The Aesthetic *Is* Relevant

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Many would agree that we need to make mathematics more relevant and interesting to students, yet most recommendations for increased relevance have ignored the aesthetic dimension of student interest and cognition. In this article, I argue that the aesthetic dimension plays a central role in determining what mathematics proves personally or epistemologically relevant to children. I present an example of a learning environment that attempts to explore this dimension – both mathematically and pedagogically – and then briefly describe a small study that examined the responses of middle-school students to this environment.

Different and sometimes seemingly opposing modes of thinking and knowing have traditionally been promoted and prized in various human endeavours. While rational, formal and logical ways of thinking and knowing dominate the sciences, the aesthetic, narrative and metaphorical modes prevail in the arts. However, researchers now argue that all abstract human thinking is metaphorical, based on our sensory-motor experiences (Lakoff and Johnson, 1999) and that humans possess an innate aesthetic sensibility that acts as one of our primary meaning-making capacities in all domains (Dissanakye, 1992; Wilson, 1998).

The aesthetic responses we have as human beings also seem to carry strong affective dimensions: we experience sensations of pleasure as a result of apprehending and discerning of information-rich patterns in the environment (Damasio, 1994; Pinker, 1997). In contrast to more traditional views, this conception of aesthetics focuses not on the formal, detached and objective judgements of beauty and/or elegance. Rather, it interprets aesthetic response as a cognisance of fit, of structure or order, perceived in part as being intuitive and recognised at an emotional level as being pleasurable. The response results from an awareness of the perceiver in relation to the environment.

The role of the aesthetic in mathematics has been explored by many mathematicians (e.g. Davis and Hersh, 1981; Penrose, 1974; Poincaré, 1956; Tymoczko, 1993). The emerging picture is that aesthetics is involved in:

(a) motivating the choice of certain problems to solve;

(b) guiding the mathematician to discovery;

(c) helping a mathematician decide on the significance of a certain result.

In a challenge to traditional epistemologies, some researchers argue that the aesthetic is, in fact, a mode of cognition used by scientists and mathematicians (Papert, 1978; Weschler, 1978; Burton, 1999). Based on these claims, and more generally on aesthetics' perceived role in learning (Dewey, 1933; Eisner, 1985; Greene, 1995), a growing number of educators have argued that aesthetic considerations should be of primary importance in children's learning of mathematics (e.g. Brown, 1993; Rogers, 1999; Silver, 1994; Whitcombe, 1988). However, we are only beginning to develop adequate understandings of how aesthetic considerations relate to mathematics learning.

Papert (1980) has perhaps provided the most compelling arguments for (and has developed examples of) mathematics learning environments that privilege the aesthetic. He offers turtle geometry as an example of an environment that resonates with a child's existing sense of aesthetics, one that allows her to use her body- and ego-knowledge to draw, explore and make mathematics. In a different vein, there have also been attempts to appeal to students' artistic and creative impulses by linking mathematics to the arts – the golden ratio through Mondrian paintings, transformations through Escher drawings, ratios through music, etc. For example, Jamison (1997) suggests that engaging students in discrete mathematics tasks with an artistic connection (e.g. modular arithmetic in music or regular polygons in eurythmy) will stimulate their aesthetic sense, pique their mathematical curiosity and reveal the artistic spirit in mathematics.

Although I do not wish to deny its virtues, such linking utilises student interest in other domains to coax them into mathematics – effectively robbing students of the pleasure of creating their own connections. A pernicious consequence of appealing to students' love of something else (whether in the arts, sports, food or money) in the hopes of increasing their interest levels in mathematics is that it endorses the belief that mathematics itself is an aesthetically sterile domain, or at least one whose potentialities are only realised through engagement with external domains of interest.

Such linking also tends to undermine the aesthetic, expressive and transformative possibilities of mathematics itself. We count on students' sensitivities and attractions to the arts to help them appreciate mathematics. But could we understand what these sensitivities are and why these attractions exist in contexts that are themselves mathematical? In other words, could we reverse the direction of the aesthetic flow, so that it originates in the mathematics?

Many of the researchers cited above have acknowledged the challenge of finding and creating meaningful mathematics that allow students to use extra-logical facets of their mathematical thinking – the aesthetic and the intuitive.

Again, Papert (1980) has argued that because software can take on many forms and appeals to many tastes and styles of involvement, computers can encourage aesthetic learning experiences for many students of many ages. While it may be true that computers provide flexible learning environments, we need a better understanding of how computers present students with opportunities for actively perceiving or constructing a sense of fit

Outside of the arena of technology, Somervell (1975) describes a compelling example of the fit between a child's physical movements and sophisticated geometric objects through curve drawing using pins and thread Through natural and rhythmic motions, students both create and discern the nature of mathematical forms such as the parabola. Similarly, Higginson describes the student's action of tesselating as:

> a direct offshoot of a common and powerful human aesthetic urge, that of 'fitting' (Upitis, Phillips and Higginson, 1997, p. 52)

As they fit shapes together, students create tiling patterns while enacting their knowledge of mathematical relationships and properties; they see both *that* and *how* things fit together.

These three examples epitomise the relationship of mathematics to basic human sensitivities, such as rhythm and fit, and to basic human activities such as artistic expression and construction. They enable a learner to become cognisant not only of mathematical structures and forms, but also of the powers of her mind that she uses everyday, those which allow her to learn language and to use imagery and symbolism. In the section that follows, I build on these three examples to generalise the types of sensitivities and activities that constitute an aesthetically-rich mathematics learning environment

Aesthetically-rich mathematics learning environments

I suggest that aesthetically-rich learning environments enable children to wonder, to notice, to imagine alternatives, to appreciate contingencies and to experience pleasure and pride. They are characterised by two facets: first, they legitimise students' expressions of innate sensibilities and subjective impressions - they 'work with' such perceptions rather than exclude or deny them. Second, they uphold Dewey's (1933) sense of the fourfold interests of children: communicating, finding things out, making things and expressing themselves artistically These dual facets - of perception and of action - permit children to become absorbed in and identify themselves with some object or idea, to become interested.

Higginson's tessellation example mentioned above includes at least three of these actions:

- the students tried fitting different shapes together, finding out which ones worked and which ones did not;
- they drew and cut out their own shapes;
- they designed and created their own tiling patterns

A learning environment that encourages students to come up with and share their questions, as well as describe and discuss their discoveries with others, would facilitate the action of communicating. The more these actions are available to children, the more they learn

What compels students to notice? I remarked above that rhythm and fit are two basic human sensitivities. By this, I mean that they are two ways in which we constantly and successfully make sense of our environments - we are good at and often engage in perceiving rhythm and fit. As Lakoff and Núñez (2000) argue, we are also good at sensing balance, motion and symmetry When gaining information about our environments, we are additionally sensitive to order, comparison and transformation.

All of these sensitivities might be referred to as capacities for patterning, making inferences and representing [1], which we exercise predominantly through visual, spatial and physical actions. In fact, we might describe using such sensitivities as aesthetic modes of cognition. Many mathematicians can also exercise these capacities through more symbolic or abstract manipulations - though some mathematicians always have a 'picture' of their objects and ideas (Burton, 1999). But in order for children to use and build on their sensitivities, they need more concrete encounters with mathematics, ones that are more sensorially pattern-rich.

The examples given above provided concrete encounters for learning geometry. I wanted to explore the possibility of creating an aesthetically-rich learning environment in the less graphical or pictorial mathematical domain of number sense. In particular, I wanted to make the pattern possibilities of the real numbers accessible to middle-school students and to explore the potential of students' aesthetic engagement with the (often abhorred) topic of fractions and decimals.

This interest was fuelled in part by my own mathematics research activity at the Centre for Experimental and Constructive Mathematics at Simon Fraser University in British Columbia, Canada, where techniques are being developed to employ the natural visual capacities of human perception to search for complex relationships and patterns in numerical distributions. [2] Though these means (such as using visual calculators, the focus of the rest of this article) were primarily aimed at looking for the fundamental underlying structures of mathematical objects, such as sequences of polynomials and continued fraction expansions, they also seemed appropriate, if modified and simplified, for the exploration of simpler mathematical objects. I therefore designed a visual calculator [3] whose aim was to facilitate the visualisation and manipulation of real numbers at the middle-school level

Numbering by colours

The Colour Calculator is a regular, internet-based calculator, 'regular' in that provides numerical results to computations, but it also offers its results in a colour-coded table. Conventional operations are provided; the division operation allows rational numbers while the square-root operator allows irrational numbers Each digit of the result corresponds to one of ten distinctly coloured swatches reflected in a legend, in the table shown in Figure 1.

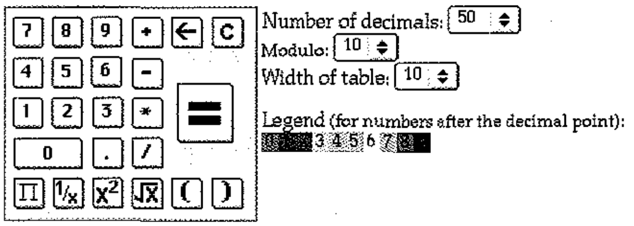

Figure 1 The Colour Calculator

The calculator operates at a maximum precision of one hundred decimal digits, and thus each result is simultaneously represented by a (long) decimal string and an array or matrix of colour swatches It is possible to change the dimension (or width) of the colour table to values between one and thirty, as well as the number of decimal places that appear, from one to a hundred. [4] Of particular interest in the Colour Calculator, therefore, are the pattern-rich real numbers, because they can be *seen and understood as patterns of colour*.

This graphical representation of number calls attention to and facilitates the perception of certain important classes – terminating, periodic, eventually periodic and non-periodic – and some of their properties as well. The calculator operations (addition, subtraction, multiplication, division, taking square roots), as well as the changeable width of the table itself, enable exploratory action on the colour patterns themselves.

In this following screenshot (Figure 2), the operation 1/7 has been typed into the calculator (with the number of decimal places to display set at one hundred, and the width of table set at ten). The Colour Calculator generates the associated coloured table.

Equation: 1/7

Results: 1428571428571428571428571428571428571428571428571428571428571428571428571428571428571428571428571428

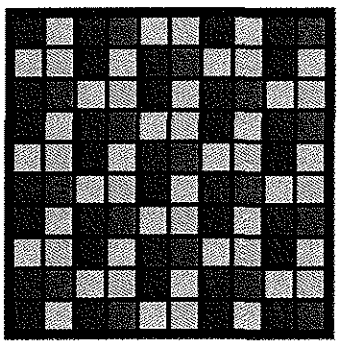

Figure 2 The Colour Calculator showing 1/7

Using the button that controls the width of the table of colours, a student can select different table dimensions which result in different colour patterns, some of which highlight interesting aspects of the number's period. For instance, Figure 3 shows 1/7 displayed using table widths of eighteen and seventeen, respectively

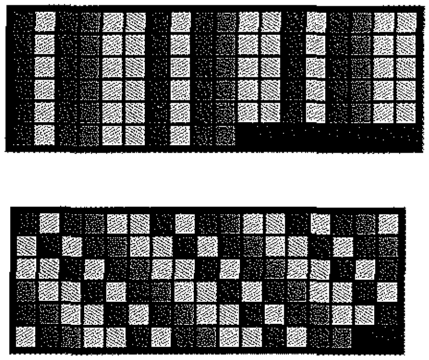

Figure 3 Different representations of 1/7

Given this basic environment engaging the visual capacities of human perception, what kinds of mathematical observations emerge? The types of questions that seem compelling to ask in this environment relate mostly to patterns in the decimal representation of real numbers. What different types of patterns exist? How does the type of pattern depend on the number? These lead to sophisticated number-theoretic questions, especially with prime numbers, that many mathematicians find compelling.

However, they are not all necessarily the type of questions or properties compelling or suitable for middle-school students But if we *begin* with this aesthetically-rich learning environment and ask what is interesting and/or significant to know about fractions and decimals – note the reversal of order – we might depart remarkably from conventional curricular objectives. For instance, rather than emphasising the importance of understanding fractions as parts of wholes or knowing how to convert fractions to decimals, the Colour Calculator emphasises the understanding of fractions as patterned objects (as representations) and the relationships between numerators and denominators that determine these patterns (relationships such as prime-, even- and oddness).

Regardless of the specific investigations that middle-school students might pursue, I designed the environment with three conjectures in mind:

1. This mathematics environment would allow students first and foremost to experiment; their sensitivity to and manipulation of visual patterns would prompt and facilitate their sense-making of some characteristics and relationships of rationals.

2. Students would be motivated to experiment because of the appeal and novelty of the colour patterns; they would take notice of the different types of rational numbers and gain fluency with the decimal representation of different fractions. This experimentation would perhaps lead students to try to figure out what the patterns mean and to pose questions about them.

3. The students' experiences of manipulating fractions and decimals as colour patterns (rather than as 'ugly' numbers) would help them to build a more positive attitude towards them.

Since the colour patterns can become objects of interest in and of themselves, it would also be possible, through various operations on numbers, for students to create new patterns – designing them through mathematical operations. Therefore, in terms of the four actions proposed by Dewey, the Colour Calculator could most saliently permit the act of inquiry, but also perhaps the act of making things.

The Colour Calculator environment is structured both by its mathematical and pedagogical design. The mathematical design dictates the domain of investigation, in this case the world of numerical computation. In this domain, the square-root operator represents an intentional design decision to locate the rational numbers (which give rise to repeating colour tables) within a larger field in which non-repeating patterns are equally accessible.

The pedagogical design dictates how students are asked and encouraged to move through the environment. In order to promote the pattern noticing and problem posing that can encourage students' intrinsic motivation and greater ownership of their mathematical activity, the environment is structured to offer only minimal instructions and a few specific questions.

In particular, in order to draw students into the pattern-rich rational numbers, they are asked to begin by trying a few fractions and observing the associated table of colours. The first fraction they are asked to try is 1/7 which gives a repeating, non-terminating result. They are also encouraged to change the width of the table and then to continue experimenting with other fractions.

These minimal suggestions present students with the potential for encountering an initial complexity – they first have to make sense of the table of colours associated with 1/7, to connect it to the fraction and to the legend provided on the screen. This has the potential to draw them into the world of the Colour Calculator. Dewey (1933) and Bruner (1969) both argue that it is the reduction of surprise and complexity to simplicity and predictability that evokes, in learners, first reflection and then pleasure. Thus, from the outset, the apparent insufficiency of textual instructions sets up trajectories of sense-making from complexity/confusion to simplicity/pleasure.

The Colour Calculator mathematical environment emphasises two aspects of learning. The first is to encourage students to make sense of mathematical ideas such as fractions, which they often find almost repellent, using some of their aesthetic sensitivities such as symmetry, repetition, rhythm and pattern. This type of sense-making is part of the cognitive processes that students use to understand the form and meaning of objects and ideas.

The second is to facilitate a process – one of exploration, research and discovery – that potentially gives rise to a chain of sensory and emotive responses (Dewey, 1934). This process is initiated by surprise (or novelty) and ambiguity. It culminates in the grasping of new knowledge that has been experientially developed. If these two aspects have the hypothesised effect of initiating and sustaining student engagement, then students might be able to have a qualitatively different experience with mathematics. They could experience pleasure from actively discovering how things fit together, transforming their previous understandings of numbers and their own dispositions toward them.

Student patterns

I conducted structured, task-based interviews [5] with fifteen grade 8 students, eight male and seven female, of mixed ability (as rated by their regular classroom teacher). The students came from lower to middle class, small-town backgrounds. The interviews were *task-based* in that each student worked through a mathematical task using the Colour Calculator as I observed and asked questions. They were *structured* in the sense of my facilitating the problem-posing and problem-solving processes for each student. Every interview began with the student reading the instructions for the Colour Calculator aloud, as follows:

> You will be able to explore fraction and decimal number patterns with this Colour Calculator. To get started, type a fraction like 1/7 into the calculator, then press the = button. Things to think about:
>
> - What do you notice in the table of colours?
> - What happens when you change the width of the table?
> - Experiment with other fractions.

The student then started on the task while I asked a series of questions designed to elicit some of her thought processes. I occasionally intervened to provide guidance, following a set sequence of prompts. These were only given when I judged that the student could no longer progress either in identifying a problem or in solving it. The interview continued until the student had concluded at least one exploration: that is, until the student had resolved one problem.

The nature of this problem varied greatly among the students, both in terms of sophistication and – more importantly – in whether the question was posed by the student, co-evolved between us or was proposed by me (though I only offered questions that had already been posed by previous students in this study). Following this, I asked each student to reflect on her experience, compare it with other mathematics tasks and comment on the open-ended nature of their own activity. Each interview lasted between twenty and thirty minutes.

The interviews were all audio-taped and then transcribed. In addition, while interviewing, I kept notes of the students' facial and bodily reactions as they interacted with the environment, particularly at the beginning when they tried their first fraction and at the end when they were approaching the resolution of their problem.

Rather than showing a group trend in response to the Colour Calculator environment, the interviews revealed enormous differences in the students' approaches to and thinking styles in mathematical learning situations, as well as their understanding of fractions and decimals. While I will present some findings in terms of my conjectures, I would first like to discuss an issue that relates to these differences, one which I believe sheds light on the nature of these students' aesthetic responses.

Attitudes affect aesthetic response

Throughout the task-based interviews, I observed three types of approach taken by the students. I have characterised each student based on these approaches as one of 'path-finder', 'track-taker' or 'floater' The 'path-finder' not only likes to find and create her own trail, she is willing to take detours as she moves through the terrain. The seven path-finders were students who were disposed to and liked to (as they said) "roam", "fiddle around" or "find my own problem". These students spent the longest amount of time inquiring, experimenting and becoming intrigued with either creating or describing patterns.

The 'track-taker', on the other hand, preferred having an established, obligatory route that was quick and straight - like a railway track. The four track-takers, though all mathematically competent, felt uncomfortable or unmotivated in the absence of a specific problem or goal. They wanted to know exactly what was expected of them, often asking "what do you want me to do now?" and wanting to know when they were finished: "I like to know when I have the right answer".

Finally, the 'floater' had neither the directed goal of the track-taker nor the exploratory goal of the path-finder; her motivation and action goals emerged slowly and were highly dependent on the winds of the environment The four floaters were the students with the lowest confidence levels and the shakiest mathematical understandings. At first, they felt uncomfortable not being told what to look for nor what to solve: but eventually, after several interventions on my part, they started to initiate some of their own experiments and pose questions. When I asked the floaters how they felt about the open-ended nature of the activity, they shrugged or started to describe what they had done during the activity, as if unaware of, or unable to express, their own preferences and aptitudes

I have described these three types of students because their different approaches greatly influenced the kind of activity they allowed themselves to become involved in with the Colour Calculator Naturally, the track-takers were engaged as long as they had a specific problem to solve and seemed more intent on achieving a result than in understanding something or enjoying themselves.

One student stands out in this group, whom I will refer to as Cameron. He is bright, articulate and socially mature, and performs very well on school mathematics tests. He seems to have decided that he will apply himself to the tasks that he is given, but will make no more effort than necessary. He is quick to find answers to problems. He is very numerically-oriented - in fact, he proved the only student who looked at and commented upon the decimal strings of digits before saying anything about the table of colours. However, Cameron seems detached from his mathematical activity, apparently refusing to engage personally, noticing only what is required in order to achieve the solution to a problem.

In his case, my conjecture about aesthetic engagement completely failed: he did not seem to use any of his aesthetic sensibilities, nor did the dynamic and visual nature of the environment draw him in. It is possible that prolonged exposure to this type of environment would produce different results with Cameron However, it is also possible that this environment does not offer much benefit to such a student. Cameron was the most extreme case of a 'track-taker', but this tentative observation applies just as well to the other three students who shared his orientation.

Revealing responses

I made three more specific conjectures about how students would interact with the Colour Calculator environment:

(1) the pattern-rich table of colour patterns resulting from the 1/7 calculation would surprise and engage the students;

(2) the students' sensitivity to visual patterns would prompt and facilitate their sense-making about certain characteristics of relationships among rational numbers;

(3) the Colour Calculator would provide a setting in which students could develop more positive relationships with fractions and decimals.

At an obvious and almost trivial level, every student declared that they had never seen fractions or decimals like this, jointly and with colours - many of them realised for the first time that a fraction and its corresponding decimal are the *same* [6], one student noting "you never see them together like this". Every student also expressed how different this type of mathematical task was from their regular classroom work, one explaining that "you actually have to *do* things", while another observed that "you have to *notice* things".

Of the fifteen students I interviewed, thirteen of them showed obvious physical signs of surprise, which they expressed through one or more of the following actions: widening their eyes, sitting upright or moving forward, making a sound such as "ooh", or saying some form of "wow" One floater student, whom I call Nadia, showed no physical surprise at all, and answered "I don't see anything" when I asked her what she saw in the table of colours. Nadia was either completely insensitive to the patterns in the table or, because of her timidity and lack of confidence, she may have been encountering too great an affective barrier even to attempt to engage. (She may also have been at least partially colour-blind.) The other student who showed no physical reaction was Cameron, who remarked flatly: "There are lots of colours and patterns there".

Of course, initial surprise is only desirable if its effect is to engage the student in sense-making: that is, if it prompts the student to try to understand something about what they are seeing This was easiest to observe with the more articulate students - the path-finders - who provided a running commentary of their thought processes, like Sean:

> Okay Ah. It looks like an abstract painting. Not exactly like a math problem. I'm trying to figure out how it calculates that. Uh. Well, it says that the results are 0.142857 and it repeats So this is a repeating pattern. I can see it because the red sticks out and the purple, and ooh the green. They kind of go in a diagonal which shows a standard repeating pattern, but I'm trying to figure out how things are working So the number corresponds to the colour.

There were a few other students who provided such spontaneous descriptions of their thought processes, but most of the students had to be prompted to share their thoughts and perceptions. All the students quickly made the connection between the table of colours and the legend (as shown in Figure 1), and between the decimal and the table of colours. A few of the students failed to see the connection between the fraction they had typed in and the table of colours, needing some further experimentation to be able to conceive of them as the same number. However, beyond suggesting these obvious relationships, I wanted to know whether the Colour Calculator environment would result in a generative engagement. For example, would the students wonder why the 1/7 fraction produced the table of colour or *why* the table showed the patterns it did?

I judged a student to be generatively engaged if, after their initiation to the Colour Calculator, they made observations or took actions that indicated an emerging question or conjecture. For example, Ann's observation that "every seventh box is a purple" indicated a conjecture that the period of 1/7 is 6, and was followed by her experimentation with the width of the table (which, perhaps not surprisingly, she first tried at 7 before realising she really wanted 6).

Sean's immediate experimentation with 1/3, then 1/2 indicated an emerging question of how other fractions contrast with 1/7. Julie took a slightly different approach by experimenting first with the width of the table of colours, describing a width of 7 as "it's like a staircase" and a width of 3 as "it's doubled up", indicating an emerging question about the types of possible patterns. She went on to characterise diagonal patterns as those that were one more or one less than the width that makes the colours of the table line up. The five other path-finders each embarked on explorations similar to the three described above.

The other students either paused, waiting for instructions or guidance, or asked me whether I wanted them to make the colours line up (as was suggested in the instructions to the task). These students, either because of their track-taker orientation, their lack of confidence or lack of interest in the task did not quickly become generatively engaged.

The floaters required some guidance and prompts, as if they needed to know *what* was interesting or significant enough to pursue. After they had formed a question or conjecture, they were able to experiment and all but one of them added a personal variation to their experimentation. For example, Robert started by following my prompt of figuring out what kinds of fractions are non-terminating, but then decided to investigate what kinds of fractions gave solid tables of colours, discovering that $n/9$ (for $0 < n < 9$) would always give a solid table in the colour corresponding to n.

The track-taker students exhibited behaviour that in other contexts might seem exemplary - they could answer every question, they did so quickly and they showed confidence. However, they experimented and solved without becoming generatively involved, without asking their own questions and without ever seeming curious, confused or excited. This raises important issues. These students are precisely the ones that educators tend not to worry about: they are both adept at learning and willing enough to work. But are they possibly also the ones that will likely remain aesthetically numb, not only in mathematics but in their other scholastic pursuits? If so, then perhaps a more humanist set of educational goals would indicate that these are the very students with whom we should be most preoccupied.

I now turn to my second conjecture about whether the students' sensitivity to visual patterns and engagement would prompt and facilitate their sense-making of some of the characteristics of and relationships among rational numbers. There were two types of sense-making exhibited by the students. The first was grouped around aspects of rational numbers that these students had encountered or already 'learned' in their regular mathematics classes. The second type centred on features of rational numbers that were new to them, which were mediated by the Colour Calculator environment.

Of course, not all the students made the same enquiries and discoveries. In fact, the wide range produced by the students revealed much about their existing understanding of fractions and decimals. Within the first type of sense-making, the majority of the students realised, some to a greater extent than others, that fractions are not just the canonical 1/2, 2/3, 3/4, 1/10 that they have often encountered during 'fraction class', but that they can have a denominator greater than 10 and they can even be *any* whole number over *any* whole number, as Steve's question shows: "You mean I can put *any* number on the bottom?"

Several students also expressed surprise at seeing the fraction and the decimal at the same time - as I mentioned above - and seemed to gain a new understanding of their equivalence. As Alice concluded: "they mean the same number". Related to this understanding of equivalence, a few of the students became intrigued with trying several equivalent fractions to see what the table of colours would depict, allaying any small doubts they had that 1/2, 5/10, 20/40 were really the same number.

A few of the students were somewhat fluent at the outset with decimals (i.e. knowing that 1/2 is 0.5 and 1/3 is 0.33...), but most of the students seemed to have very little sense of which decimal would result from a given fraction, even with fractions whose denominators were multiples of 10. This is perhaps due to the situatedness of their fraction-decimal knowledge in classroom worksheets, but it would be interesting to see what impact their brief exposure to fraction-decimal pairs had had on their future classroom work with fractions and decimals. These insights highlight some of the basic conceptions with respect to fractions and decimals that students rarely have a chance to develop, yet that are almost assumed to be part of their ability to operate on fractions, to convert and estimate them.

I now turn to sense-making of the second type. Since many of the students experimented with changing the width of the table, they were able to see what the period of a fraction is, how long the period of 1/7 is, and how any multiple of the period of the fraction makes the colours in the table line up. These are not the kind of rational number characteristics and relationships typically taught in school curricula, but are both accessible and interesting for this group of students in the Colour Calculator environment.

Other than making these common realisations, the students embarked on quite individual, entirely studentgenerated investigations, in that I only proposed questions during slow-starting interviews that had already been posed by other students in this study. The table below presents a list of student investigations, in their words, along with a corresponding version I reformulated (and at times generalised) as a question in more conventional mathematical discourse.

Student question/ observation	My reformulation
How can I make the diagonals go in the opposite direction?	How are diagonal patterns related to the period?
Maybe the period is always one less than the denominator.	How is the period of the fraction related to its denominator?
I think that all the fractions with odd numbers on the bottom will repeat.	What values of the denominator yield non-terminating decimals?
The decimals stop when there is a two, five or ten in the denominator I wonder what happens with eight.	What values yield terminating decimals?
So with twelve on the bottom, there's an extra number before the repeating	When is a decimal eventually periodic?
I want to take away those colours that don't fit with the rest of the pattern.	How do you move the decimal point to the right?
Aren't there some numbers that have a totally random pattern?	What kinds of numbers neither terminate nor repeat?
I want to get the table all red	What fractions have a repeating expansion of a single digit?
I wonder what happens if I try three over seven now	How are fractions that have the same denominator related?
So what do I get if I add one over two to this one over three?	What is the effect when you add two fractions with different periods?
I think that since one over nine has a period of one, like one over three, then one over forty-nine should have a period of six too	What is the effect when you square a fraction that has a certain period?

This environment certainly prompted the students to make new understandings of fractions and decimals and, in particular, to explore characteristics and relationships they are not usually encouraged to explore. Students were engaged in problem posing, they experimented and used problem-solving strategies and expressed their findings to me: they were doing mathematics. The Colour Calculator environment highlighted some of the incomplete fraction and decimal understanding that these students had and helped them gain a new understanding of what a fraction *is*, as opposed to what you can *do* to fractions – add them, generate equivalent ones, etc.

Additionally, the Colour Calculator appears to be an environment in which students are interested and motivated to discover certain things about numbers, using fractions and decimals, that differ from the emphases in current school curricula. These ideas are not easier than the ones we typically emphasise, but, in this Colour Calculator environment, they are more relevant to students' personal and epistemological interests.

This brings me to my third conjecture, namely whether the Colour Calculator would provide a setting in which students could develop a more positive relationship with fractions and decimals. The only observations I collected related to this are the students' reflections at the end of their interviews. In these reflections, I asked them how they compared what they had just done with their usual mathematics tasks. I found it difficult to determine the cause of their unanimous belief that this environment provided them with a more positive experience

Comments such as it is "fun because you can work with patterns" or "good because it helps you out more" or "creative because you can make patterns" or "fun because you don't just have to look at numbers" suggest that the colourful patterns were enjoyable, but do not determine whether the students now had a different relationship with fractions and decimals than they did before. Some students may also have had positive experiences simply because they like working on the computer or because they like having an adult's attention and help And, for others still, the fact that they were not faced with a task at which they would eventually either succeed or fail (as is often the case in mathematics class) may have made their experiences more enjoyable.

That this third conjecture remains unclearly substantiated is due both to the paucity of data in this particular study and partly to the methodological challenges of assessing students' emotional responses. One way to assess a student's change of attitude towards fractions would be to inquire about her attitude both before and after interaction with the Colour Calculator – this might have provided some useful clues. However, a more satisfying way of exploring my third conjecture would be to try to ascertain whether a student who had developed some mathematical understanding of fractions and decimals in the Colour Calculator environment is subsequently more interested or confident in doing and learning mathematics that involves fractions and decimals (even outside this environment). This would be a significant achievement, given students' aversion to fractions and decimals, a significant topic they confront at precisely the time when they are consolidating their motivational attitudes toward mathematics (Middleton and Spanias, 1999).

Aesthetic possibilities

Aesthetic possibilities emerge both in terms of what mathematics students learn and the processes by which they learn it. Though the decision to continue each interview until the student had concluded an investigation was primarily one of convenience, in retrospect it was perhaps a crucial component of the process. The students were, as a result of this decision, given the opportunity to undertake their own complete investigations, from beginning to end, without the usual constraints of classroom pacing, synchrony and bells.

This privileged interaction resulted in a whole, unified and intrinsically satisfying experience for the majority of the students. Such an experience, especially if it also helps to transform a student's perception and meaning of objects or events, resonates strongly with aspects of Dewey's (1934) notion of the aesthetic experience, a clarified and intense form of his 'educative' experience. The qualities of experience afforded simply by allowing students to 'live' through a process during which their sensibilities are awakened through aesthetic engagement suggest possibilities for aesthetic experiences in mathematics learning.

I raised a question for debate of whether the kinds of mathematics that emerge from aesthetically-rich learning environments *should* or *could* form a worthwhile curriculum. If we adopt Bruner's (1969) definition of what is worth knowing:

> whether the knowledge gives a sense of delight and whether it bestows the gift of intellectual travel beyond the information given (p 39)

then only the *could* question remains Could such an aesthetically relevant curriculum achieve the coverage or continuity we demand of our curricula now? Would it challenge the mathematical structures that currently dictate the basic guidelines for identifying the content of the curriculum?

Acknowledgements
I wish to thank Bill Higginson for helping me develop my conception of aesthetic relevance in mathematics, Nicholas Jackiw for comments on earlier drafts and the National Research Council of Canada for financial support

Notes
[1] This notion was suggested to me by Martin Schiralli
[2] See http://www cecm sfu.ca for further details
[3] The calculator is part of a larger project called *Alive Maths* (on-line at http://math.ai.iit nrc.ca) which, as an internet-based environment, is accessible by students both at school and at home, and allows students to chronicle - in writing and through activity recording - their discoveries and questions on their own personalised web pages This work is being funded by SchoolNet through the National Research Council of Canada and owes much to the programming expertise of Stéfan Sinclair
[4] The calculator also had a function that converts each digit of a decimal string according to the chosen modulus (from 2 to 10). For example, 1/32 = .03125 becomes 01101 modulo 2 and is represented accordingly in the table of colours.
[5] See Goldin (2000) for a more detailed description of structured, task-based interviews and their methodological advantages
[6] It is interesting to recall that a regular calculator replaces its input with its output, so that a student calculating 1/7 on the calculator never actually sees both the fraction 1/7 and its decimal expansion simultaneously Though I am sure that the students think that a fraction and its decimal are equal, they seemed struck here for the first time by an ontological equivalence

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