MATHEMATICS FOR LIFE: SUSTAINABLE MATHEMATICS EDUCATION

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We live in troubling times. The ecological systems that sustain life on earth are stressed gravely and degrading rapidly. History tells of several societies - the Mayans, Easter Islanders, and Sumers - whose inability to shift course, even when they recognized the harm caused by their unsustainable practices, led to their demise (Wright, 2004) Humans today are confronted by the challenge of managing multiple large-scale ecological problems simultaneously, including climate change, loss of biodiversity, and depletion of natural resources (IPCC, 2008; Millennium Ecosystems Assessment, 2005)

As a mathematics teacher and researcher, I am experiencing a growing disconnect between the preoccupations of my professional life and the increasingly loud calls around me to attend to the problems of ecological sustainability. For years it was easier to busy myself with metaphors of multiplication than to contemplate imminent environmental catastrophes But over time, I have realized that I cannot be a genuine educator and also avoid the greater challenges that will confront my students in the future Yet, it has been quite difficult to conceive of how my practice should change in order to respond appropriately to the challenges that we all face. I believe that many other mathematics educators are also facing these critical questions. And so I wonder: how should we reconcile the urgent need to act for the future with the practices of mathematics education of today?

This paper is an initial attempt to answer this question in my own practice I write it in the hope that it contributes to a much-needed sustaining conversation among mathematics educators about reorienting our shared practice I begin by examining the role that sustainability has played in education in general and in mathematics education in particular. I present a model of possible responses to sustainability in mathematics education and apply the model to two extended examples: *large numbers* and *chaos*. Finally, I reflect on the two examples to outline some possible features of *sustainable mathematics education*

Ecological sustainability and mathematics education

Ecological sustainability is a broad term with multiple contested meanings As Esbjörn-Hargens and Zimmerman's (2009) survey shows, numerous approaches to *ecology* exist, many of which are at odds with each other. These approaches are enacted simultaneously at various scales - the biological, the personal, the cultural, the social, the economic, and the biospheric. The term *sustainability* generally refers to the ability of living systems to endure over time; since the 1980s, it has been widely used to describe humans' long-term survival and wellbeing (see WCED, 1987).

Although environmental issues have attracted considerable attention in education since the 1960s, ecological sustainability became a major focus of education only in the 1990s (Palmer, 1998) Following the 1992 UN Conference on Environment and Development, environmental education assumed a more activist and future-oriented stance, as evidenced by the rise of *education for sustainable development* (Hopkins & McKeown, 1999), *transformative education* (O'Sullivan, 1999), *futures education* (Hicks & Slaughter, 1998), and *sustainable education* (Sterling, 2004). These trends share a common critique of the ways in which current educational systems perpetuate an unsustainable industrial/modernist model of growth (see Orr, 2004).

School mathematics has traditionally organized some of its applications around the needs of the moment This is why examples drawn from commerce, such as giving change and buying carpet, are so common in classroom teaching. Occasional references to the environment can also be found in past and present curriculum documents. But by and large, ecology has played only a negligible role in mathematics pedagogy. Sustainability has likewise attracted little attention in mathematics education research Why is this so? I believe that it is the legacy of Platonism. Mathematics is popularly conceived of as a pure body of knowledge, independent of its environment, and value-free (*e g* Hardy, 1940). From the Platonist perspective, connections between global warming and the topics found in mathematics textbooks, such as fractions or quadratic equations, are not readily apparent.

In the past two decades, social constructivist readings (*e g*, Ernest, 1998) and critical mathematics education (Skovsmose, 1994) have challenged Platonic assumptions about mathematics by underscoring the political and sociological dimensions of its teaching and learning. The Rethinking Schools movement (Gutstein & Peterson, 2006), for instance, has been instrumental in raising awareness about the ways in which mathematics pedagogy is implicated, both culturally and ethically, in issues of social justice, such as racism, equity, gender, and democracy. A critical stance could also be constructive for mathematics educators who wish to approach issues of the environment, such as climate change (Barwell, 2010) To date, however, these issues have not been a major focus of critical mathematics education.

And so ecological sustainability and mathematics education remain largely unconnected in the research literature Yet, many connections can be made as the following statement about food production illustrates:

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> The efficiency with which various animals convert grain into protein varies widely. With cattle in feedlots, it takes roughly 7 kilograms of grain to produce a 1-kilogram gain in live weight. For pork, the figure is over 3 kilograms of grain per kilogram of weight gain. For poultry it is just over 2, and for herbivorous species of farmed fish (such as carp, tilapia, and catfish), it is less than 2. As the market shifts production to the more grain-efficient products, it raises the productivity of both land and water (Brown, 2009, p 226)

This statement is qualitatively different from examples about giving change or buying carpet in that it could implicate learners in responsibility for the earth and compel them toward an ethic of conservation It discloses the reality that a bite of beef stresses the earth's limited agricultural resources 3 5 times more than does a bite of chicken It suggests that a diet of vegetables and herbivorous fish may provide a ready solution for eliminating over 65% of the pollution caused by protein production

Like most other information communicated about the environment, the statement relies on mathematical reasoning and numbers. Making sense of it requires some sophistication in proportional reasoning—a major strand of school mathematics.

The mathematics is not simple and any conclusions need to be worked out in larger systemic contexts that include science and society Issues of sustainability call for an interdisciplinary conversation Mathematics educators can bring important perspectives to bear on this conversation, due to their familiarity with the vast network of metaphors, exemplars, applications, and algorithms that underlie proportional reasoning (see Confrey *et al.*, 2009).

Integrating the environment into the discourse of the mathematics classroom signals the possibility of a more genuine mathematics education - one that is not so much about acquiring certain competencies but about noticing the world differently, seeing proportional reasoning in multiple contexts, making connections, and moving to ethical action as a result of increased awareness

Educational approaches to sustainability

Given the imperatives of sustainability, how might mathematics educators react to this call for change? Judging from the track record of environmental education outside mathematics - which runs the gamut from avoidance to transformation - a developmental stage model may be useful for anticipating the responses to sustainability in mathematics education The model that I propose adapts two existing stage models of approaches to sustainability to settings of mathematics education.

The first stage model is Sterling's (2004) model of educational responses to sustainability. It derives from Bateson's (1972/2000, p 279) logical categories of learning: *first-order learning,* which proceeds within agreed boundaries and does not challenge basic values; *second-order learning,* which reflects critically on the assumptions that govern first-order learning; and *third-order learning,* which involves a creative shift of consciousness made possible by deep awareness of alternative worldviews Sterling's model consists of three broad stages: *accommodation, reformation,* and *transformation*.

The second of the stage models is Edwards's (2010) model of organizational approaches to sustainability It applies to contexts of organizational transformation in general, rather than to educational contexts in particular It employs a developmental lens to identify seven narrower stages: *subsistence, avoidance, compliance, efficiency, commitment, local sustaining,* and *global sustaining*

Table 1 shows my combined reading of both models as applied to contexts of mathematics education In comparing the two models, I found that their stages correlate quite easily, and that the two models are complementary. Sterling's focus on education and educational biases clarifies how knowledge about sustainability is interpreted by educators at different stages. Edwards's focus on organizational change reveals stakeholders' power positions and worldviews on sustainability at different stages.

One practical difficulty that follows from Table 1 is that it is not easy to imagine how mathematics education might be enacted beyond one's present stage. In my case, for example, I could not quite see how the Local Sustaining level would be enacted. What would it mean for a mathematics educator to "value sustainability as a way of developing education into the future" and to "devise and implement transformation strategies for moving towards goals that support host communities"? These words were just abstractions until I shifted my thinking to specific examples. I will next discuss how mathematics teachers might approach two examples - large numbers and chaos - through the model's interpretive lenses of *accommodation, reformation,* and *transformation*

Large numbers

Humans emit 29 trillion (2.9×10^{13}) kilograms of carbon to the atmosphere each year. Like most other numbers that describe ecological quantities, it is a large number But how much carbon is this? We cannot readily imagine this amount, let alone have a felt bodily sensation of it This quantity is an abstraction that we put into the category of *large numbers*

Barrow (1992) distinguished between the *notion of counting* and the *notion of quantity*. Whereas *number sense* refers to humans' ability to transact numbers appropriately, *quantity sense* refers to humans' ability to comprehend magnitude and size. When it comes to large numbers, our number sense is almost entirely divorced from any quantity sense. Humans' ability to count - that is, to use numbers as symbolic representations of quantities - provides us with a powerful mechanism for storing, recalling, and manipulating cultural information. But humans' inability to feel large numbers is very problematic in our dealings with ecology and the environment.

Emotions play a crucial role in decision-making and human action (Damasio, 1994) If we do not *feel* numbers, then our emotional access to the physical phenomena they represent is much diminished The emphasis on number sense in mathematics education has led Wagner and Davis (2010) to caution that current curricular and pedagogical methods may exacerbate students' deficit in comprehending quantities They called for "mathematics classroom

Type of Educational Response (Sterling, 2004)	**Stages of Organizational Sustainability (Edwards, 2010)**	**Description**
Accommodation = Education *about* sustainability • Has a content/knowledge bias. • Can be assimilated easily within existing educational paradigms. • Assumes that knowledge about sustainability is uncontested and can be codified and transmitted	Subsistence	Sustainability is considered only as it relates to survival of current educational practices. Stakeholders are concerned solely with perpetuating current practices (e.g., *"Don't bother me. I have to prepare these students for the SAT."*)
	Avoidance	Sustainability is seen as an attack by opposition groups on the status quo. Stakeholders exhibit ignorance and apathy towards the negative impact of current educational activities on the environment (e.g., *"Math has nothing to do with sustainability. It's the science teacher who should be thinking about it."*)
	Compliance	Sustainability is seen as an imposition. Stakeholders conform to traditional ethics, and comply begrudgingly with top-down regulation as a way of circumventing more demanding regulation (e.g., *"If I solve a couple of mathematical problems about population explosion, then perhaps no one will bother me."*)
Reformation = Education *for* sustainability • Includes a values and capability bias • Involves some reformation of the existing paradigm, but essentially leaves it intact. • Still assumes that we know the values/knowledge/skills needed for sustainability, but includes critical thinking.	Efficiency	Sustainability is considered to be a source of potential profit/benefit (e.g., *'I can use examples drawn from sustainability to motivate my students to learn about logarithms."*)
	Commitment	Sustainability is valued for balancing educational, social, economic, and environmental concerns. Schooling is seen as connected with the outside community in a societal network. Stakeholders are committed in principle and go beyond regulatory compliance (e.g., *"Sustainability is the most important issue that our society faces. Topics of sustainability should be a large part of the curriculum in my math class."*)
Transformation = Education *as* sustainability • Knowing is seen as approximate, relational, and provisional. • Involves a transformative epistemic learning response by the educational paradigm. • The process of sustainable development is essentially one of learning, while the context of learning is essentially that of sustainability.	Local Sustaining	Sustainability is valued as a way of developing education into the future. Stakeholders devise and implement transformational strategies for moving towards goals that support host communities (e.g., *"Mathematics education itself is a living complex system. We should promote maximum vitality in the system for the benefit of students and their communities."*)
	Global Sustaining	Sustainability is embodied within all aspects of the educational process and is seen in global and intergenerational terms. Stakeholders make connections between multiple layers of purpose that include: physical, economic, environmental, emotional, social, and spiritual. (e.g., *"Based on my understanding at this moment, I would like to reshape mathematics education as an integral project which addresses every student's body, mind, and spirit, for the benefit of society and the planet at large. However, I realize that my actions might actually exacerbate the problem in ways I cannot see or understand."*)

Table 1. A stage model of approaches to sustainability in mathematics education

experiences that can help students feel the weight of number" (p. 48). Their call becomes all the more urgent in the context of sustainability. Mathematics educators can respond to it through accommodation, reformation, or transformation.

An *accommodating* response might be to include a new unit of study in the curriculum, under the heading *Orders of Magnitude*. Middle-school students would be taught and then tested on the use of small integers to describe the sizes of large numbers. For example, the number 1,250,000,000 is of order 9. Note that this educational response does not overcome the separation of number sense from quantity sense. The students are still engaging in activities that develop their number sense only.

A *reforming* response might recognize that much of our appreciation of scale is processed through our visual system and devise suitable experiential classroom activities. The film *Powers of Ten* (Eames and Eames, 1977), for instance, tries to impart a sense of the scale of the universe through a series of images. Wagner and Davis (2010) described how they used grains of rice in various containers to represent numbers of various magnitudes. Chemist Nate Lewis offered a particularly effective analogical account of carbon pollution:

> Imagine you are driving in your car and every mile you drive you throw a pound of trash out your window. And everyone else on the freeway in their cars and trucks is doing the exact same thing, and people driving Hummers are throwing two bags out at a time – one out the driver-side window and one out the passenger-side window. How would you feel? Not so good. Well, that is exactly what we are doing; you just can't see it. Only what we are throwing out is a pound of CO_2 – that's what goes into the atmosphere on average, every mile we drive. (Friedman, 2008, p. 34)

Lewis's analogy is powerful because it compares one type of pollution with another, one pound of carbon with one pound of trash. The image of freeways piled up with garbage arouses a physical sense of disgust, and is likely to open up an opportunity for critical discussion of carbon pollution among students.

A *transforming* response subsumes the accommodating and reforming responses and goes beyond them. It will see the value in learning about orders of magnitude, while discounting the strong focus on computational accuracy. It will embrace teachers' ingenuity at devising meaningful experiential activities that open up a space for critique. But it will also recognize that teachers are not the only source of ingenuity in mathematics classrooms, and that deconstruction and critique ought to be followed by innovation and transformation.

One transformative approach to the problem of feeling large numbers is to pose it directly to students as an intractable problem in mathematics. A generative prompt might be: *"Many adults are having a hard time comprehending large numbers and as a result find it difficult to relate to issues of the environment. How would you explain the meaning of some large numbers (for example, the number of kilograms of carbon emitted daily into the atmosphere) to adults in your life in order to move them to action?"*

This prompt suggests a new kind of school mathematical problem solving. Most mathematical problem solving in today's classrooms relies on the unchallenged assumptions that each problem has one correct answer and that the teacher knows this answer. Students' creativity is therefore limited to replicating solutions that are already known by an adult. In contrast, the solutions to many problems of sustainability are not known *a priori,* and in some cases there is no certainty that solutions can be found at all. A different order of ingenuity is required to approach these problems, one that we may call *radical creativity*. The prompt also shifts the responsibility of knowledge production from the teacher to the entire classroom collective. It connects knowledge with political action and can empower students to act locally to bring about change in their own communities.

Chaos

Chaos theory and fractal geometry provide another ready way to connect mathematics education to the environment. Chaos theory is the mathematics of complex dynamic systems, and fractal geometry is often described as the geometry of the natural world. Between them they provide formal and visual metaphors for understanding the nonlinear dynamic patterns of living systems.

Since the enlightenment, the way humans conceive of the world has been guided by the reductionist scientific paradigm, which maintains that complicated systems can be disassembled and reassembled at will. The mathematical equations of classical physics provide a fully dissociated description of nature and suggest that natural phenomena are predictable and can be controlled. Newton's second law, $F = ma$, for example, predicts with complete certainty that if the mass, m, is increased by a factor of 3, then the force, F, will also increase by a factor of 3. This linear mode of reasoning is at the basis of every mathematical equation that we teach at school.

But complexity science, whose early roots can be traced to Poincaré and the invention of chaos theory (Waldrop, 1992) shows that complex systems are holistic, indivisible, and do not lend themselves to piecemeal analysis. They are open, evolving systems that maintain their identity in the face of constant environmental flux through the iterative processes of self-organization (autopoiesis) and emergence. Autopoiesis employs two types of feedback: negative feedback regulates activity and keeps it within a set range, while positive feedback amplifies and can drive the system towards instability. Unstable systems far from equilibrium may reach bifurcation points at which new forms of organization emerge. We can think of the emergence of increasingly more complex novel structures as the creative dimension of living systems.

Self-organization and emergence are nonlinear dynamic processes. While linear systems change smoothly in response to small influences, nonlinear systems can be very sensitive to initial conditions and tiny perturbations because of the amplifying effects of feedback. Nonlinearity places complex systems beyond human capacity to predict and control. As Meadows (2005) observed, the most we can do is try to encourage the structures that help complex systems run themselves.

Complexity science sees nature as whole: interconnected, seamless, and organic. Current structures of school mathematics generally do not reflect or support this vision. The binary *right-or-wrong* logic enacted repeatedly in school mathematical discourse presupposes absolute certainty. The overriding emphasis on quantification and measurement reinforces the belief that apsects of our world that can be quantified are more important than those that cannot (see Baker, 2008). The systemic connections between the measurable and non-measurable – *e.g.* between rapid economic expansion and ecological values – are rarely made explicit. And since values themselves cannot be measured, mathematics comes to be regarded as value-free. Again, chaos can be taught through an accommodating, a reforming, or a transforming approach.

Chaos theory derives from the study of nonlinear differential equations that is far beyond the level of high school mathematics. Any *accommodating* curriculum of chaos that focuses primarily on the mathematics will have to treat the theme of chaos broadly, rather than emphasize mathematical detail. A good example of such treatment is provided by Burger and Starbird (2005), who present non-linearity through difference equations and Julia sets.

The theme of chaos very much lends itself to a *reforming* approach that connects mathematics with the environment. Reforming teachers could critique linear notions about sustainability by explicating and elaborating the meanings of complex dynamics. For example, scientists warn that the earth will warm up by 1.1–6.4 degrees Celsius by the end of the century (IPCC, 2008). A common response to this warning is to dismiss it with the thought, *"That's not too bad. I actually prefer a slightly warmer winter."* This line of thinking is strictly linear in presuming that a small rise in the earth's average temperature would lead to a small fluctuation in daily climate. Unfortunately it does not apply to the world's nonlinear weather systems, since it does not take into account the greater extremes that increases in standard deviations bring. It also does not take into account the vast impact of wider climate fluctuations on ice sheets, oceans, storms, and crops. A surprising, yet telling, statistic is that the difference between the earth's temperature today and in the last ice age is only 5–6 degrees Celsius.

This example suggests that a descriptive modeling approach may be an effective means with which to explore chaos with our students. This approach would rely on technology to facilitate simulations of phenomena in multiple variables, such as weather patterns. [1] The models could accept probability distributions or even difference equations as inputs, and set in motion iterative simulations that employ a mix of stochastic and chaotic processes. Since multivariate models overcome the common restriction in school algebra of using only single-variable functions, their descriptive power far exceeds that of algebraic equations.

Descriptive modeling is a powerful problem-solving tool in cases where a single approximating formula does not suffice. It would support a problem-based pedagogy in which teachers and students search for the right mathematics required to make sense of real-life problems. The focus of teaching and learning would shift from prescribed lists of mathematical topics to identifying, selecting, using, and evaluating appropriate mathematical processes. The students' toolbox would be extended beyond algebra and allow far greater versatility. It would include stochastic processes, which are arguably more suited to understanding natural phenomena than deterministic ones (Eigen & Winkler, 1993). Information within the models would be represented both numerically and visually, and students would employ both quantitative and qualitative reasoning techniques to draw conclusions from it.

A shift in mathematics education from algebraic equations to qualitative visual reasoning and descriptive analysis will clearly be *transformative*. At the same time, it is easy to think of other transforming opportunities that teaching chaos affords. Concepts such as nonlinearity, emergence, and wholeness carry deep metaphorical meanings that can reshape our understanding of causality, creativity and spirituality (Briggs & Peat, 2000; Juarrero, 1999). It is up to us, educators, to choose to what degree we are prepared to engage analogical reasoning to enable new understandings of humanity's place within the environment.

The metaphors of chaos show that humans, far from being separate life forms in a controllable universe, are systemically implicated at every level of life on earth (Briggs & Peat, 2000). As an example of how these metaphors may transform human action, consider the common belief that ecological problems are just too great for any one person to do anything about. This belief is founded on a linear argument that proceeds by quantitative comparison: *"The problem is very big. I am small. Big is greater than small. So there's nothing I can do."* This line of reasoning may lead to resignation and inaction on the part of individuals. Chaos can help change the way we think about power and influence. It teaches us that complex systems cannot be controlled, but can be accessed and perhaps influenced through the myriad of feedback loops they contain. This notion has been illustrated by Lorenz's (1972) metaphor of the butterfly whose flapping wings in Brazil could set off a tornado in Texas.

The metaphor of butterfly power is very empowering. It suggests that each of us individually can make a difference, and that the consequences of our actions may be far more profound than we expect. Since the implications of our current actions cannot be predicted, butterfly power also calls on us to act with humility. And so, the mathematical notion of chaos gives rise to a new ethic: attentiveness to the present as a way to act right for an uncertain future (see Varela, 1999).

Sustainable mathematics education

The extended examples of large numbers and chaos allow us to discern some likely defining characteristics of *sustainable mathematics education*.

Sustainable mathematics education is the project of reorienting mathematics education towards environmentally-conscious thinking and sustainable practices. It is a change effort that we cannot afford to ignore. Even though sustainable mathematics education is motivated by urgent issues of survival, it need not adopt the pessimistic tone of many writings on ecology (*e.g.*, Deffeyes, 2001). Dire projections are typically founded on the idea that humanity has already passed, or will soon pass, an ecological point of no return – a climate tipping point, peak oil, and the like. While pessimism may be an appropriate response to current conditions, it is neither helpful nor constructive for educators Since school is a social institution situated at the intersection between present society and the promise of what society may become, educators are more likely to succeed in their work with messages of hope and possibility. Some writers (*e.g.*, Hawken, 2007; Edwards, 2005) have suggested that humanity is at the threshold of a sustainability revolution no less significant than the Industrial Revolution It will transform our unsustainable industrial practices and set humanity on a new course of ecological harmony for the future

The notion of *transformation for sustainability* provides mathematics education with a clear generative purpose What are some pathways for mathematics educators into the sustainability revolution? Brown's (2009) comprehensive survey of practical solutions to problems of sustainability serves as one example of a possible launching point for a sustainable mathematics curriculum of action and hope Mathematics plays an integral role in many of these solutions: renewable power, smart energy grids, reforestation and carbon sequestering, changes in food production and consumption, and a cradle-to-cradle, zero-waste, new materials industry The current lack of political will or urgency to implement sustainable solutions on a large scale can be understood, according to Sterling's and Edwards's stage models, as arrested development in the way we see nature and our role in it I believe that this is where mathematics teachers can make a real difference. Sustainable mathematics education can help evolve the ways in which we see the world by evolving the ways in which we understand and use mathematics.

Sustainable mathematics education is about seeing the world anew through renewed mathematics. It is concerned not only with feeling large numbers, but also with feeling the global situation. It trades linear metaphors of certainty and separation for complexity metaphors of possibility and connection It helps us relinquish our desire for deterministic predictability and embrace the contingency and stochastic probabilities of each living moment. It turns mathematics from a collection of objects, or a series of competencies, into an open-ended state of observing the world. It aims towards a more complete and appropriate mathematics, and from this position it calls on us to engage in ethical action for healing the world

Having been shielded by the perceived neutrality of our discipline, mathematics educators are latecomers to environmental education One benefit is that we may sidestep mistakes made by those who preceded us. We can recognize from the start that what we are aiming for is a paradigm shift and that accommodating responses alone will not be enough:

> [T]he crisis/opportunity of sustainability requires second - and where possible - third order learning responses by cultural and educational systems. There is a double learning process at issue here: cultural and educational systems need to engage in deep change in order to facilitate deep change - that is, need to transform in order to be transformative (Sterling, 2004, p. 15)

Some mathematics educators are likely to adopt accommodating approaches at the start and they are to be welcomed for taking steps in the right direction. But in setting the more ambitious end goal of transformation early on, and in promoting awareness around it, we may create the conditions necessary for the emergence of second- and third-order learning responses.

Sterling's (2004) quote points to the interdependency of multiple co-implicated systems in sustainable mathematics education: learning systems, ecological systems, cultural systems, and systems of mathematics and science The nested, self-similar nature of these systems suggests that we should promote maximal vitality and co-enact sustainable practices in all of them simultaneously. A paradigm shift of mathematics education, founded on metaphors of chaos and complexity, would recognize that the mathematics class itself is a living complex system, integrally embedded and open to exchanges with its environment (see Davis & Sumara, 2006).

Just as the borders between class, school, community, society, and ecology are likely to be continually challenged and blurred, so will the disciplinary boundaries between mathematics and other fields The descriptive modeling approach discussed earlier, for instance, demands interdisciplinarity if its models are to be useful for the analysis of real-life phenomena. Admittedly, many of today's teacher education programs tend to favour disciplinary specialization and thus may leave classroom teachers poorly equipped to act as interdisciplinary authorities. Complexity thinking suggests that a new, and perhaps more effective, kind of interdisciplinarity is needed, one that does not depend on one individual to be knowledgeable in every field. The new interdisciplinarity, which we may call *transdisciplinarity*, consists of decentralized networks of specialists who work in concert towards a common goal (Davis & Sumara, 2006). A joint collaboration of mathematics, science, and social studies teachers on a common modeling project would be an example of transdisciplinarity in a school environment. Transdisciplinarity is further enabled by network technologies, such as the Internet, which allow ready access to diverse communities of disciplinary experts.

If mathematics education is to undergo transformation, we would be wise to start by transforming the way reform itself is done One of the lessons of chaos is that creative emergence cannot be controlled top-down. It is a bottom-up project that involves the diverse contributions of many interacting participants. We are these participants - educators, researchers, and students who are passionate about mathematics and the role it can play in the world. [2]

A new ethic has presented itself to energize our practice with purpose and meaning - the ethic of mathematics for life.

Notes

[1] *E.g.*, the Microsoft Excel add-on Crystal Ball.

[2] The website www.sustainableMathEd.com has been established as a forum for all who are interested in the emergent dialogue about sustainable mathematics education Please join us there to share your thoughts and resources

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If the view is adopted that knowledge is a conceptual means of making sense of [one's] experience, rather than a representation of something that is supposed to lie beyond it, this shift in perspective brings an important corollary: the concepts and relations in terms of which we perceive the experiential world we live in are necessarily generated by ourselves [...] it is we who are responsible for the world we are experiencing.

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