent in mathematics education research and are aspects of its epistemology and methodology.

The methodology of mathematics is a dimension of epistemology, although it also belongs to the philosophy of mathematics. It is the systematic study or theory of the methods of mathematics, and can be either prescriptive or descriptive. Prescriptive methodology offers a systematic account of the methods that can or should be used in the present or future in making mathematics, including concept definition, conjecture and theory formulation, and proof construction. Of course one can never proscribe new methods emerging in mathematics research; they themselves are one of the products of creativity. In contrast, descriptive methodology provides an account of the methods that have been used by mathematicians in mathematical practice and in the history of mathematics.

Once there were high hopes that mathematical logic would fulfil the function of a prescriptive methodology of mathematics. However, mathematical logic does not capture the proof methods used by mathematicians in practice, not even the formulations of natural deduction (Kneebone, 1963; Prawitz, 1965) or tableaux systems (Bell & Machover, 1977) intended to serve this purpose. We also know that even at its strongest, formal mathematical proof is incapable of warranting all mathematical truths (Gödel, 1931/1967). Anyway, the methodologies used in mathematical practice go well beyond proving, and include making definitions, making and testing conjectures, analogical reasoning, problem solving, theory construction, and so on (Corfield, 2003; Lakatos, 1976).

In the history of mathematics, the distinction between prescriptive or descriptive methodologies of mathematics has not been clear-cut. Scholars since Pappus of Alexandria (Boyer, 1989), who distinguished the methods of analysis and synthesis, have described strategies employed, and used them to make claims about potentially fruitful methodologies for mathematics. Thus al-Khowarizmi's *Al-jebr …* translates as [*the method of] the reunion of broken parts.* This treatise is known as a milestone in the history of algebra and is, indeed, the source of its name. However, the original name illustrates how the heuristic mathematical methods addressed by the prescriptive methodology of mathematics become mechanized into algorithms or decision-procedures. In both the history and the psychology of learning mathematics, methods that are first encountered or invented as heuristic problem-solving strategies subsequently become routinized and made specific, and thus turn into mechanical algorithms. This suggests a possible reason why mathematical methodology has never developed into a significant discipline in its own right, for if heuristics turn into algorithms, then what once were methodological rules lose this character and become absorbed into mathematical content as mathematics advances.

Not all heuristics suffer this fate and, following on from landmark books such as *Rules for the Direction of the Mind* (Descartes, 1628/1951) and *How to Solve It* (Polya, 1945), there is a myriad of publications which, although not helpful to research mathematicians, have become very prominent in mathematics education. Such treatments of mathematical discovery have prompted the problem-solving and investigations movements.

Imre Lakatos is one philosopher who tried to develop the methodology of mathematics. But his *Proofs and Refutations* (Lakatos, 1976) has multiple functions: philosophical, historical, methodological, and pedagogical (Ernest 1998); and while it is of great significance and impact, its breadth of approach is not intended to further the claims of epistemology (via methodology) to be the first philosophy for mathematics education.

The second plank of the claim of epistemology as a first philosophy for mathematics education concerns learning theories. This topic has been one of the most hotly debated areas within the mathematics education community for the past twenty-five years. Various theories of learning including radical constructivism, enactivism, social constructivism and socio-cultural theory have been hotly debated. As von Glasersfeld (1983) said, "To introduce epistemological considerations into a discussion of education has always been dynamite" (p. 41).

Insights derived from this range of learning theories have undoubtedly deepened and enriched both research and the practice of teaching and learning mathematics. Some areas of attention foregrounded by the controversies between competing learning theories include:

- learner errors and alternative conceptions;
- learners' previous learning, constructions and perceptions as a whole, cognitive and affective;
- bodily movements and gestures in learning, and their meanings;
- root metaphors as the basis of learners' meanings and understanding;
- social context and interpersonal relations;
- the value of an ethnographic view of mathematics teaching and learning as social practices;
- language, texts and semiosis in the teaching and learning of mathematics;
- the problematic nature of mathematical knowledge, as well as learner knowledge;
- the fragility of all research methodologies.

Thus, debates around learning theories have powerful implications for teaching and learning mathematics. However, philosophers would argue that learning theories and their consequences belong to the fields of psychology and pedagogy, and not to epistemology.

The third plank of the claim of epistemology as a first philosophy concerns research methodology, which, as the means of validating knowledge, is a central concern of epistemology. Following Habermas (1972) three main clusters of research methodologies have been distinguished: the scientific, interpretative and critical theoretic research paradigms. Table 1 shows that research paradigms include many aspects of epistemology beyond the warranting of knowledge,

PARADIGM	SCIENTIFIC	INTERPRETATIVE	CRITICAL
INTEREST	Prediction and control of material world	Understanding and making-sense of the human world	Social justice, emancipation
FOCUS	Validation of laws and theories	Exploration of meanings and actors' understandings	Reform of social practices and institutions
WORLDVIEW	Scientific worldview	World of human meanings	Social world and power relations
ONTOLOGY	Objects in physical space	Subjective/intersubjective reality	Persons in society/institutions
VIEW OF KNOWLEDGE	Objective knowledge	Personal /socially constructed knowledge	Objective multi-perspectival knowledge of all participants
METHODOLOGY	Experimental seeking general laws	Case studies of particular contexts	Critical action research in/on social institutions
METHODS	Mainly quantitative using predetermined instruments and categories	Mainly qualitative focusing on textual and spoken responses from individuals	Collection of all data types from overall context and all participants
INTENDED OUTCOME	Objective knowledge and truth in form of laws	Illuminative and illustrative case studies	Social reform, social justice intervention

Table 1. Simplified summary and comparison of the three main research paradigms.

including the interests behind knowledge inquiries; the focus of investigations; the assumed worldview and ontology; the view of knowledge, methodology/methods used in the inquiry; and the overall intended outcome.

The breadth of research methodology and research paradigms in the social sciences means that they encompass several branches of philosophy, including ontology and ethics, thus going beyond epistemology as philosophers understand it. Thus a consideration of research paradigms does not univocally point to epistemology as a first philosophy.

I have explored some of the dimensions of epistemology that are important for mathematics education. However, most of these dimensions are not acknowledged by epistemology proper as belonging to the field. Thus, although epistemology seems to have a case for being first philosophy, much of the basis for its claim is disowned by philosophers. It therefore seems hardly proper to put forward epistemology as a first philosophy for mathematics education.

Ontology

Ontology inquires into the kinds of objects we take for granted as populating the universe we study, and live and work in, as well as the worldviews associated with these objects. Table 1 contrasts three research paradigms and their ontologies:

1. the scientific world, comprising material objects in physical space;
2. the world of subjective and intersubjective reality, comprising human meanings;
3. the social world and its power relations, comprising persons in relationships and within institutions.

Each underlying ontology constitutes a profound set of assumptions for mathematics education. More generally, in considering the claims of ontology as first philosophy, a number of fundamental questions are raised. What are mathematical objects? What are the objects of education and mathematics education? What overall theory of existents and existence are we assuming in our research, either overtly or covertly? Without presupposing essentialist answers to these questions, what is the character of these objects?

The primary objects of study in mathematics education are human beings and their activities and relationships. Ontology poses the question: what is a human being? Philosophically this is a very fundamental question. Answering it with respect to our field of study brings up issues of identity, subjectivity, agency, and human "nature" and development. What is a *human* being? And what is human *being*? Currently, identity and its historical trajectory, in particular the learning career of students, is a growing area of research in our field (Boaler, 2002). This focus goes beyond scrutiny of separate elements of learning to consider the character of the learner as a whole.

Heidegger (1962) develops a complex metaphysics of being based on the idea that our understanding of ourselves and our world presupposes something that cannot be fully articulated, a kind of knowing-how rather than a knowing-that. At the deepest level, such knowing is embodied in our social skills, in how we interact with and share experiences and practices with others, rather than in our concepts, beliefs, and values. Heidegger argues that it is these cultural practices that make our lives meaningful and give us our identities. Although socio-cultural theory already emphasizes the import and tacit nature of the social practices in which all of our being, including the teaching and learning of mathematics, is enacted, there is potentially much more to be gained from the applications of Heidegger's thinking to research in mathematics education. For example, in one formulation he distinguishes three types of being: human, objects, and "ready to hand" objects. These last are tools, shaped objects in our world embodying human intentions. Already this ontology suggests a novel way of conceptualizing technology.

Many find Heidegger, distasteful because he flourished under Nazism. However, I believe his work should be judged on its own merits, and he is only one of many philosophers in the domains of ontology and metaphysics seeking to ground our understanding of being and other fundamental assumptions.

Such considerations suggest that metaphysics, and in particular ontology, might potentially provide a rich philosophical basis for mathematics education. However, any claims for ontology to be a first philosophy need much more development. We need theories of being, including what it means to exist as a human being, that are applicable within mathematics education research. But such theories are not developed and are, as yet, nothing more than an unrealized possibility. Thus I reject the claims of ontology as the first philosophy for mathematics education.

Ethics

The last candidate for a first philosophy for mathematics education is ethics, which enters into mathematics education research in several ways. First, we need to be ethical in our research, ensuring that our research is based on the informed consent of any participants, causes no harm or detriment, and respects the confidentiality of all involved. Any research that does not conform to these standards is ethically flawed and its knowledge claims suspect. Unlike stolen money, which is apparently just as good in the shops as honest money, unethically derived knowledge is epistemologically as well as ethically tainted [3]. Knowledge rests on trust. In accepting knowledge we assume it has been warranted in good faith. The trustworthiness, reliability, validity, proof, testing and warranting of knowledge require that the warrantors and authorizers are honest, trustworthy, and acting ethically throughout. Accessible mathematical proofs and scientific results are checkable and subject to replication, but even with such safeguards, we have to trust the scientific and research communities to produce honest knowledge. Thus, there is a hidden dependence of epistemology, and perhaps of all philosophy, on ethics.

Second, educational researchers are participants in the great, age-old human conversation that sustains and extends our common knowledge and cultural heritage. By sharing our thoughts, we are part of the public conversation from which we and others benefit and grow. Oakshott's (1967) great conversation is an end in itself, and is inescapably ethical because it requires valuing the voices of others; it requires valuing the young who represent the future of the conversation, protecting its integrity; and it requires acknowledging that the conversation is greater than ourselves. Mathematics education is one of the strands in the great conversation and we can be proud that our predecessors and our own efforts have built and are extending it.

Third, as human beings we are irreducibly social creatures. We are essentially interdependent, emerging into the world after development within our mother's bodies. We must experience love and care in our early years to become fully functioning human beings. We must acquire language (or equivalent systems of communication) and acceptable behaviour with others to participate in social life and practices. Without such skills we cannot survive and further the human race. Our species depends for its very survival on our ethical and cooperative behaviour with regard to our fellow humans.

Fourth and last, I come to Levinas. His abiding concern is for the primacy of the ethical relation to the other person and his central thesis is that ethics is first philosophy. Levinas begins with a reconsideration of the subject-object relation and the role of knowledge within it. He adopts a phenomenological position in which the primary reality of the subject is its presence in the world, confronting the objects and inhabitants of the world. He locates the origin of the very idea of "an object" in the concrete life of a subject. A subject is not a substance in need of a bridge, namely, knowledge, in order to reach an object; the secret of its subjectivity is its being present in front of objects. Privileged in such relations is the subject's relation with the other (another human being). Levinas argues that this relation cannot be reduced to comprehension. The other is not a phenomenon but an enigma, something ultimately refractory to intentionality and opaque to understanding. If we could fully comprehend the other, we could reduce it to an object. The other has a right to be herself, unlimited by our expectations and understanding. When I confront the face of the other, I respond, instead, to the face's epiphany of first, their right to be, and second, of my infinite responsibility for the other. Thus, according to Levinas, we owe a debt to the other that precedes and goes beyond reasons, decisions, and our thought processes, and precedes and exceeds any attempt to understand the other. Our infinite responsibility to the other person is, of course, ethical: "Ethics precedes ontology [...] ethics primarily signifies obligation toward the other, that it leads to the Law and to gratuitous service, which is not a principle of technique" (Levinas, 1987, p. 183).

Thus, as social creatures our very nature presupposes the ethics of interpersonal encounters, even before they occur, and before we form or reflect on our practices, let alone our philosophies. This is why Levinas asserts that ethics is the first philosophy, presupposed by any area of activity, experience or knowledge, including mathematics education. If we accept his reasoning, as I do, then our quest is at an end. Ethics, in this bare, stripped down sense, is the first philosophy of mathematics education. Following Levinas's arguments, ethics trumps the other candidates for first philosophy for mathematics education.

Conclusion

Accepting Levinas's arguments, I have arrived at the endpoint of my quest, identifying ethics as the first philosophy of mathematics education. My quest for the first philosophy of mathematics education enables me to review the contributions of several branches of philosophy and show how they are woven into the fabric of mathematics education. Where they do not impact directly on the practicalities of our research, they shine a light on it from an unexpected exterior space, enabling us to rethink and re-evaluate some of the taken-for-granted commonplaces of our practices.

Identifying Levinas's version of ethics as the first philosophy of mathematics education opens up a new domain of thinking and implication for us in theorizing our field. For example, it has immediate implications for individualistic learning theories. For acknowledging the primordially social character of human beings weakens these latters' claims. Thus, for example, radical constructivism's account of the learner as a cognitive alien making sense of a world of experience, constructing other persons as regularities in that world, in effect denies the social and ethical foundation of human being (Ernest, 1994). In its broadest claims it fails. Do more recently adopted theories, such as enactivism, pass this test? I leave this question open, for others to address.

Notes

[1] An earlier version of this paper was presented at the 33rd Annual Conference of the International Group for the Psychology of Mathematics Education, Thessaloniki, Greece, July 2009.

[2] *Encyclopedia of Marxism:* www.marxists.org/glossary/index.htm (retrieved, 12 March 2009).

[3] Perhaps this criterion provides grounds for rejecting Heidegger's philosophy.

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