EXAMINING MATHEMATICAL DISCOURSE PRACTICES

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What are the features of Discourse practices? Are there characteristic mathematical Discourse practices? Can we distinguish everyday and academic mathematical Discourse practices? This article [1] considers these questions from a socio-cultural and situated perspective of mathematical Discourse practices (Moschkovich, 2002a, 2004) [2]. To ground that discussion, I first present an excerpt of a classroom discussion about quadrilaterals.

The excerpt comes from a lesson in a third grade (students are 8-9 years old) bilingual classroom in an urban California school. The students have been working on a unit on two-dimensional geometric figures. During the past weeks, instruction had included technical vocabulary such as the names and definitions for different quadrilaterals. Students had been talking about shapes and the teacher had asked them to point to, touch, and identify different quadrilaterals. In this lesson, students were describing quadrilaterals as they folded and cut paper to form Tangram pieces (see Figure 1).

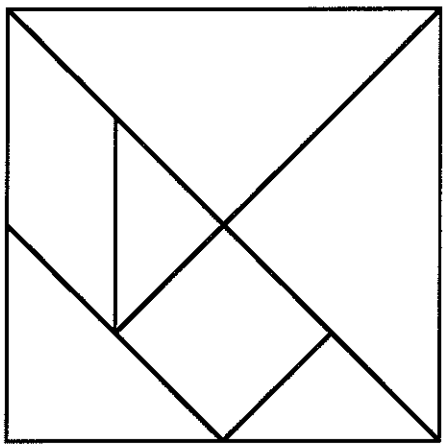

Figure 1: Tangram.

Towards the end of the lesson, the teacher asked the students to consider whether a trapezoid is a parallelogram, posing the following question:

> Teacher: What do we know about a trapezoid. Is this a parallelogram, or not? I want you to take a minute, and I want you at your tables, right at your tables I want you to talk with each other and tell me when I call on you, tell me what your group decided. Is this a parallelogram or not?

The students discussed this question in their small groups, followed by a whole-class discussion:

> Teacher: [To the whole class] OK. Raise your hand. I want one of the groups to tell us what they do think. Is this [holding up a trapezoid] a parallelogram or not, and tell us why. I'm going to take this group right here.
>
> Vincent: These two sides will never meet, but these two will.
>
> Teacher: How many agree with that. So, is this a parallelogram or not?
>
> Students: Half
>
> Teacher: OK. If it is half, it is, or it isn't?
>
> Students: Is
>
> Teacher: Can we have a half of a parallelogram?
>
> Students: Yes
>
> Teacher: Yes, but then, could we call it a parallelogram?
>
> Students: Yes.

This discussion drew my interest for several reasons. One salient aspect of this interaction is how stubbornly each side of the conversation (the teacher on one side, the students on the other) cling to their position. The teacher re-phrases the original question several times and the students stick to their claims. No matter how the teacher rephrases the question, students insist on maintaining their position as valid. What might be the sources of this repeated disagreement? When polled, the students seemed to agree with Vincent's statement that "these two sides will never meet, but these two will," thus seeming to agree that the trapezoid has only two sides that are parallel to each other. It seems that the students and the teacher agree with Vincent's claim. So then, what is the disagreement here?

This puzzling (and on the video even humorous) interaction led me to consider what Discourse practices might be involved in this discussion as a way to illuminate the teacher's and the students' perspectives. I will show how this interaction provides a window into mathematical Discourse practices. I will examine how the teacher's and the students' perspectives of this situation reflect two important features of discourse practices, the meaning for utterances and the focus of attention.

Before proceeding to an analysis of the mathematical Discourse practices involved in this discussion, I first briefly outline the theoretical framework for that analysis.

Theoretical perspective

The theoretical framework builds on previous work examining classroom mathematical discussions that involve conjecturing, explaining, justifying, and evaluating solutions (Forman, McCormick and Donato, 1998; Pimm, 1987; Radford, 2001). I use a situated perspective of learning mathematics (Brown, Collins and Duguid, 1989; Greeno, 1998) and the notion of Discourses (Gee, 1996) From this perspective, learning mathematics is a discursive activity (Forman, 1996) that involves participating in a community of practice (Forman, 1996; Lave and Wenger, 1991; Nasir, 2002), developing classroom socio-mathematical norms (Cobb, Wood and Yackel, 1993), and using multiple material, linguistic, and social resources (Greeno, 1998) This perspective assumes that participants bring multiple perspectives to a situation, that representations and utterances have multiple meanings for participants, and that these multiple meanings are negotiated through interaction.

I use the phrase "Discourse practices" to emphasize that I am not conceptualizing Discourse as individual, static, or referring only to language Instead, I assume that Discourses are more than language, that meanings are multiple and situated, and that Discourse practices are connected to multiple communities Discourses certainly involve using language, but they also involve other symbolic expressions, objects, and communities. Gee's definition of Discourses (1996) highlights how these are not just sequential speech or writing:

> A Discourse is a socially accepted association among ways of using language, other symbolic expressions, and 'artifacts,' of thinking, feeling, believing, valuing and acting that can be used to identify oneself as a member of a socially meaningful group or 'social network,' or to signal (that one is playing) a socially meaningful role. (p. 131)

Discourse practices involve not only language but also perspectives and conceptual knowledge. Discourse is not disembodied talk; it is embedded in practices Words, utterances, or texts have different meanings, functions, and goals depending on the practices in which they are embedded. [3] Discourses occur in the context of practices and practices are tied to communities. I view Discourse practices as dialectically cognitive and social On the one hand, mathematical Discourse practices are social, cultural, and discursive because they arise from communities and mark membership in Discourse communities On the other hand, they are also cognitive, because they involve thinking, signs, tools, and meanings.

Cobb, Stephan, McClain and Gravemeijer (2001) define mathematical practices as the

> taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas. (p. 126)

In contrast to social norms and socio-mathematical norms, mathematical practices are specific to particular mathematical ideas. While Cobb *et al.* (2001) see mathematical practices as emergent in classroom activity, I see mathematical practices as simultaneously emergent in ongoing activity and those socially, culturally, and historically produced practices that have become normative.

The excerpt above involves a common mathematical Discourse practice, using definitions. I view the two perspectives on defining a trapezoid as both emergent phenomena during this classroom discussion as well as already established ways of working with definitions within communities that regularly use definitions of trapezoids and parallelograms I take the view that Discourse practices are constituted by actions, meanings for utterances, focus of attention, and goals and that these actions, meanings, focus, and goals are embedded in practices. [4]

I take the notion of focus of attention from a socio-cultural framework that uses appropriation for describing learning. The central features of appropriation as described by Rogoff (1990) are that appropriation involves achieving a shared focus of attention, developing shared meanings, and transforming what is appropriated. Rogoff suggests that intersubjectivity may be especially important for learning to participate in practices that are implicit:

> Intersubjectivity in problem solving may (also) be important in fostering the development of 'inaccessible' cognitive processes that are difficult to observe or explain - as with shifts in perspective as well as some kinds of understanding and skills. (Rogoff, p. 143)

In a previous article (Moschkovich, 2004), I described how mathematical practices involve goals, meanings for utterances, and focus of attention. In this analysis I examine how meanings for utterances reflect particular ways to focus attention This discussion provides an example of how utterances can have multiple meanings depending on the focus of attention during joint activity. Language, utterances, and meanings are not mathematical in themselves but as they are embedded in mathematical practices. Mathematical practices involve not only meanings for utterances but also focus of attention. Mathematical practices are not simply about using a particular meaning for an utterance, but rather using language in the service of goals while coordinating the meaning of an utterance with a particular focus of attention.

Features of Discourse practices: attention and meanings

Returning to the vignette at the beginning of this article, I will first examine the students' perspective, describing how the statement "a trapezoid is a half a parallelogram" can be interpreted to make sense. I will then use this example to describe two features of Discourse practices - focus of attention and meanings for utterances - that are evident in this discussion I use the following questions (loosely following Gee, 1999):

1 What are the situated meanings of some of the words and phrases that seem important in the situation?

2. What perspectives are involved, especially where is the focus of attention?

The example illustrates two features of Discourse practices. Discourse practices are an "insertion into an intellectual practice requiring a social use of signs and the understanding of their meanings" (Radford, 2001, p. 261). Discourse practices also reflect particular perspectives and one way to examine these perspectives is to analyze the focus of attention. The analysis that follows illustrates how the teacher's and the students' contributions to this discussion reflect contrasting Discourse practices in terms of both the meanings for utterances and the focus of attention.

The standard definition of a trapezoid is *a quadrilateral with one pair of parallel sides* and the standard definition of a parallelogram is *a quadrilateral with two pairs of parallel sides*. The students' response to the question "Is this a parallelogram or not?" was "Half", implying that a trapezoid is half of a parallelogram.

First, let us consider how "a trapezoid is half a parallelogram" might be a reasonable response to the question. A parallelogram has two pairs of parallel sides and a trapezoid has one pair of parallel sides. A trapezoid can be seen as a half of a parallelogram because a trapezoid has half as many pairs of parallel sides as a parallelogram (see Figure 2)

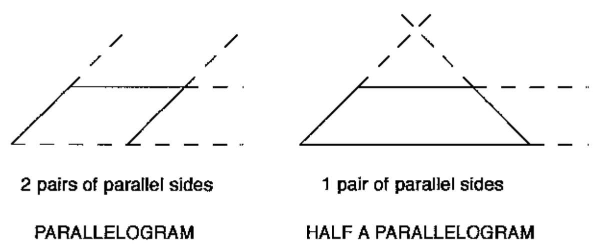

Figure 2: One way to see a trapezoid as half a parallelogram.

Another way to think about a trapezoid as "half a parallelogram" is illustrated below in Figure 3

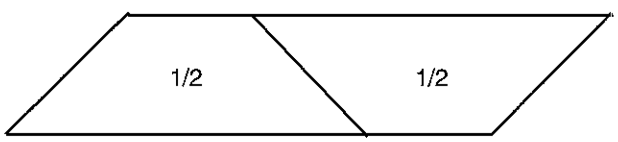

Figure 3: Another way to see a trapezoid as half a parallelogram.

Because Discourse practices involve more than single words and their meanings, I will not examine the meanings for any single word. Instead, I will consider the situated meaning of the question "Is this a parallelogram or not, and tell us why?" No question has a single meaning. One way to uncover the multiple meanings of a question is to explore how different participants respond to the question. For the teacher, "half a parallelogram" was not an acceptable response to the question, as evidenced by his repeated attempts to help students change their response. In contrast, for the students "half a parallelogram" was an acceptable response, as seen by their repeated reluctance to change their response. The teacher's expectation is that the response to the question "Is this a parallelogram or not?" will be either a 'yes' or a 'no': a trapezoid either is or is not a parallelogram. The teacher was invoking a formal dictionary definition of parallelogram that is binary, either the figure is a parallelogram or it is not a parallelogram. Using this meaning of the question, "Yes, it is half a parallelogram" is not an acceptable response, as evidenced by the teacher's response to this answer and his persistence in attempting to lead the students away from this answer.

Where do the teacher and students seem to be focusing their attention? Students might be focusing their attention on whether and how these two figures possess the *property* of having pairs of parallel lines. In contrast, the teacher was focusing his attention on whether the figures belong to one of two *categories*: *figures with two pairs of parallel lines* or *figures with one (or no) pair of parallel lines*. These are two very different views of quadrilaterals, in general, and trapezoids and parallelograms in particular.

Mathematical Discourse practices

Does the discussion above reflect mathematical Discourse practices? If so, how? Before addressing these questions, I would like to explore what I mean by "mathematical Discourse practices." We should not imagine that classroom discussions involve one single set of Discourse practices that are (or are not) mathematical. In fact, we might imagine the classroom as a place where multiple Discourse practices meet. As teachers and students engage in conversations, they bring in multiple meanings for the same utterances and they focus their attention on different aspects of any situation. These meanings and ways of focusing attention may reflect the meanings and ways to focus attention that are common in more than any one Discourse community. When discussions serve as a way to negotiate what utterances mean and where one might focus one's attention, different Discourse communities are, in some sense, meeting.

There is no one mathematical Discourse practice (for a discussion of multiple mathematical Discourse practices see Moschkovich, 2002b). How do mathematical Discourse practices vary socially, culturally, and historically? Mathematical Discourse practices vary across different communities for example between research mathematicians and statisticians, elementary and secondary school teachers, or traditional and reform-oriented classrooms. Mathematical arguments can be presented for different purposes such as convincing, summarizing, or explaining. Mathematical Discourses also involve different genres such as algebraic proofs, geometric proofs, school algebra word problems, and presentations at conferences

The labels used to refer to different mathematical Discourse practices, such as *everyday*, *professional*, *academic*, and *school*, can be misleading. The terms are complex and contested and the categories are not mutually exclusive. Both professional and school mathematics can be considered everyday practices – the first, for mathematicians, and the second, for teachers and students, in that these are everyday activities for these participants. Professional mathematics practices are workplace practices and school mathematics has been described as a subset of academic mathematics (D'Ambrosio, 1985) Nevertheless, making distinctions between these different Discourse practices can serve to clarify how we conceptualize mathematical Discourse practices

I will use these labels as if they were mutually exclusive, even though in fact they are not. The purpose of using them in this manner is not to describe distinct categories but to juxtapose and contrast multiple mathematical Discourse practices and examine the relationship among multiple Discourse practices I will use *professional* to refer to the Discourse practices of academic mathematicians, scientists, and professions such as engineering, architecture. *School* will refer to the practices of students and teachers in school. [5] *Academic* will refer to the mathematical practices we expect learners to engage in order to become mathematically literate *Everyday* will refer to the mathematical practices adults or children engage in (other than school or professional mathematics).

I make a distinction between school and professional mathematical practices, even though researchers, mathematicians, and teachers have called for bringing school mathematics closer to what mathematicians do Because many traditional mathematics classrooms do not reflect the practices of mathematicians (Stodolsky, 1988), I assume that school mathematical practices and mathematicians' practices remain distinct. I also make a distinction between school mathematical practices and what I am calling academic mathematical practices because many classrooms do not reflect the practices that lead to mathematical literacy or literate mathematical Discourse. I will assume that these two practices remain separate, although more recent instructional practices in some classrooms may be bringing them closer together

Everyday mathematical practices have been studied ethnographically in several settings (Carraher, Carraher and Schliemann, 1985; Lave, 1988; Nunes, Schliemann and Carraher, 1993; Saxe, 1991; Scribner, 1984). This body of work provides a systematic and detailed account of how "just plain folks" (Lave, 1988) carry out everyday problem solving and use tools and symbols, including language Autobiographical accounts of academic mathematical practices (for example Davis and Hersch, 1982; Schoenfeld, 1992) provide mathematicians' own descriptions of what mathematicians do [6]. Mathematicians report that their practices involve aesthetic values, such as elegance, simplicity, generalizability, certainty, and efficiency. However, autobiographical descriptions of mathematicians' practices are contested within the community of mathematicians either because there are different practices across different subfields, or because there are fundamental disagreements about what it is that mathematicians actually do (Restivo, 1993).

School mathematics has its own objects of study, social organization, and discursive practices For example, answering questions to which the teacher knows the answer is a typical Discourse practice in school but not necessarily outside of school (Heath, 1983) Another typical school Discourse practice is the IRE (Initiation-Response-Evaluation) format of many interactions (Mehan, 1979). Reading and discussing traditional word problems is a typical discourse practice in many mathematics classrooms (Stodolsky, 1988)

Although we might agree that mathematical Discourse is reasoned discourse (Høyrup, 1994), mathematical Discourse practices vary across communities, time, settings, and purposes. Current inquiry into the practices of mathematicians concludes there is not one mathematical practice, one way of understanding mathematics, one way of thinking about mathematics, or one way of working in mathematics:

> Out of the interviews with research mathematicians, I have a clear image of how impossible it is to speak about mathematics as if it is one thing, mathematical practices as if they are uniform and mathematicians as if they are discrete from both of these. (Burton, 1999, p. 141)

Professional mathematical Discourse is also historically situated. For example, over time, the roles played in professional mathematical Discourse by dialogue (Mendez, 2001) and mathematical arguments have changed:

> What was a good argument in the scientific environment of Euclid was no longer so to Hilbert; and what was nothing but heuristic to Archimedes became good and sufficient reasoning in the mathematics of infinitesimals of the seventeenth and eighteenth centuries. (Høyrup, 1994, p. 3)

Even mathematical definitions have changed over time. For example, the definition of a function has changed throughout history from the Dirichlet definition as a relation between real numbers to the Bourbaki definition as a mapping between two sets:

> At some early stage, functions were restricted to those which could be expressed by algebraic relationships. Later, the concept was extended to encompass not only correspondences which can be expressed algebraically, and later still to correspondences not involving sets of numbers at all. (Arcavi and Bruckheimer, 2000, p. 67)

Mathematical definitions can also differ across national contexts. For example, in Spanish

> the word trapezoid is reserved for the quadrilateral without any parallel sides, whereas trapezium is used when there is one pair of parallel sides. This is the opposite of American English usage. (Hirigoyen, 1997)

Mathematical Discourse practices also vary depending on purposes. Richards (1991) describes four types of mathematical discourse:

> *Research mathematics* of the professional mathematician and scientist.
>
> *Inquiry mathematics* as used by mathematically literate adults.
>
> *Journal mathematics*, emphasizing formal communication, is the language of mathematical publications and papers (Richards describes this type of mathematical discourse as different from the oral discussions of the research community because written formal texts reconstruct the story of mathematical discoveries)

> *School mathematics*, being the practices typical in the traditional mathematics classroom, may share with other classrooms the initiation-reply-evaluation structures of other school lessons (Mehan, 1979). Richards points out that school mathematics has more in common with journal mathematics than with research or inquiry mathematics.

Characteristics of academic mathematical Discourse practices

Academic mathematical Discourse practices can be understood in general as using language and other symbols systems to talk, think, and participate in the practices that lead to literate mathematical Discourse practices that are the "the objective of school learning." These practices involve much more than the use of technical language Gee (1996) uses the example of a biker bar to illustrate the ways that any Discourse involves more than technical language. In order to look and act like one belongs in a biker bar, one has to learn much more than a list of vocabulary words Knowing the names of motorcycle parts, makes, and models may be helpful. However, it is clearly not enough. In the same way, knowing a list of technical mathematical vocabulary is not sufficient for participating in mathematical Discourse

Can we describe the general characteristics of practices we identify as academic mathematical Discourse practices? In general, particular modes of argument, such as precision, brevity, and logical coherence, are valued (Forman, 1996). Abstracting, generalizing, and searching for certainty are also highly valued practices in mathematical communities. The value of generalizing is reflected in common mathematical statements, such as 'the angles of any triangle add up to 180 degrees,' 'parallel lines never meet,' or '$a + b$ will always equal $b + a$' Making claims is another important mathematical Discourse practice. What makes a claim mathematical is, in part, the attention paid to describing in detail when the claim applies and when it does not. Mathematical claims apply only to a precisely and explicitly defined set of situations as in the statement 'multiplication makes a number bigger, except when multiplying by a number smaller than 1.' Many times claims are also tied to mathematical representations such as graphs, tables, or diagrams. Although less often considered, imagining is also a valued mathematical practice Mathematical work often involves talking and writing about imagined things - such as infinity, zero, infinite lines, or lines that never meet - as well as visualizing shapes, objects, and relationships that may not exist in front of our eyes [7]

Everyday and academic mathematical Discourse practices

In closing, I would like to return to the vignette in the introduction to consider how our definition of academic mathematical Discourse practices informs how we label student contributions It may be tempting to describe the teacher point of view as reflecting academic mathematical Discourse practices because he was using formal dictionary definitions. And to describe the students' perspective as reflecting everyday Discourse practices because they were not using a formal definition. However, is a formal textbook definition necessarily the only way to participate in academic mathematical Discourse practices? The answer depends, once again, on how we define and imagine academic mathematical Discourse practices. If using dictionary definitions is the only practice we imagine that mathematically literate persons participate in, then the teacher's definition of a trapezoid is the only mathematical definition in this discussion. Instead, we might consider other ways of using definitions as academic mathematical Discourse practices.

O'Connor (1999) discusses different types of definitions and how they can be used in mathematical discussions in classrooms. She includes stipulative, working, dictionary, and formal as different types of mathematical definitions. Stipulative and working definitions are developed as part of an interaction or an exploratory activity; dictionary and formal are given by a text For O'Connor, constructing shared definitions

> is a signal example of what we mean by authentic intellectual practices of mathematics and science. (O'Connor, 1999, p. 42)

From this perspective, using dictionary or formal definitions is not the only way to participate in academic mathematical Discourse practices The definition students were using can be described as a working or stipulative definition. Using this perspective of definitions, we can see that these students were participating in an activity that may, in fact, be closer to the practice of scientists and mathematicians than to conventional school practices. We should not confuse 'mathematical' with 'formal' or 'textbook' definitions Formal definitions and ways of talking are only one aspect of academic mathematical Discourse practices Since there are multiple mathematical Discourse practices, rather than one monolithic mathematical Discourse, we should clarify the differences among multiple ways of talking mathematically and specify the goals for academic mathematical Discourse

In describing everyday and academic mathematical Discourse practices, it is important to avoid construing this as a dichotomous distinction. This distinction is not a tool to categorize utterances as originating in particular experiences During mathematical discussions students use multiple resources from their experiences both in and out of school. It is difficult, if not impossible, to use this distinction to describe the origin of student talk It is not always possible to tell whether a student's competence in communicating mathematically originates in their everyday or school experiences

Everyday Discourse practices should not be seen only as obstacles to participation in academic mathematical Discourse. The origin of some mathematical Discourse practices may be everyday experiences and practices. Some aspects of everyday experiences can provide resources in the mathematics classroom. Everyday experiences with natural phenomena can be resources for communicating mathematically. For example, climbing hills is an experience that can be a resource for describing the steepness of lines (Moschkovich, 1996). Other everyday experiences with natural phenomena may also provide resources for communicating mathematically.

In addition to experiences with natural phenomena, O'Connor (1999) proposes that students' mathematical arguments can be at least partly based on what she calls argument proto-forms:

> Experiential precursors (arguments outside of school, the provision of justification to parents and siblings, the struggle to name roles or objects in play) may provide the discourse "protoforms" that students could potentially build upon in the mathematical domain. (p. 27)

These precursors are related to academic mathematical Discourse practices such as arguing, making and evaluating a claim, providing justification, or co-constructing a definition.

It is not possible to unequivocally identify the origin of the resources students use in the classroom and it is possible that there are precursors from experiences outside of school for a contribution that sounds mathematical. We may be tempted to label phrases such as 'Let's assume ...' and 'if ..., then ... ' statements as mathematical and claims such as "a trapezoid is half a parallelogram" as everyday. These labels reflect our definitions of these two Discourse practices, rather than evidence that these aspects of students' talk originated in experiences inside or outside mathematics classrooms.

How we hear student contributions depends on how we define everyday and academic mathematical Discourse practices and imagine the divide between them. As the example above illustrates, before we label student contributions as everyday or academic, we need to be explicit in our descriptions of the practices and goals for academic mathematical Discourse. Before we label student talk as everyday or academic, we need to consider what we include or exclude in our definition of academic mathematical Discourse practices. If we assume that academic mathematical Discourse consists only of textbook definitions or the practices mathematicians use in formal settings, we may miss the mathematical competence in student contributions

Notes

[1] This work was supported in part by grants from the National Science Foundation (REC-9896129 and REC-0096065)

[2] Gee distinguishes between "discourse" and "Discourse." In keeping with this distinction, I will use the term Discourses

[3] I distinguish practice (repetition), practice (for example "my teaching practice"), and *practices*. I use the terms *practice* and *practices* in the sense used by Scribner (1984) for a practice account of literacy to

> highlight the culturally organized nature of significant literacy activities and their conceptual kinship to other culturally organized activities involving different technologies and symbol systems. (p. 13)

[4] In this article I will put aside a discussion of goals and focus here on meanings and attention. For a description of how discourse practices involve actions and goals and an analysis of the role of goals in the appropriation of mathematical practices, see Moschkovich, 2004.

[5] Note that this is different from Richards' (1991) use of "school mathematics."

[6] There are also accounts of mathematical practices using historical, philosophical, and cognitive methodologies. For example, Schoenfeld's (1985) account of mathematicians' problem solving is a cognitive analysis based on think-aloud protocols. Other accounts of mathematical practices, such as Lakatos (1977) and Polya (1957), rely on a combination of introspection, historical data, and philosophical methods

[7] Note that some of these may be the characteristics of the end point of producing mathematics, such as making a presentation or publishing a proof, while others are characteristics of the discourse practices involved in the process of producing mathematics (K. King, personal communication).

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This brief excursion into a semiotic analysis of a teaching/learning incident highlights the importance of understanding both the levels at which *forms* become *signs* or *symbols* and the affective forces which bring about changes in the relations between the *forms*. It is especially the case in mathematics where the power of the symbol in provoking smooth action can blind the user to another's non-comprehension. A teacher who knows the dynamics involved in his [*sic*] task as an "interpreter of signs" is in control of ways of inducing the power of the symbol in the learner. This is a common characteristic of a large number of teachers of mathematics. What is not so common is to find those teachers who know that they must also help the learner to interpret for himself the signs that the form becomes when, for some reason, the form as symbol loses its power. This was Peter's state when faced with '=' and instead of a single form on the right being 'the answer', he had a form which was no longer symbolic. And, for him, the whole needed re-interpreting

The skill in using semiotic analysis in a problem in communicating lies in knowing when to stop. Analysis is needed up to the resolving of a difficulty. Consider, in the incident, what could be called an anomalous act where the boy said, "It means that that's the answer." He had pointed to the form on the right hand side of '=', but in 'doing' the first twenty, 'the answer' for each had clearly been the number which could replace the form '?'. He knew this and had accepted '?' as a symbol guiding his action. Whatever caused the distinction which had stopped him 'doing' the second twenty, it had been strong enough to prevent the symbolic power of the form '?' from operating as it had on the first occasion. The change in direction caused by the father's offering that the form '=' could also mean 'is the same as', seemed to allow a coalescing of the symbolic power of the '?' with the newly interpreted 'sign-becoming-symbol' '='. This dynamic could be seen as powering the 'sign' to become 'symbol'. It does not tell us what the effect of the previous symbolic meaning of '=' had in governing the operation of the symbol '?' so that it operated in the first instance but not in the second. All we know is that the father's action released an inhibition without knowing how the boy had grasped the implications of the form '?'.

(Members of the Association of Teachers of Mathematics (1980), *Language and mathematics*, Derby, UK, ATM, p. 34. From the preface: 'It has been the practice in ATM for many years to publish work as a group. Individual contributions are drafted and submitted to group discussion; in some sense the authors take joint responsibility for what is finally written. At the same time the Association does not publish representative reports which are then supposed in some way to reflect the views of all members. This publication is offered in the same spirit; it offers some current thoughts from a working group of some ATM members. *Bill Brookes, John Dichmont, Eric Love, Judy Morgan, Dick Tahta, Jim Thorpe*.)
