Mathematics and Australian Aboriginal Culture

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In this paper we explore the status and role of knowledge for a non-Western culture, and examine aspects of that culture bearing on its accessibility to functional mathematics knowledge. We understand functional mathematics knowledge to be a most valuable commodity. Mathematics knowledge is a commodity because it not only permits persons to control vital aspects of day to day living, but possession of mathematics knowledge allows one to enter otherwise restricted fields of knowledge and endeavour. Possession of mathematics knowledge means power, because such knowledge can be traded in the market-place of problems and ideas

The traditional view of learning considers the mind as an empty bucket, and the stuff that fills it — knowledge — has an objective existence. However we believe mathematics knowledge must be considered from at least two frames of reference — from the point of view of the learner, and from the point of view of the teacher with whom the learner has sustained communication [Steffe, 1985b] A learner's mathematical concepts and ideas develop dynamically over time, and knowledge acquisition can be precipitated by appropriate teacher-initiated activities [Vygotskii, 1978] Knowing what activities might be appropriate to precipitate reorganisation of mental structures depends in part on the teacher knowing some general characteristics of learners with respect to the particular mathematics involved, and specific understanding of the knowledge state of the child in question. Learning occurs not only as a result of the interactions and communications between teacher and student, but also in a broader social setting, where powerful social, contextual, attitudinal and emotional forces can influence the child's progress in learning mathematics. [Bishop, 1985; Cobb, 1985a] As Saxe and Posner [1983] have concluded:

> The formation of mathematical concepts is a developmental process simultaneously rooted in the constructive activities of the individual and in social life. [p. 315]

We have suggested that the child's knowledge of mathematics and the teacher's mathematics knowledge are different. Teachers themselves differ in their knowledge of mathematics content, and in their conception of the essence of mathematics. One's conception of what legitimately can be considered to be mathematics will profoundly effect the way one teaches, as well as effect one's observations of others. Harris [1982] has noted misconceptions about Aboriginal number systems documented in the literature. For example, some writers have only accepted those number words which are regarded to be not etymologically related to body parts such as "hand," "fist" or "foot". An explanation for some of these misconceptions is that some observers rejected what others accepted as evidence of number knowledge because that evidence did not fit their conceptions of what constituted mathematical behaviour. It is useful to distinguish between the products of millenia of scholarly effort to logically organise, formalise, and document what we think of as mathematics, and the processes of individuals as they have pursued solutions to their problems and theorems We consider mathematics to be a human activity. There are similarities between a child's progression from understanding "ten" as a collection of singleton objects to conceptualising ten as a unit in its own right, and the graduate student of mathematics progressing from the notion of "group" as instances of particular phenomena exhibiting particular properties, to group as an algebraic class of objects invested with various relationships and properties.

We assume that there are broad processes of learning with their associated mechanisms, whatever they are, that are general for all peoples, for we have all come to survive, more or less, in our respective environments. While admittedly these environments may differ quite markedly, we simultaneously react to them, and act upon (that is, change) them. It is the details of those environments that determine in part what problems are seen to be needing solution In addition, it is a community's conceptualisation of itself and the very meaning of life that provides the energy and framework for identifying problems and constructing tools for their solution Advancement of the cause of mathematics learning of Aboriginal people lies at the intersection of research into mathematics learning processes and understanding Aboriginal world-views and how Aboriginal communities conceptualise their environments.

Aboriginal knowledge, cognition, and number

We began by asserting the value of mathematics because it can be considered to be a commodity that can be exchanged by those who have it for other benefits Palmer [1981] has argued that Aboriginal religious knowledge (which we understand to be the core of traditional Aboriginal knowledge in general) is used as if it were a commodity. He uses the metaphor of shareholders in a business to suggest that transferring knowledge is like redistributing shares between a great number of shareholders:

> A commodity is an item, used in an exchange transaction (or which may be used in an exchange transaction) over which identifiable individuals have rights of allocation."
>
> Identifiable individuals have rights over the allocation of certain items of religious knowledge. The allocation of religious knowledge is used in exchange transactions. He who passes on religious knowledge may do so by virtue of his rights to transmit that knowledge to others. The recipient is indebted to the bestower and is obliged to discharge the debt so incurred by making a return presentation."
>
> Aboriginal knowledge is exchanged, not at the expense of giving up one's holding, but at the cost of having a diminished proportional share in the business subsequent to exchange. One man's generosity in this diminishes the value of the knowledge he shares with all prior co-holders. In this sense (which is a special sense) transfers of knowledge in Aboriginal society can be likened to transfers of equity made up of shares where shares are regarded as values that can be reapportioned to incorporate more shareholders into the business at hand. [Palmer, 1981, p. 9]

Palmer goes on to say that since a body of religious knowledge is never completely transferred at any one ritual event, competence in such knowledge is acquired only after many years of ritual experience so that men and women who control the knowledge are usually middle-aged or older. At Yandearra, in the Pilbarra region of the northwest of Western Australia, where Palmer [1981] conducted his field work, a necessary condition of the transmission of some religious knowledge was that the young should "hunt for meat".

Aboriginal knowledge is bound up with the Dreaming. Palmer [1981] explains the Dreaming as:

> an ontological concept that is framed in metaphysical terms and is symbolic of the motive forces that characterise human interaction. By an appeal to an historically creative period, during which precedents for social and cultural action were inaugurated, Aborigines place their continuing practice of traditional action within a metaphysical context. Similarly, by appeal to a contemporary spiritual motive force they also place daily exchanges and interactions in the context of a metaphysical realm of discourse. The dreaming is then the concept, symbolic of the mythical source of all life, that both justifies and sustains the contemporary reality for Aborigines. [pp. 59, 60]

Sets of statements about the Dreaming are established as absolute truth by force of tradition and by social processes whereby people together may agree that something has the special status of knowledge when it is considered to be "unimaginably old, ordained before the Dreaming and has "always" been so". [Palmer, 1981, p. 61] The logic for establishing the truth of some knowledge is before an audience to refer to the place, object, animal, bird, or way of behaving as the "proof" that the truth of an assertion is established. Palmer offers as an example a person who makes a statement about the supposed actions of a mythological being at a particular place during the Dreaming:

Statement:	A mythological being, in the Dreaming, made a damper (bush bread made from flour and water) at this place. Accidently she trod on it.
Attestation:	A rock at the place is that damper. The rock has a mark at one corner that is the footprint of that ancestor.
Knowledge:	The rock is the metamorphosed damper, with a metamorphosed footprint. The reported actions of the Dreaming ancestor are true. [Palmer, 1981, p. 62]

Provided that all in a given population endorse the validity of the statement it will continue to be accorded the status of knowledge.

A three-part structure underlying the way the Yolnu people of Elcho Island in the Arnhem Land region of the Northern Territory perceive discrete quantities has implications for their development of number knowledge. A careful study of classificatory and evaluative systems used by the Yolnu by Rudder [1983] shows that the way they order the world of perception and experience is consistent with their supernatural-metaphysical world-view. Rudder analysed systems of classification in each of four domains: measurement, number, colour, and ethno-biology. Across these domains structural similarities were found, which indicated the existence of deeper common structures functioning at a cognitive level. Rudder identified four fundamental structures: taxonomic structures, anatomical structures, three-part structures, and paired relations. Taxonomic structures provide a framework for the perception of unity and harmony in the diversity of the perceived world. Anatomic structures complement the taxonomic, and are used in classifying perceptions of varieties of elements which make up each unified whole. Three-part structures and paired relations are used primarily in the qualitative evaluation of identities.

The three-part structure was found to be ubiquitous across all four domains as a framework for evaluation (see Figure 1). Rudder argued that there is a very strong relationship between the three categories and the Yolnu perception of the nature of existence (see Figure 2):

> In three of the ethnographic chapters (two, four, and five) I have shown that the Yolnu also superimpose a metaphysical three-part evaluation over the pragmatic one. These metaphysical evaluations clearly express the notions that entities, properties or qualities which exist in the physical dimension also have a spiritual existence, and there are transformations from one to another, and that the spiritual existence

is both the ultimate reality and the source of the physical [p. 202]

	Partial Expression	General Expression	Ideal/Intense Expression
size	moderately big	big	intensely big
relationship			
:between items	solitary item	pair	trio
between sets	solitary set	pair	trio
	(almost but not yet a relationship)	(relationship)	(complete or intense relationship)
non-precise grouping	moderately non-precise grouping	non-precise grouping	very much non-precise grouping
red (or any other colour)	moderately red	red	very red
living things	movement and power	breathing and reproducing	having and keeping laws
emu	emu chicken	young bird	mature bird
turtles	hatchling	young turtle	large adult (mature)
yam	small leaf	large leaf	old leaf

Comparison of three-part classifications made in different domains [Rudder, 1983]

Figure 1

The three-part structure matches qualitatively the assumed three dimensional expression of existence.

> By evaluating the successive changes of an entity as different expressions of a single quality across time it also provides support for the Yolnu perceptions of changelessness. The simple fact that the three-part structure is everywhere observable in the physical world forms an intellectual justification for the metaphysical assumptions. The relations between the three classifications at all times supports the perception of spiritual reality as ultimate. That is, in the classifications applied to the categories of life and existence, this support is clearly displayed. The pragmatic classification of the world enables the perception of three different categories of the quality "life". Superimposed over this the metaphysical classification enables the perception of the ultimate reality to be displayed as self evident, supported at all points by the evidence of the classifications of the physical world with its identical structure [Rudder, 1983, pp 210, 211]

a size

pre-expression of big	physically big			supernaturally big
not physically big	moderately big	big	very big	not natural

b animal

existence before birth	physically turtle			supernatural turtle
not turtle but turtle egg	hatchling	immature turtle	fully adult	beyond natural size

c colour

pre-expression of colour	physically colour			supernatural colour (pigment)
not colour (different colour)	moderately colour	colour	intense colour	solid colour (more than just visually colour)

d life-forms

not expressing life	with physical life			supernaturally alive
not alive	movement and power	breathing reproducing	keeping laws	beyond physical life

Relations between metaphysical and physical evaluations [Rudder, 1983]

Figure 2

Yolnu cognition and number

The three-part structure is apparent in the way Yolnu people perceive discrete quantities. Rudder [1983] differentiates between what he calls primary and secondary terms for identifying groups of discrete elements. There are primary terms for a single item, a pair, and a trio of items. A primary term refers to a precise grouping and can be used to classify sets of other differently sized groups. In contrast, a secondary term, while referring to a precise quantity, would not be used to categorise other groups. Examples of secondary terms are the words gon (hand) and rulu (a particular arrangement of five turtle eggs). There are also categories of non-precise relationships where the terms are concerned with groups in which the relationship between the items is non-precise, vague, or indistinct in definition, and the quantity of items in the group is not precisely determined. Figure 3 shows the structure of terms used to evaluate relationships between items within groups. There is an analogous relationship between the two sets of terms

In addition there is a relationship of transformation Rudder [1983] explains it thus:

> This occurs as the quality changes through a progression of categories of increased complexity, to the point where it is perceived to transform into a different kind of expression of the quality [p 64]

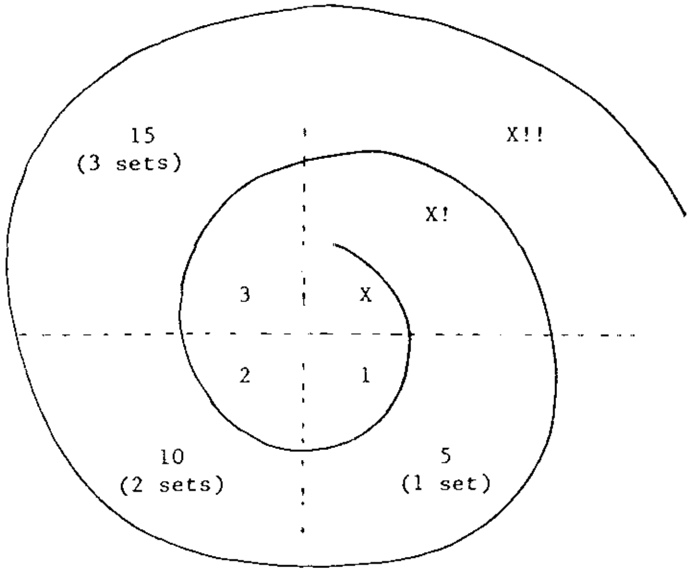
No Relationship		bäyŋu (nothing/none)	
Precise Relationships	waŋgany (solitary) (part of quality of relationship)	märrma' (pair) (quality of relationship)	ḻurrkun (trio) (full or complete quality of relationship)
Non-precise Relationships	märr dharrwa (moderately many) (moderately non-precise relationship or quantity)	dharrwa (many) (non-precise relationship or quantity)	mirithirr dharrwa (very many) (very much non-precise relationship or quantity)

Analogous structures of sets of terms used in evaluating relationships between items within groups [Rudder, 1983]

Figure 3

In the case of evaluations of discrete quantities the link between the two dimensions of existence has been portrayed in diagrammatic form in the shape of a spiral. Figure 4 shows how the continuation of the same quality merges into a new state of existence The feature marked "X" takes the place of the word baynu (nothing /none) Rudder suggests that in this case the X refers to the absence of any established relationship, or non-expression of the quality "quantity," rather than being a reference to the non-existence of quantity. In the context of evaluating quantities of turtle eggs, the solitary egg, and the solitary set of five eggs (or rulu — a square pryamid possessing a base of four eggs and a "head" of just one egg) are the basic units. A set of four eggs ("headless") is perceived as a condition of pre-existence (to the new state of existence, that of precise solitary set), and so corresponds to the condition "X".

In analysing the evidence for a Yolnu counting system Rudder [1983] has argued that one system, originally attributable to Chaseling [1957], is not a true counting system on two grounds — one being that it is not consistent with the Yolnu system of categorisations (see Figure 5) A crucial difference is that the five or hand set has been used as a unit of classification. A more recent system devised by Yolnu teachers is based on the turtle egg distribution system (see Figure 6) That system, Rudder argues, more accurately reflects the cognitive evaluative structures of the Yolnu, and represented a significant conceptual shift. For example, in the earlier system four items were seen as a duplication of a set of two solitary items That same quantity is now seen as a distinct set, not related to solitary items, but to the rulu (set or group) as an almost complete group As Rudder explains:



Relation between quantity perceived as solitary items and quantity perceived in sets [Rudder, 1983]

Figure 4

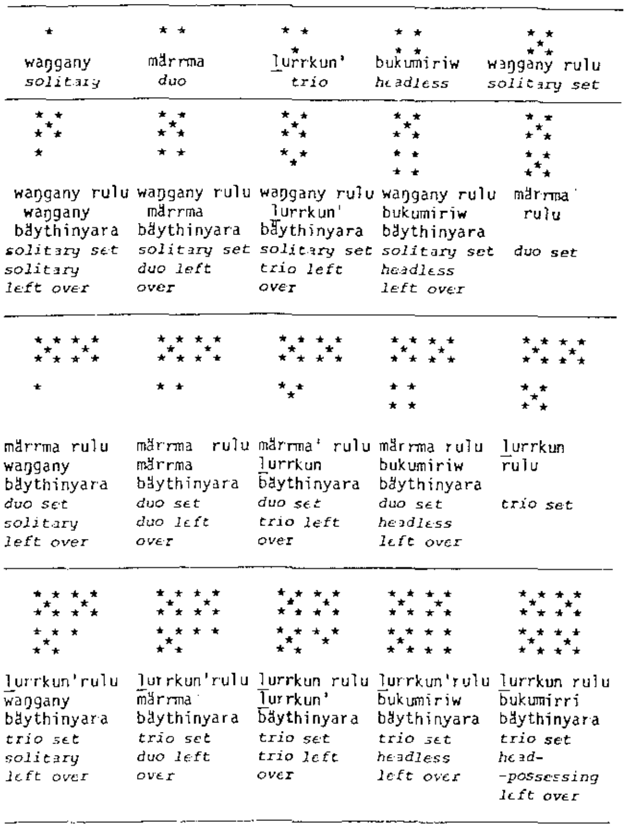
> Thus four items, a problem at the boundary of the two systems of classification, (i.e., by sets or by units), is treated in the first system as being "more than" the basic set of primary precise quantity units, and in the second system as "almost" or "not quite" one of the precise quantity sets The second difference between this system and the earlier one is that reduplication as a means of describing a larger quantity has been dispensed with. [p. 85]

We agree with Rudder that neither of the systems reflect an ordinal quality in essence Discrete quantities are evaluated on the basis of patterns of perceptual items rather than by counting processes, using a mechanism called subitising [Klahr & Wallace, 1976; von Glasersfeld, 1982] Western number words have imbued in them meanings which include the dual possibilities of both cardinal and ordinal qualities, and there exist elementary mathematics curricula in Japan and the United States which stress cardinal patterns to develop early number concepts [Hatano, 1982; Wirtz, 1980] How the strands of the spatial-figural (patterns) and the sequential-operational (counting) are integrated in the development of a mature concept of number has yet to be uncovered. [Cobb, 1985; Hunting & Korbosky, 1986; Steffe, 1985a] To what extent particular cognitive mechanisms allied to such strands are related to cultural milieu, and typify specific neural functioning, await clarification. The data reviewed here indicate that, for Aboriginal people at least, beliefs and culture provide a powerful organising framework for looking at the world,

Symbolised Quantity	Yolnu Term	Literal Translation	Supposed English Equivalent
	bäyŋu	nothing/none	0
*	waŋgany	solitary/single alone	1
**	märrma	duo	2
***	lurrkun'	trio	3
** **	märrma märrma	duo (and) duo	4
*****	goŋ waŋgany	hand solitary	5
*** *** or ***** *	lurrkun' lurrkun or goŋ waŋgany ga waŋgany bäythinyara	trio (and) trio hand solitary and solitary left over	6
***** **	goŋ waŋgany ga märrma' bäthinyara	hand solitary and duo left over	7
***** ***	goŋ waŋgany ga lurrkun' bäythinyara	hand solitary and trio left over	8
***** ****	goŋ waŋgany ga märrma' ga märrma bäythinyara	hand solitary and duo duo over	9
***** *****	goŋ märrma	hand duo	10
***** ***** *	goŋ märrma' ga waŋgany bäythinyara	hand duo and solitary left over	11
***** ***** **	goŋ märrma ga märrma bäythinyara	hand duo and duo left over	12
***** ***** ***	goŋ märrma ga lurrkun' bäythinyara	hand duo and trio left over	13
***** ***** ****	goŋ märrma ga märrma' ga märrma' bäythinyara	hand duo and duo duo left over	14
***** ***** *****	goŋ lurrkun	hand trio	15
***** ***** ***** *	goŋ lurrkun ga waŋgany bäythinyara	hand trio and solitary left over	16
. . .	. . .	. . .	
***** ***** ***** *****	goŋ märrma ga goŋ märrma	hand duo and hand duo	20

System purporting to enable counting from one to twenty [Rudder, 1983]

Figure 5



Patterns and terminology used in a Yolnu attempt to develop a counting system [Rudder, 1983]

Figure 6

including the categorisation of quantities of various kinds. Lancy [1983] has reported similar finding. Yolnu cognitive structures used to evaluate and classify elements of their environment may constrain certain possibilities for learning Western mathematics but may allow others. A challenge for the mathematics educator here is the question: How can whole number knowledge be taught if there are inhibiting factors which make a conception of ordinality difficult? That is, what can be done if physical collections of five are not usually considered to contain, at the same time, a collection of four, which in turn contains a collection of three, and so on? How is it possible to speak of five preceding six, or four "coming after" three? What does it mean to have a meaningful understanding of whole number addition or subtraction without the integration of cardinal and ordinal number relations? At the level of language, even in a bilingual program, how can we be sure that English and Aboriginal number words have similar meanings?

On the broader front, how does one teach the concept of five with its full relativistic meaning in a culture that views knowledge as absolute and changeless. Five in its abstract sense is not related necessarily to any particular physical attribute. It is a floating concept. If one is used to the idea of establishing the truth of some knowledge by attesting to its manifestation in a particular physical embodiment, what can five be for an Aboriginal? If knowledge of five is established by showing a physical arrangement of certain objects, how can five be something else? Indeed, why should it be?

Directions for research and development

Addressing the challenge of learning and teaching mathematics across cultures demands that we attend to the tacit assumptions about life and existence that, in the case of the Australian Aborigine, are very different from those of Western upbringing. The world view of a culture determines the way in which the data of experience is organised and classified. The work of Palmer and Rudder demonstrate this. Inevitably cultures and communities of individuals find themselvs having to adapt to changing circumstances. New problems and new distractions are an outcome of change, whether it be conquest, independence, drought, disease, new technology, or political power. We believe that in the long term mathematics will be more accessible to Aboriginal people if trouble is taken to fit that mathematics sensitively onto and around the beliefs, values, thinking patterns and problem solving processes contained in Aboriginal cultures.

To formalise a research program that will focus on the activities and processes of Aborigines that have potential for connections with mathematical concepts, techniques, and procedures we propose the following questions:

1. What problems arise in traditional environments which require application of mathematics knowledge for their solution?
2. What is the nature of the mathematical processes used to solve those problems?
3. How does the mathematics of a culture or community change in response to changes brought about by contact with a different culture or community?

A suitable term to describe data arising from such avenues of inquiry is ethnomathematics — mathematics used by a defined cultural group in the course of dealing with environmental problems and activities. Gay and Cole [1967] called it indigenous mathematics. In an earlier paper we discussed the potential of card playing as a platform for developing numeration knowledge [Hunting & Whitely, 1983]. Saxe [1982, 1983] has investigated how the Oksapmin of Papua New Guinea have adapted a body part counting system to solve problems resulting from the introduction of a money economy in the context of the Indigenous Mathematics Project [Lancy, 1983]. The Indigenous Mathematics Project documented over 200 different counting systems used in Papua New Guinea. Rudder [1983] has documented a number system developed by Yolnu teachers, and has shown how features of that system are more compatible with Yolnu thought than an already existing system of indeterminate origin. Rudder's study of the Yolnu people shows that basic premises of life and existence are not universally agreed upon, and suggests that Western and Yolnu logic and classificatory processes are not similar or even compatible. How is a Yolnu learner in a Western-influenced world to succeed with even those mathematical concepts and processes implicit in a local Arnhem Land lifestyle? The emerging field of ethnomathematics addresses these concerns. We would expect that the focus of such a research program would be to identify possible platforms for establishing number and measurement concepts, since these are basic to Western economic and technological knowledge. In this regard traditional activities involving quantifying, sorting, grouping, and sharing would be of interest. However spatial visualisation and geometric abilities are known to be highly developed in Aboriginal cultures, as demonstrated, for example, by Kearins [1981], Lewis [1976], and McCarthy [1960], and there may be links to number that can be developed via these means.

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