SEEING MORE, SEEING DIFFERENTLY AS MATHEMATICS TEACHER EDUCATOR RESEARCHERS

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The following visualisation has been used by Tracy with different groups of prospective secondary teachers of mathematics during a session with a focus on algebra. The words are spoken aloud, and the prospective teachers are invited to work with what they see, their mental images. We invite you to do the same.

Imagine a cube, the cube is solid

make it grey, inside and out so that if you were to dissect it, every exposed surface would be grey

make your cube bigger

make it smaller

spin it around in different directions

make sure you get to see all of its vertices

now paint your cube red on the outside only and fix it in one position, the way it would sit naturally if placed on a table

make a series of cuts through your cube

the first set of cuts are horizontal, parallel to the table and equidistant from one another

make two further sets of equidistant cuts, this time vertical and perpendicular to one another

your cube should now be divided into a set of equal sized cubelets

tease the cubelets apart so you can see the grey insides of the original cube exposed, play around with this new structure trying to see the different types of cublets, their positions and relative numbers of red faces

What did you see? Perhaps you experienced a sense of ambiguity, as was the case for some of the prospective teachers in Tracy's algebra session? Perhaps you are left with a clear image?

Beginning this article with a visualisation as we have, you might be expecting a discussion of the use of visualisation as a pedagogical tool, perhaps with teachers of mathematics or students of mathematics. This would indeed be commensurate with much of Laurinda Brown's work that has undoubtedly influenced our use of visualisation. However, the focus of this article is on our learning as *mathematics teacher educator researchers* (MTERs); we turn to the use of this visualisation as one context for our learning later in our discussion. Teaching on the same one-year course for prospective secondary teachers of mathematics, we are committed to improving our practice as MTERs and in this article we aim to articulate a way of working together that supports our learning and development of expertise. We hope that this article makes available some of the enactivist principles that guide our work as MTERs. We do not present our approach as a well-defined way of working, to be imitated, but as an account of our emergent thinking as we interrogate our practice.

Learning as seeing more, seeing differently

In her discussion of the process of researching as an enactivist mathematics education researcher, Laurinda identifies learning as "seeing more, seeing differently, in a recursive process linked to actions in the world giving feedback leading to adapted actions until the behaviours become effective" (L. Brown, 2015, p. 192). In the journey to becoming expert, whether engaging with mathematics or with those engaging with mathematics (as a mathematics teacher) or with mathematics teachers (as a mathematics teacher educator), we are expanding the set of what is available for perception and action in the moment, whilst being able to recreate the awarenesses that lead to in-the-moment perceptions or actions. Effective behaviours are those which are "good-enough-for" (p. 192) us to function without registering a problem; when our actions are other than good-enough-for, our thinking shifts to the problem itself. Such problems are brought into awareness through perturbations that trigger the recursive process linked to actions which, for us, develops our expertise as MTERs. We identify instances in our practice with groups of prospective secondary teachers of mathematics that we have experienced as perturbations, feelings of discomfort, of not knowing what to do. Reflecting on these instances will often provoke dialogue between the two of us, which we sometimes audio record in order to work on specific points in our practice. In this article we present three vignettes, each taken from the transcript of one such recorded dialogue, which have been edited for clarity. The first vignette relates to Tracy's use of the visualisation in one particular algebra session with prospective teachers.

We see several purposes for presenting this as our first vignette: to point to how an instance of practice experienced as a perturbation can provoke dialogue; to exemplify dialoguing as a process; to offer one way of how we come to see more, see differently as MTERs and how we might support our prospective teachers in working on their seeing; and to illustrate how we work on a problem to inform future actions which might be good-enough-for. We appreciate that through articulating these purposes, we inevitably lose something of the complexity involved in learning as MTERs. Nonetheless, we see value in separating these ideas for the purpose of analysis and so we will comment on the first vignette under each purpose in turn before re-engaging with some of the associated complexities.

Vignette 1

Tracy We talked before about an algebra session where I used a visualisation. I hadn't spent long enough working through what I was going to say in setting up the visualisation, so it wasn't a surprise that there were a few different versions of what people were seeing.

Julian What people were seeing as the image?

Tracy Yes, their mental images. Having said that, if I had somehow set it up so perfectly that everybody saw the same thing, then we wouldn't have spent that period of time testing out one another's images, I had to work quite hard to make sure that happened and that was a good discipline for them to experience I think.

Julian I think you said that you'd commented along those lines during the session, something about it being important to spend time making sure everybody was seeing the same thing.

Tracy Yes. I think I might have said something similar doing this in a mathematics classroom though so that's not something new.

Julian So, what is new, what is different now?

Tracy Someone from the group asked a question while I was setting up the visualisation and I refused to answer it, I think maybe I gestured something to communicate that, when it happened, and then much later on, I returned to it and addressed my not answering someone's question explicitly. Then I think I said something like, "something you need in the classroom, if you're going to do visualisation, is to establish the rules, and I don't think I established the rules clearly". I don't think I would have said that in my classroom at school. I am being explicit about my decision making.

Julian For me there is also something there about when to step in and step out of the mathematics.

Tracy Yes, a splitting of my attention in that moment, I knew it had to be dealt with, but not at that time, it got logged as something that had to be returned to. It's not that they must do visualisation, but that if they choose to, there are some rules, rules that I had conviction about as a teacher. I guess my conviction now, as a mathematics teacher educator, comes from these experiences in the classroom.

Perturbation provokes dialogue

Through the dialogue in the vignette there is a sense of an instance of practice that had perturbed Tracy, a feeling of discomfort staying with her. As with interactions fostered and valued in our work with prospective teachers, we live the process "of staying with the detail of an experience without judgement or justification to allow new awarenesses to arise" (L. Brown, Helliwell & Coles, 2018, p.112). We have come to notice feelings of discomfort as indications of something unresolved that might need to be worked on retrospectively. In the instance of practice referenced in the vignette, the opening turn indicates that this particular episode had already appeared in our dialogue, ("We talked before about…"), but it arises again. The re-emergence indicates (for us) that the perturbation Tracy had experienced in the session still carried (for her) the sense of something unresolved, and her educated awareness of this sense made possible the dialogue presented above.

Dialoguing as a process

As MTERs reflecting on ourselves and our practice, we engage in a "coming-to-know process based in dialogue" (Hamilton & Pinnegar, 2014, p. 153) which we see as both a way of supporting our learning as MTERs and as producing data which we interrogate in order to conceptualise our learning. In the vignette, we see as central to the process of our dialoguing Tracy's awareness of a sense of something unresolved, supporting each of us in seeing differently. Our focus on the *process* of dialoguing arises from our enactivist positioning, to which we turn later. We see a significant feature in the shifting points of attention as the dialogue proceeds, with what comes into awareness being contingent on the unfolding interactions. Tracy locates her sense of discomfort in the group not getting to a shared image; Julian offers a prompt to see differently, ("What is new, what is different now?") and Tracy reconstructs the detail of her practice before drawing out a new awareness, ("I don't think I would have said that in my classroom at school. I am being explicit about my decision making."); this provokes Julian to articulate a new awareness, related to changing frames and, in turn, Tracy responds with a further different way of seeing the episode, articulating a conviction about rules in the classroom ("I guess my conviction now, as a mathematics teacher educator, comes from these experiences in the classroom."). We acknowledge that the account in this vignette might raise other questions about the use of visualisation but, as we have noted, our focus here is on offering an example of the dialoguing process.

Seeing more, seeing differently with others

As has been identified, we see our dialoguing as leading to learning, *i.e.,* a way in which we engage in seeing more, seeing differently. Our work with groups of prospective teachers attunes us to such speaking and listening, where "we are explicit about a way of working where they share details of their practices and listen to others to extend their range of strategies, not judge what someone else offers. Over time, the group learns to trust this process and shares more openly in learning conversations" (L. Brown, Helliwell & Coles, 2018, p.114). We do not seek to define a singular way of working with conversation that must be applied universally, since we see each person bringing a distinct system of actions, perturbations and responses. We do, however, see these ways of doing and knowing, synonymous in an enactivist framing (Maturana & Varela, 1987), as generating opportunities for seeing more, seeing differently at every scale of working with others. One example offered by Laurinda arises from the well-established context of a group of researchers and teachers viewing the same episode from a video recording of a mathematics classroom. Through the discipline of all observers reconstructing what had been viewed, it is possible to move beyond what we can see individually, allowing "us to work on our multiple perspectives and work with each other to see what each other sees" (L. Brown, 2015, p. 193).

Working on a problem

In sharing that "I hadn't spent long enough working through what I was going to say in setting up the visualisation", Tracy engages with the provocations of dialogue, supported by Julian. Through the process of dialoguing, we experience an openness in both talking and listening. According to Kahane (2004), "if talking openly means being willing to expose what is inside of us, then listening openly means being willing to expose ourselves to something new from others" (p. 73). As Tracy talks openly about her experiences of working with the visualisation, Julian listens openly, aware that his listening will be shaped by his own history but always valuing what is being said; we each come to the process with a commitment to interrogate the judgements that "co-emerge" (Davis, 1996, p. 10) in the space of the dialogue. Seeing more, seeing differently emerges in this case not so much as a strategy that might inform future action but an awareness of a sense of conviction, centred here on the significance of setting up visualisation in a classroom, shown particularly in the final turn of the vignette. When such awarenesses are carried into subsequent instances of practice, we move forward in the recursive process that might, again, see us adapting our actions. We offer our second vignette here to further illustrate the recursive process of working on a problem together.

Vignette 2

Tracy I just didn't feel like I could do it. I didn't feel like I could just go and tell this 25-year-old man to stop rocking on his chair, whereas if he was a 16-year-old boy, I wouldn't have any issue doing it.

Julian So, you saw this as an issue with teaching adults?

Tracy I didn't know what else it could be, I mean, they're adults and they're children, that's the difference right, but it's not that, it just didn't make sense at the time.

Julian And now?

Tracy And now I'd quite happily tell someone not to rock on their chair in the PGCE group, and I think the difference is how I see myself, in that role, not as authoritarian or something, but that I'm modelling being in a classroom, and suddenly then it's okay. So, I'm doing it for a different reason. I'm not trying to teach him not to hurt himself. I mean he might do, he might well hurt himself, but, it's then not about me and my issue that they're adults. That's not the issue anymore. The issue is that I want that room to feel like I want their classrooms to feel; safe and respectful.

Julian Modelling being in a classroom feels like another one of those tenets.

Tracy Yes, it is! and suddenly, I've got this conviction and now I can go and tell people to stop rocking back on their chairs.

In this vignette, Tracy arrives at an articulation based on a sense of an emergent conviction. A problem for Tracy in her practice had arisen when she felt unable to act when presented with a situation (a prospective teacher rocking back on his chair); this was experienced strongly by Tracy as a perturbation, giving her feedback in that moment that her (non-)action was not good-enough-for. The dialogue presented in the vignette illustrates us working on this problem, seeking a resolution, or a possible future action that is good-enough-for: "Suddenly, I've got this conviction and now I can go and tell people to stop rocking back on their chairs." We see working together as positioning ourselves openly in order to work on changing our understanding and our practice as MTERs, as "intimate scholars" (Hamilton, 1995, p. 30).

Intimate scholarship

In framing our approaches to learning as seeing more, seeing differently, we borrow the term intimate scholarship as a label. Intimate scholarship is based in "relational ontology and oriented toward the particular" (Hamilton & Pinnegar, 2015, p. 69). By 'ontology' we are referring to fundamental assumptions about reality, in our case, the reality of our practices as MTERs. As Slife (2004) suggests in his examination of the character of practice, practices are "inextricably intertwined with their concrete contexts and cultures, they cannot be abstracted from them" (p. 158). By using 'relational ontology' we discern that practice exists only in relation to the particular situations and contexts in which they occur. As enactivist researchers, we place emphasis on such relationships between ourselves and the environment so that through

these relationships and interactions, "both our structures and the structures of the environment change" (Lozano, 2015, p. 224) through a process of co-emergence. Within a relational ontology, practice originates from non-deliberate understandings rather than rational calculations of theory and in this sense, practices are more "pre-theoretical than theoretical, more concrete than abstract" (Slife, 2004, p. 157). Thus, our research is grounded in relational ontology such that we make no claims to knowledge (epistemology) but instead are concerned with understanding and accurately portraying our own realities (ontology) as MTERs, as we are learning. Along with using dialogue as a process of coming-to-know, a grounding in ontology, and bringing openness to interpretation, Hamilton and Pinnegar draw out two further characteristics of intimate scholarship: valuing the particular and positioning in vulnerability. Because of our orientation toward our particular practice, we are positioned as both open and vulnerable; only then are we able to uncover our embodied knowing as MTERs through a coming-to-know process based in dialogue. In other words, only through interrogating our practice can we work on changing ourselves.

Characterising our research: an enactivist self-based study

One corollary of researching as intimate scholars, is that intimate methodologies are utilised. Self-based methodologies such as autoethnography, action-research, narrative writing or autobiography are some of the most commonly cited versions. During the symposium on learning in honour of Laurinda, Olive Chapman gave a seminar on the use of self-based methodologies in research on MTER learning. She talked of the recent meeting of PME-NA where she had been involved in a working group (Suazo-Flores, Kastberg, Ward, Cox, & Chapman, 2018) whose interest was in developing use of studies within mathematics education that privilege the self. Olive brought to our attention that journal articles documenting self-based methodologies in 'prestigious' mathematics education journals seem to be the exception (*e.g.,* Hjalmarson, 2017). One plausible reason for this dearth of self-based research within mathematics education journals relates to the well-versed criticisms of any research that values the particular, especially when the particular is the researchers themselves and their own practice. The criticisms we refer to here concern the value and usefulness of such research, the applicability to the research community more broadly, and the legitimacy or trustworthiness of any research outcomes.

In contrast to the shortage of papers within mathematics education journals, self-based study is a well-established genre of research within the teacher education community more broadly. Much activity originated from a group of teacher educators who engage in what is now known as 'Self-study of teacher education practices' (S-STEP) and who have published abundantly within this genre. For scholars within the S-STEP community, one response to criticisms of the value of such research comes through addressing the so-called "so-what question" (Bullough & Pinnegar, 2001, p. 15). We interpret this so-what question as along the lines of *how does what we have learned about our own practice move scholarship on and extend the research conversation in teacher education and educational research more widely?* As intimate scholars, one way of answering the so-what question is to provide an "intimate portrait that reinvigorates curiosity" (Hamilton & Pinnegar, 2015, p. 119), in the wider research community about the practices and expertise of MTERs. Such intimate portraits can uncover new insights or offer deepened understandings about phenomena that may have previously gone unquestioned within our field. In other words, these portraits can disrupt the status quo more generally, and support a seeing more, seeing differently within the field of educational research. For enactivist researchers "the researcher and the object of research are seen as deeply connected, and constantly changing" (Lozano, 2015, p. 224). This co-dependence is intensified further when the researcher's practice is the object of research. We argue, as we set out below, that careful attention must be paid to *decentring* the self within self-based research whilst, at the same time, acknowledging the co-dependency of the researcher and the researched.

Decentring our-selves

Paying exclusive attention to three of the five characteristics of intimate scholarship: valuing the particular; positioning in vulnerability; and bringing openness to interpretation, would correspond to acting from the centre, since valuing the particular approximates to a localised view, and vulnerability and openness can be seen as both qualities of the researcher and the research. Taking seriously the so-what question requires us to decentre ourselves from our self-based research by paying attention to all five characteristics of intimate scholarship, the additional two being grounding work in relational ontology; and using dialogue as a process of coming-to-know. We look at both of these characteristics more closely from the perspective of decentring ourselves.

Taking an ontological stance that orients our research toward the relationships between us, our environment and interactions between ourselves as researchers working on understanding our practice as MTERs, our gaze is not upon ourselves, as in forms of psychological research on consciousness. Instead, our focus is on the emergent activity within the relations and interactions just described. Our way of staying with the emergent activity is to draw on *co/autoethnographic* (Coia & Taylor, 2005) methods that include us engaging in the process of dialogue as coming-to-know, allowing us to create spaces for exploring connections between the personal and the cultural, the individual and the shared, ourselves and our practice. We form a process by which we are examining how we, as MTERs, are learning, with the 'co' of co/autoethnography indicating that *together* we have drawn on autoethnographic methods whilst opening up to shared analysis. The set of labels used for such autoethnographic applications is large (Chang 2008), provoking close consideration of the details of one particular way of working. As we move into a space of co/autoethnography, we write and speak into one another's lives, "investigating our own selves and engaging in self/other analysis" (Coia & Taylor, 2005, p. 26), in order to understand and develop our approaches as teacher educators (Coia & Taylor, 2009; Butler & Diacopoulos, 2016). For us, the

co/autoethnographic process does not enable our learning; our combining in the process is itself the learning. In the space of intimate scholarship, we engage in dialogue in order to hear ourselves and each other. This dialoguing, in itself, creates for us new meanings, new ways of seeing more, seeing differently.

We look to the co-emergence of shared meaning: we are co-implicated in our process of learning as MTERs and we strengthen our decentring by combining key features of enactivist research, namely the "importance of working from and with multiple perspectives, and the creation of models and theories which are good-enough *for*, not definitively *of*" (Reid, 1996, p. 207).

Utilising multiple perspectives

As a key feature of enactivist research, Reid offers four methodological ways that multiple perspectives can emerge, summarised here as:

> Multiple researchers looking at the same data but through their own theoretical lens, with their own goals and agendas.

> Multiple revisitations of data, using different theories.

> Examination of a wide range of data.

> Communicating research to others and inviting new interpretations.

The research we articulate in this article makes particular reference to the second and fourth ways of utilising multiple perspectives. Having presented our initial ideas at the symposium, new interpretations of our data were offered that motivated us to revisit our data, looking with a different theoretical lens, always with the purpose of deepening our own understanding of our practice as MTERs through seeing more, seeing differently.

As enactivist researchers, learning about ourselves and our practice as MTERs, we do not look for theories *of* that "purport to be representations of an existing reality" (Reid, p. 208). Instead we look to bring forth new ways of seeing and understanding by using theories that are good-enough *for*. Our awareness of the importance of decentring ourselves from our own ways of seeing is a perturbation which motivates us to seek out different ways of seeing the same data, so that we can see more, see differently. We see the use of theories as good-enough *for* as connected to our use of effective behaviours that are good-enough-for in our practice. The problem we are seeking to resolve by using theories that are good-enough *for*, however, is that of how to better understand and articulate our practice. For us this is an ongoing endeavour. Using different theories *for* the same data however is not unproblematic.

In the process of formulating our practice through seeing more, seeing differently, we can never seek to use different theories without preconceptions. Any revisiting of our data becomes a seeing more, seeing differently but from a basis of what was seen before. Taking our history of acting into account, we are sure to find traces of 'old' theory in amongst the 'new'. What, then, does it mean for us to use multiple theories for the same data?

One image for using multiple theories or perspectives arises out of a term first articulated by Deleuze and Guattari (1987) who introduce their 'book' as neither signifier nor signified, but as "a little machine" (1987, p. 4), to be considered in terms of functions and transmissions in connection with other things. Drawing on Deleuze and Guattari's image, Jackson (2013) considers *data* (in this case, interview transcript) as a machine. Plugging this machine (the interview transcript) into other machines (*e.g.*, theory) leads to a *productive force* that generates connections, such that newness arises. The image of 'plugging in' theory is one with which we resonate strongly; each time a different theory is plugged in to our data, new ways of seeing our practice are generated.

In working with multiple perspectives, we create opportunities to explore the co-emergence of meaning that we look for. We see more, see differently through an "attentiveness to how each constitutes the other and how each, as supple, sprout as something new in the threshold" (Jackson & Mazzei, 2013, p. 265). Such a 'threshold' space might be seen as a gathering of our dialogues, our questions, theories brought from ourselves or others and us as researchers. A threshold might be crossed (in more than one direction), reached or exceeded; here, we see it as a space in which to dwell for a time; we see it as both created by plugging in and a space in which to work on plugging in. In both cases, the plugging in itself is not a space we occupy, we occupy the threshold space, part of the complex system which we form together. We present our final vignette as an example of dwelling in a new threshold space.

Vignette 3

Julian I've felt really positive about the opportunities that have come up to work on mathematics with the school-based mentors when I've gone into schools for joint lesson observations.

Tracy During the lessons?

Julian Yes, but also afterwards. There have been one or two cases that I guess have stayed with me, of the three of us, [the prospective teacher, school-based mentor and university tutor], talking after the lesson and carrying on working on the mathematics, like moving from area of compound shapes to think about conversion between units of measurement in area, and then in volume. The sharing of different images to illustrate those felt really powerful between the three of us.

Tracy Say more.

Julian So, part of it was me exploring the wisdom of the course that in the debrief conversations we as university tutors are really working with the mentors. I suppose at this point, it has been partly me trying to inhabit the convictions that were spoken in the context of the course. But now I can see that it also really connects with

something that emerged for me strongly when I moved from the [initial teacher training] course to start in my first school, which was about working on mathematics together as teachers.

Tracy   So you were carrying this on from your role as a teacher, with other teachers?

Julian   Well, yes. But really, that move to my first teaching job was a sense of loss, of no longer having those spaces to work together on the mathematics. And it's something I've tried to grow again ever since. So, working with the mentor feels like modelling as well as working on the mathematics, creating a space together to unpack what's going on. It has developed another layer of significance for me. I feel a conviction about the value of creating those spaces with the mentor, who might then expand the spaces throughout their work with our prospective teacher.

Julian is seeing differently, himself in relation to working on mathematics, as a teacher of mathematics and then as a MTER. Through dwelling in this aspect of his practice, occupying a new threshold space, Julian comes to a new seeing, around his role in supporting school-based mentors so that, in turn, they will more effectively support the prospective teachers in school.

Occupying new threshold spaces

As we write this, we find ourselves now describing ways of seeing more, seeing differently as MTERs as plugging our different 'texts' (data, theory, experiences, contexts, histories, memories *etc.*) into other texts, within a threshold space. In returning to a previous piece of writing (Helliwell & J. Brown, in press) where we articulated our learning as the emergence of convictions, we have come to a greater appreciation of how we are transforming through occupying new threshold spaces. In such spaces, we are learning to see more, see differently through working with theory and data, such that we can no longer see the vignettes as we saw them in our original formulating; we are no longer able to access directly our original formulation, or the formulation presented as part of the symposium. This does not mean we are unable to make sense of previous ways of seeing but that our making sense of them is different to what it was. Changes resulting from occupying new threshold spaces mean that we see differently what we have seen before and we experience these changes within our practice as MTERs. In talking with others about the practice of teaching, we find ourselves using different words to describe familiar scenarios or phenomena, scenarios that we have described many times before, and not only are the words different, but in saying them, we experience a new strength of conviction.

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