WHAT COULD CRITICAL MATHEMATICS EDUCATION MEAN FOR DIFFERENT GROUPS OF STUDENTS?

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Ideas of critical mathematics education have been expressed through general notions like empowerment, social justice, and autonomy. These ideas can be condensed into the expression “reading and writing the world with mathematics”. Following Paulo Freire (1972), “reading” refers to critical interpretations of the world and “writing” to some kind of political engagement. “Reading and writing the world with mathematics” can also characterise mathematics education for social justice. In fact, I do not distinguish between critical mathematics education and mathematics education for social justice. I believe we are dealing with two largely overlapping educational approaches [1]. In this article, however, I will mostly discuss critical mathematics education.

General conceptions, including those that characterise critical mathematics education, get some of their meaning through the meta-narratives of modernity. One of these narratives tells of progress: that we are witnessing a process of historical development towards freedom and social justice. This narrative can be used to justify certain political actions, namely those that aim to overcome oppression and injustices. With respect to education, the assumption has been that particular approaches are justified when they are associated with such aims. In this way, critical mathematics education may come to be conceptualised within the modernist outlook.

Jean-François Lyotard has emphasised that the movement towards postmodernity includes a change in attitude towards meta-narratives. While in modernity meta-narratives were used to legitimise actions, the postmodern condition is characterised by a decomposition of such meta-narratives (see Lyotard, 1984, p. 18). I do not follow postmodernism in its many ramifications, but I am interested in Lyotard’s use of Ludwig Wittgenstein’s notion of language games. Thus the decomposition of meta-narratives draws on Wittgenstein’s rejection of the existence of any unified conception of language and his recognition of a variety of language games.

This decomposition also applies to the notions that characterise critical mathematics education. Thus we should not expect the existence of any well-defined meaning of, say, “reading and writing the world with mathematics”. When leaving the outlook of modernity, one comes to operate in an open landscape of diverse, even contradictory, interpretations of any such notions.

With these remarks in mind, I consider what critical mathematics education could mean for different groups of students. Much discussion and research has addressed students who are at social risk—students in poor living-conditions missing basic resources, students in unstable situations due to violence and conflicts, or students subjected to preconceptions perhaps linked to immigration or deportation. My point, however, is that critical mathematics education concerns other groups as well: for example, students in comfortable positions, blind students, elderly students, “Other” students, university students, as well as engineering students. By considering such different groups of students, I will show that “reading and writing the world with mathematics” opens into a range of different interpretations, which bring critical mathematics education into an open conceptual landscape.

Students at social risk

As a first step, consider what critical mathematics education could mean for students at social risk. Such risks can be caused by poverty, violence, or preconceptions. We can think of students living in modern hyperghettos or in favelas. We can think of students living next to a war zone.

Eric Gutstein’s work illustrates what working with students at social risk could mean. He emphasises the importance of reading and writing the world with mathematics, and he refers to a range of topics that can be addressed, such as: elections, immigration, deportation, the spread of HIV-AIDS, criminalisation of young people of colour, racism, and sexism (see Gutstein, 2012). In each case, mathematical investigations can help to reveal particular features of oppression, exploitation, and injustice. The overall idea of working with such examples is condensed by Tonya Gau Bartell in the following way: “[M]athematics can be effectively used to teach and learn about issues of social injustice, assisting students to develop a critical consciousness that supports them in deepening their knowledge (and understandings) of the sociopolitical contexts of their lives” (Bartell, 2012, p. 114).

In Brazil, Denival Biotto Filho (2015) has worked with children living in a kind of orphanage, as their parents may have been involved in crime, drug abuse, prostitution, or have simply disappeared. These are children at social risk. Biotto Filho engaged these children in project work, one of which had to do with football. The children, both girls and boys, chose this topic with much dedication. In Brazil, there are many dreams associated with football, and several times Marta Vieira da Silva, widely known in Brazil, has been
nominated by FIFA as the best female football player in the world. The football project included a range of issues, including one about the working conditions of professional players. It was revealed that more than half of the professional football players in Brazil earn less than the legal minimum salary. We know about the glamorous life of some professional players, but the reality for the majority is completely different. This is an example of reading the world that became established through the football project.

The results of this project were presented by the children to a mixed group, including some of Biotto Filho’s colleagues, teachers, and people from the university. The presentation took people by the heart. The children were very well prepared. They welcomed everybody and introduced the project; they presented topics from the project; and they answered questions. They mastered the whole situation, which came as a surprise to several who attended. The project was conducted by a group of children from whom nobody really expects anything. However, it not only brought about a competence in reading with mathematics; it was also empowering in a direct, personal way. The children got an experience of “Yes, we can do it!”

From the perspective of critical mathematics education such experiences are important. Thus empowerment need not be interpreted solely in socio-political terms; it could also be interpreted in terms of personal experiences—as having, for instance, to do with self-esteem.

**Students in comfortable positions**

The literature of mathematics education for social justice has mainly focussed on students at social risk. The idea has been to ally with these students and to bring them into a position from which they can address issues of social injustice.

However, one can consider a very different group of students, which I refer to as students in “comfortable positions”. I have in mind students whose parents are, if not rich, then at least economically well off: students who belong to the well-protected layers of society. They go to private schools, if this will ensure better access to further studies. They enjoy all the advantages that education might offer.

What could reading and writing the world mean for such students? Let me refer to an example. João Luiz Muzinatti is working with students in comfortable positions. His idea is to address how mathematics-based arguments can help to point out preconceptions broadly assumed among middle class students in Brazil, for instance concerning how much the state spends on people with low or no income. He tries to provide a reading of the world by means of mathematics that challenges general socio-political assumptions that make up part of a traditional middle-class outlook.

With reference to the Brazilian context, Muzinatti addresses an example of such assumptions related to the Bolsa Familia. This system of family support was introduced during the presidency of Luiz Inacio Lula da Silva (2003 - 2010) and extended during the presidency of Dilma Rousseff (2011-). This example of social welfare has faced much middle-class critique, such as “we are paying an awful lot of money to people that, as a consequence, do not want to work”. By means of mathematics, Muzinatti addresses the content of such general claims. It can be clarified how much money is, in fact, involved in this programme; how many families receive this support; how much of total tax paid in Brazil goes to Bolsa Familia; and how much the individual taxpayer contributes to Bolsa Familia. There are many calculations students can complete in order to provide a deeper reading of Bolsa Familia.

With a small reformulation of Bartell’s (2012) summary, we get the following: mathematics can be effectively used to teach and learn about issues of social injustice, assisting students, including also students in comfortable positions, to develop a critical consciousness that supports them in deepening their knowledge and understandings of the socio-political contexts of their lives.

It is crucial that we consider the development of critical consciousness with respect to any group of students. However, the contexts for doing so can be very different. Students in comfortable positions may not be subjected to social risks; they may even benefit from social injustices and economic inequalities. Still, to them reading and writing the world with mathematics might establish radical new perspectives, and this is a concern of critical mathematics education.

**Blind students**

Renato Marcone (2015) recently completed a study of mathematics education for blind students in Brazil. I was his supervisor, and I remember clearly the start of his project. As a point of departure, we wanted to identify previously conducted studies in mathematics education with respect to blind students. We searched around the world, but found very little. We also observed that in contributions explicitly referring to mathematics education for social justice, we could not identify any study regarding blind students. We were surprised, since it seemed obvious to us that we were dealing with an issue concerning social justice. Apparently research conducted in Brazil is providing an opening for this research area [2].

What could reading and writing the world with mathematics mean for blind students? Naturally one can interpret this expression in just the same way as for students at risk or for students in comfortable positions: as referring to socio-political readings and writings. However, in the most literal sense, reading and writing the world with mathematics is a challenge for blind students. For example, difficulties arise from the relationship between Braille and mathematical symbols and concerning the visualisation of mathematical concepts and operations.

As part of his study, Marcone coined the notion of deficientialism [3]. It refers to the construction of deficiency by normality. One inspiration for this notion comes from orientalism, as elaborated by Edward Said (1979). Orientalism refers to the conception of the East by the West and for the West. Orientalism was formed over centuries; it accompanied the brutal processes of colonisation and got its principal expression through the worldviews that accompanied the formation of the British empire. The crucial claim of orientalism is that people from the East are inferior compared to people from the West. In particular, people from the East lack the capacity for self-government. As a consequence, it became a noble thing that the British empire assumed the responsibility for governing these people, who were much
better off being ruled by the British, rather than being left alone to their own poor destiny.

Deficiencialism nominates some groups as suffering from a disability and provides conceptions of what this group is able to do and not do. In the case of blindness, it appears all too obvious: there are a lot of things a blind person cannot do: driving a car, becoming a dentist, or mastering mathematics. Let us, however, observe that the implications of visual impairments are ever changing, not least due to technology. A classic example is the construction of glasses, but today we find a huge amount of other technological devices that modifies the implications of visual impairments. Ronald Vargas Brenes (2012) talks about the social construction of blindness to emphasise that we are not dealing with a simple biological fact, but with a social construction that can always be reconstructed.

With respect to blind people, one particular feature of deficiencialism concerns mathematics. It has been broadly assumed that mathematics, and certainly more advanced studies in mathematics, is not for blind students. However, this deficiencialism can be challenged and deconstructed. What a blind person can come to master with respect to mathematics depends on the available technology and the form of interaction that constitutes the learning process.

Deficiencialism forms a web of conceptions that may construct devastating boundaries. However, conceptions of what a certain group of people can and cannot do, can never be taken as a given. We must be ready to reconsider assumed boundaries with respect to reading and writing the world with mathematics. Boundaries are contingent affairs, and challenging deficiencialism in all its possible expressions is crucial to critical mathematics education.

Elderly students
Not only have blind students been ignored by the literature on mathematics education for social justice, so have elderly students. The first study I know of that addresses this group of students was conducted by Luciano Feliciano de Lima (2015).

The group of students that Lima worked with included retired people that, after a life of hard work, had got some spare time. It included housewives who had taken care of their husbands, but were now alone. It included former bank assistants and shopkeepers. It was a mixed group. Lima’s project was not organised through any formal educational programme and the students joined out of personal interest. If we compare Lima’s group with the groups that Freire engaged in his literacy programme, there are some differences. Lima’s students were not illiterate in any general sense. They experienced a form of reasoning they had not imagined to be within their reach, for instance with respect to the Möbius band. Coming to master more elementary parts of mathematics, such as the conception of different geometric shapes, also became an important personal satisfaction. Many of Lima’s students are grandparents, and becoming familiar with mathematical notions might open new possibilities for communicating with their grandchildren, and helping them with their mathematics homework.

Let us listen to some of the elderly students’ comments (all names are pseudonyms):

I cannot come to think of anything I did not like. All interested me. Because I learned, I learned in my brain. (Roberto, quoted from Lima, 2015, p. 92)

It opened our minds, me and my wife. It opened our reasoning. We began working with the mind. Got it? We really wanted to understand and to try to do. When I cannot solve something […] it gave me a kick here in the head. You know? It’s a difficult feeling to explain. (Davi, quoted from Lima, 2015, p. 93)

Then, among the activities […] I like Tangram, Bingo, Möbius band. […] This improves the quality of people’s life. (Sueli, quoted from Lima, 2015, p. 95)

Here we hear their experiences of coming to master new forms of reasoning. We hear local stories about engaging with mathematics. The elderly students read new dimensions of news about political and economic issues, as they no longer felt the need to skip texts involving numbers. They also experienced being able to master new family situations, such as reading and writing homework together with their grandchildren. Thus, with reference to this group of students, we have to acknowledge that the defining notions of critical mathematics education may operate in an open landscape of different interpretations.

Other students
In orientalism, we meet conceptions of the East by the West and for the West. We have conceptions of the Other by the We and for the We. Such conceptions includes a ranking of superiority and inferiority between the We and the Other.
In a profound way, anti-colonialism challenges such conceptions. A presentation of anti-colonialism is found in the works of Franz Fanon (2004, 2008). The title of his book Black Faces White Masks points out the pathological relationship between the coloniser and the colonised, between the oppressor and the oppressed. The oppressed seem forced to overcome their stipulated inferiority by imitating their oppressors. Such imitations operate as a general pattern in a colonised world. But even if colonialism belongs to the past, many of its oppressing structures are still in full operation. Thus today we find conceptions by the We of the Other that assumes layers of oppression.

Ethnomathematics challenges such conceptions by exploring cultural diversities in mathematics. Thus Ubiratan D’Ambrosio (2006) points out that any form of mathematics is culturally embedded, and that one can talk about the mathematics of any culture. However, even though ethnomathematics profoundly challenges stereotypes regarding the We and the Other, there are still many issues concerning “our mathematics” and “their mathematics” that need to be addressed. Different forms of reading and writing the world may be contradictory.

Although Marcone’s conception of deficiencialism was developed with particular reference to blind students, it can be interpreted more broadly. It can be interpreted as a conception provided by the We about the limitations of the Other. It explains why some groups of students will perform poorly in school. Deficiencialism can express itself through many discourses, such as, for instance: “children with a certain background are not able to perform well in school due to their lack of cultural capital”.

Students at social risk may experience deficiencialism. Nobody expects much from them, as with Biotto Filho’s students. Deficiencialism represents a negation of the capacities of coming to read and write the world with mathematics. Not only may students at social risk be captured by such deficiencialism, so might any minority group. To challenge deficiencialism is crucial to critical mathematics education. We have to be careful not to rush to any conclusions about what Other students can and cannot do, and about what might serve the them best [4]. In particular, we should not rush into specific interpretations of how Other students must read and write the world.

University students in mathematics
Let us now consider university students in mathematics. What has critical mathematics education to do with them? Over centuries, mathematics has developed in close connection with interpretations of nature and religion. In The World, ready for publication in 1633 but only published posthumously in 1664, René Descartes presented a general world perspective. He outlined some basic laws according to which nature operates. The first law states: “[E]ach particular part of matter always continues in the same state unless collision with others forces it to change its state” (Descartes, 1998, p. 25). The two other laws are of the same nature, and together they reflect a mechanical interpretation of nature. According to Descartes, we are governed by laws that God imposed on nature. Furthermore, Descartes pointed out that these are the only laws that God imposed. In Principia (1999/1687), Isaac Newton formulated three basis laws that resonate with Descartes’ formulations. Newton agreed with Descartes: the basic laws of nature are established by God. They constitute a defining part of God’s creation. Newton’s laws have a mathematical form, meaning that we, human beings, can grasp God’s creation by means of mathematics. Mathematics represents the rationality of God. In mathematics, we meet a divine form of knowledge that can be trusted and celebrated. This trust and celebration defines part of the modernist meta-narratives that present mathematics as the paradigmatic case of human knowledge.

These narratives establish the modernist conception of mathematics, which is broadly assumed, even today. This conception includes two elements: (1) mathematics provides a powerful reading of the world; it provides the principal descriptive tools in natural science and in disciplines such as, for instance, economics; (2) mathematics provides a principal way of writing the world, as mathematics is an integral part of technological enterprises.

This modernist conception of mathematics stipulates and simultaneously celebrates the unique powers of mathematics-based readings and writings. To a large extent, this celebration defines the format of university studies in mathematics. Thus, these studies are, typically, completely focused on content matter issues. We do not usually find space for reflection on the socio-political roles of mathematics; instead, the principal educational task is defined in terms of students’ mastery of mathematics.

From the perspective of critical mathematics education, it is crucial, however, that university studies in mathematics leave the protective cover provided by the modernist conception of mathematics. In fact, an explicit negation of (1) and (2) opens up a critical perspective on mathematics: (1) readings of the world by means of mathematics can be highly questionable; they can provide simplifications and distortions; they can be unreliable; and they can represent particular interests; (2) writings of the world by means of mathematics can be questionable, and mathematics-based technological actions can have any kind of qualities: they can be risky; they can be unsustainable; and they can represent particular priorities.

To critical mathematics education it is important, as an integral part of university studies in mathematics, to address critically the way mathematics provides readings and writings of the world. Such studies cannot just concentrate on mathematical subject-matter, but also needs to address the possible socio-political roles of mathematics. It is important to address the possibility of different and also of contradictory ways of reading and writing the world with mathematics.

Engineering students
Mathematics operates as an integral part of technological constructions and devices. One can think of techniques for automating production procedures; for surveillance; for implementing new techniques in medical treatment; for constructing drones that can deliver small parcels to customers; for making any kind of more advanced weapon work efficiently; for ensuring personalised electronic advertising as facilitated by web crawlers; for managing page-rankings
with respect to internet searches; for managing economic risk-taking; for identifying strategies in the stock market; etc.

The world that is addressed in an engineering project is subjected to a mathematical reading. It is conceptualised in terms of numbers, figures and diagrams. Thus the departure for any engineering project is a mathematical representation of some parcels of the world. The very engineering design takes the form of a mathematical writing of the world. The blueprint for any such design has a mathematical format.

According to the modernist perspective on mathematics, the application of mathematics ensures that such engineering blueprints have some intrinsically good qualities. From a critical perspective, however, mathematics-based technological actions can have any kind of qualities. They can be effective, risky, cost efficient, benevolent, horrifying, cynical, well-intended, etc. They can create profit for some, new workplaces for others, and cause unemployment for many. Any mathematics-based action is a contingent affair in need of reflection.

In most cases, the development of such reflections is not part of the educational programme of technical disciplines. This observation brings me to characterise what I refer to as one-dimensional expertise. In a typical university curriculum for technical disciplines, like engineering, we find the desired professionalism decomposed into a range of modules. Mastering such modules provides professional know-how in doing things. It is much more seldom to find study-activities that contribute to a professionalism of reflecting on what is done and could be done. I find, however, that a principal challenge for any university programme with respect to technical disciplines is to develop a double professionalism: a professionalism in doing, and a professionalism about doing [5]. This means a professionalism in reading and writing the world with mathematics, as well as a professionalism in reflecting on such readings and writings.

To develop a double professionalism becomes crucial when the meta-narratives of modernity are decomposed and no longer provide an overall celebration of mathematics-based technology as being a reliable motor of progress. As a consequence, students of any technical discipline can become a focus of critical mathematics education.

A variety of readings and writings
To critical mathematics education it is crucial that students at social risk come to act as readers and writers of the world. But critical mathematics education must address other groups of students, such as, for instance, students in comfortable positions. It is important to challenge presumptions and preconceptions, and in this way to provide revisions of some readings and writings of the world.

Critical mathematics education must address any group of students, including blind students, elderly students, and “Other” students. Critical mathematics education must address university students in mathematics and engineering students in general. These students come to master powerful readings and writings of the world that might be in urgent need of critique.

To critical mathematics education it is important to address critically any form of reading and writing with mathematics. This not only concerns academic mathematics and engineering mathematics. It concerns any form of mathematics. It concerns any version of ethnomathematics. It also concerns the readings and writings which are part of Gutein’s projects, and which is considered paradigmatic to critical mathematics education. Such readings and writings may also represent limitations, simplifications, and presuppositions. Thus auto-critique is an integral part of any critical mathematics education. There are no modernist meta-narratives to provide critical education with any incontestable foundation; nor are there any readings or writings of the world with undeniable qualities. Critical mathematics education needs to operate in an open landscape of diverse, even contradictory, interpretations also of its defining notions.

Notes
[2] Sociedade Brasileira de Educação Matemática, SBEM (Brazilian Society for Mathematics Education) has established a working group with the title Diferenca. Inclusao e Educação Matemática (Difference, Inclusion and Mathematics Education). This group coordinates research not only with respect to blind students, but with respect to any group of students with disabilities. The group is coordinated by Lulu Healy and Miriam Godoy Penteado. See http://sbempaulista.org.br/gt-no-13-diferenca-inclusaoe-educacao-matematica/
[3] The Portuguese word deficiência means disability. However the notion of deficiencialismo I have translated directly as deficiencialism and not as disableism, being aware that the English notion deficiency includes a range of negative connotations that disability does not include.
[4] These observations can be further explored through the notion of foregrounds—see Skovsmose (2014a).

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References
Freire, P. (1972) Pedagogy of the Oppressed. Harmondsworth, UK: Pen-
[Pragmatists] see both intellectual and moral progress not as a matter of getting closer to the True or the Good or the Right, but as an increase in imaginative power. We see imagination as the cutting edge of cultural evolution, the power which—given peace and prosperity—constantly operates so as to make the human future richer than the human past. Imagination is the source both of new scientific pictures of the physical universe and of new conceptions of possible communities. It is what Newton and Christ, Freud and Marx, had in common: the ability to redescribe the familiar in unfamiliar terms.