

Transformations

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In the Open University course *Learning and Teaching Mathematics* two video cassettes are provided for students to keep. Each of the cassettes has three hours of excerpts from classrooms which show some of the excitement as well as difficulties in mathematics teaching. As a non-mathematician, I was invited to make some observations about the excerpts to provide another perspective for teachers to consider. In my contribution to the course material [1], I discuss what I saw in the behaviour of learners and teachers through their experience of mathematics. I noticed links with mathematics being continuously developed, and I became more and more interested in the difficulties people have in forming these links.

Various authors have considered the emotional factors involved in learning [2]. Barret and Trevitt, in particular, have considered emotional blocks to learning mathematics as well as other subjects. They write that without a clear sense of "I am" it may be difficult to master concepts of number beginning even with the number 1. Furthermore, it may be difficult for some children to make estimates if their inner world of relationships is insecure. To move towards something new may be difficult without adequate experience of reliable goals.

In this article, I focus specifically on themes of identity and change, and consider some of the difficulties that might be involved by trying to interpret some selected excerpts from the video recordings. The excerpts considered are vivid and should be easy to imagine from the brief descriptions given. Further information about the videos and where the excerpts quoted can be found on them is given at the end of the article [3]. I have discussed the theoretical perspective in more detail in a recent publication [4].

Subjective space

The distinction of what is inside the self and what is outside helps us see how empathy links one person with another through a process of projected feelings. When a relationship become threatening in some way a child may withdraw and become isolated or find it difficult to make transformations. One of the most fundamental transformations is from doing something physical to forming some more abstract symbol, including words, about the action. In this first excerpt we might reflect on why it is difficult to respond to a question even when the answer is obviously known.

In the first video excerpt, *Teddies and shapes*, the teacher asks the children to choose a teddy. One girl, Amy, says "yellow and yellow" as she picks up two teddies, but is hesitant to answer when the teacher asks her to describe them. Amy tells her teddies to sit and stay and then lifts them in the air. As the teacher continues to question Amy, another girl, Jessica, tries to help by suggesting "light and dark yellow" as a response to the teacher.

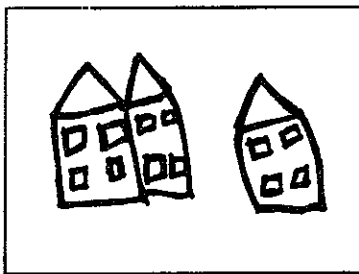
In this deceptively simple situation, we can see many of the important features of any learning situation. The task set by the teacher is part of a context which is safe and supportive. For most of the children, selecting a teddy and describing colour (as a basis for selection) is not difficult. But Amy, who talks to her teddies, appears to be engaged in telling a story linked to some other time and place. We can only guess about what she is feeling and thinking, but it is important to try to understand her context. She may be imagining herself in a teaching or parental role telling the teddies what to do and making a nice safe place for things to happen, rather like the teacher is doing. Or she may be a little insecure about what the teacher really wants to know. Jessica offers an answer to the teacher's question, perhaps concerned that Amy does not know enough answers. We see here the basis of empathy, being able to identify oneself with another and wanting to help. Through these spontaneous behaviours we can begin to infer more about the complexity of inner experience of children in school.

The second part of the excerpt focuses on five children a year older. The teacher asks them to select a "logi-block" and describe it. She encourages their awareness of four different attributes - size, colour, thickness and form - and expresses surprised approval as they identify three and then four attributes. She leaves them to continue with the domino game in which shapes added to a line have to be described. The girls appear to be delighted to assume the teacher's role, encouraging and then requiring Jason to describe more attributes of the shape he has added to the others on the table - "and something else?" the girls say. Daniel suggests, with some feeling, an answer to help him. It is not obvious what his feelings are - humour, breaking rules, embarrassment, joining forces?

It is easy for experienced teachers to see a particular task in a very intuitive way, but their feelings in a situation rarely get acknowledged. Children do not hide their feelings like adults and their directness may sometimes be uncomfortable. In order to avoid making premature judgements we need to be in touch with our own mixed feelings. Otherwise, hasty decisions may be evidence of how anxieties colour the reality of the situation one is attempting to understand.

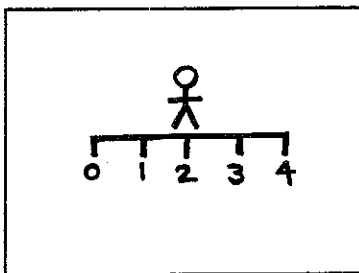
In *Accommodation base 4*, children are throwing a dice to see how many windows they get. When they have four they can exchange them for a house. Four houses can be exchanged for a street. The teacher expects the children to cooperate, but they are more excited about obtaining a higher number to get more windows. One child suggests there is a battle going on. There can so easily be unintended consequences in a carefully planned lesson. In this sequence the children are absorbed at a level of action and are not moving to the more abstract task of transforming four houses into one street. Instead the excitement is focussed on competition between individual or group.

Tasks set by a teacher may have many meanings for children; competition may be linked to worries about being left over, left being or left out.



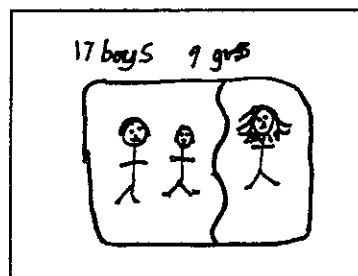
Sometimes active behaviour may have a covert aim not to change. Beginnings may remain just that with no end or middle place for thinking. Practice until a problem can be solved may become locked into a circular process which does not go anywhere. In *Divide and share*, pupils are making three groups of five with one left over. Although the lesson seemed that it had somewhere to go, it did not because making groups continued to be repeated.

In *Linesman*, thinking about negative numbers appears to be unexpectedly difficult. The teacher introduces a human stick figure standing on a line and facing right in a positive direction. The class is encouraged to think of a negative number in terms of behaviour – the figure facing in the opposite direction or walking backward. Might these terms have unintended associations which are difficult to limit? Opposition may become equated with conflict, or focus on negative aspects of experience discouraged.



The sequence *Graphical calculators* shows two girls comparing written equations with how they look in a graph. One girl who appears not to understand does not feel confident or safe enough to ask questions. She may not be clear about any question and reluctant to reveal her lack of understanding. She attempts to obscure her difficulty by mechanically entering data in her book. We get only a hint of her feelings at the moment she moves the calculator and allows it to fall with some impact. Is this a physical outlet through the body of intellectual blocks? Unconsciously, she might like to break the calculator into fragments – an acting out which would reflect how she feels internally. Because the teacher appears to give more attention to the girl who appears to understand better, the message might be that clever ones get rewarded. It may be more difficult to cope with struggles and more attractive to focus attention on the responsive pupil. Since we only see a small fragment of the lesson, we cannot conclude that this message is consistent – only that it appears to be so at one moment and that it may affect the experience of one girl.

In the next example, children attempt a task of classification which may have too close associations with personal relationships. The emotions stirred up may be affecting what is and can be learned. In *Dividing the class*, the teacher asks the children to divide their class into two sets in which all the members of each sub-set will have one thing in common. Some children choose hair colour – light and dark. Others select whether or not children go home for lunch. But they experience difficulties with their classifications – not all the members of the class are included, or they get three sub-sets instead of two. The use of very personal attributes for an abstract classification task may have become a source of confusion. Hair colour is just one of many ways the children see each other and just to select one attribute out of all those possible means that many others are left out. The separation of inside from outside is of fundamental importance in developing a clear sense of self.



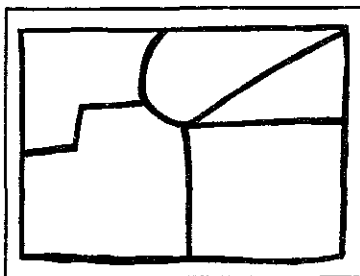
The association of food as something taken inside oneself may lead to some hiccoughs when it is used to think about some children going outside, home for lunch. These speculations suggest how different an experience might be for one child compared to others. The challenge for teachers is to learn to recognize the evidence for how different an experience may actually be compared to what it seems to be.

Transformations

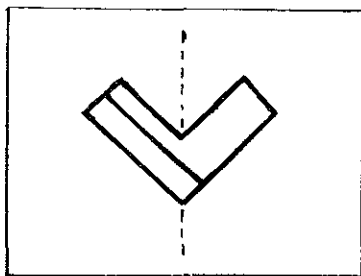
The previous examples introduce a number of simple transformations as well as resistance to making them. In this section, the notion of time appears to become more important as the examples become more complex, and the link between thinking and acting more strained (or a focus for more stress). Furthermore, anxieties about being left alone, being at some disadvantage and feeling hopeless, are powerfully expressed through the learner-teacher relationship as well as between peers. When a sense of self is secure enough, enquiry is possible along with the taking of risks and giving up control of relationships which are separate or in the future.

What could be more simple than starting with just straight lines and circles to make a world of shapes which one can change? Such a complete world within itself might well be described as “hermetic”, ie a world cut off from outside views. In *Tak-tiles* two girls are giving values to the area of a complex shape by dividing it into smaller shapes. To keep track of what they are doing the shapes are given a letter – A, B and so on. Thus the concrete shape and area becomes represented by a letter and a series of letters linked by operations (added or subtracted) written in particular sequences (indicated by brackets). The

expressions are more abstract compared to the physical attributes of the shapes. Difficulties with the task may be increased by emotional associations and the transition from physical attributes of the shapes.



Towards the end of the excerpt one girl refers to a missing "number" rather than a "letter". This may reflect difficulties in keeping transitions clear - from wholes to parts, from values to symbols. To think about parts of a object, as well as the whole, may be particularly difficult for some children, especially if keeping things the same or whole has some emotional significance. Gaining confidence to create abstract ideas may be entwined with personal relationships in a confusing way. Doing things to shapes and changing them brings up the possibility of constructive or destructive actions and problems about locating the source of the action



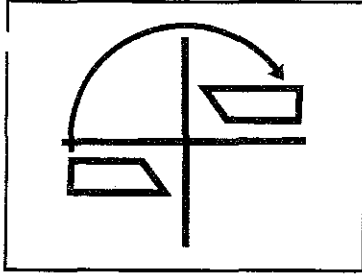
In *Symmetries*, two boys are handling two solid shapes, one large and complex and the other a smaller rectangular shape referred to as a spare piece. The task is to place the spare piece in a position so that the large shape becomes symmetrical. Because of the complexity of the object, how to balance the two halves is not at all obvious. The teacher demonstrates how to find the axis of symmetry with a mirror placed so it reflects just half of a symmetrical shape drawn on paper. One child expresses his frustration by yawning and tapping the edge of the mirror on the shape with some force. The most difficult aspect of the task is keeping in mind the reflection they see in the mirror and imagining where the spare piece must be placed behind the mirror to make a symmetrical shape. Visual perception and creative imagination are linked from our earliest experiences of life. When something we have experienced is hidden, an image or symbol must be created to represent the absent object, to keep it in mind and maintain a continuity of relationship in space and time. Absences associated with frustration are initially difficult to accept when associated with someone who is valued. One aspect of creativity is

reparation - to heal the feared destructive consequence of bringing hate and love together in one place. The idea, word or symbol we form to represent what is missing provides a link which helps shape future relationships - connections between thought and feeling, inner and outer reality.

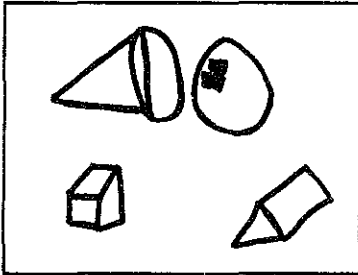
There are two important points to be made here. First is that, for any individual, a simple task may have many unsuspected associations or evoke mixed feelings about relationships outside a classroom. Ambivalence about relationships is closely linked with early experiences, the presence and absence of important figures, and may inhibit as well as facilitate learning. The other thing to note is that shape, position, and movement are physical aspects of experience which, without our awareness, may have strong associations with emotions. A child unable to complete a task about symmetry may express this block in learning through the body by over-excited or lethargic responses. Focusing in detail on one aspect of behaviour may help us locate the difficulty more precisely.

The three dimensional aspect of working with solid objects in *Back into shape* is clearly challenging. Triangular shaped prisms are placed on top of cubes to make a greater variety of shapes. There are different kinds of boundaries, external and internal - those around solid objects and others inside hollow objects. Inside and outside do have emotional associations. We do not want to be left outside or excluded. In an effort to help the pupils, their teacher describes a solid shape as a "true" triangle and distinguishes it from a triangular space which he calls an "interior" triangle. Boundaries are often difficult to think about because they are so important in defining structures - for example, between self/other, mind/body, imagination/reality. For infants and adolescents, the emotional aspects of boundaries are being extended from home to the "extended family" of groups in institutions.

The difficulty of comparing the position of one object with another may be related to the importance of being centred within oneself inside a familiar context. In *Transformations*, the idea of mirror image is taken further and connected with position (x and y axes) and movement (angular rotation). A girl, Rosina, is presented with two drawn triangles and she has to determine if one has been flipped over (giving a mirror-image) or rotated in relation to the other. The challenge is to orient herself in relation to the triangle and to develop a way to think about the task. She demonstrates that she understands the idea of rotation by turning one triangle. But the jump from comparing positions to a quantitative estimate (how many degrees?) may be blocked by feelings of vulnerability, lack of confidence or self doubt. The teacher helps her by showing the importance of keeping a fixed point in mind around which a line rotates. The angle moved through describes the amount of rotation as well as being essential to identifying what line to compare in the second shape. She does not appear to be pleased or relieved when she says she understands. The focussing of a camera on her struggles probably makes this a very uncomfortable experience for her.



Emotions can be alarming as well as exciting and incline people towards a preference for more static situations compared to dynamic changing ones. In *Rolling shapes*, - the children are asked to describe how an object moves (for example, rolling in a straight line or circle) as well as the physical attribute of number of "faces". The teacher then hides one object and asks the children to guess which one she chose on the basis of them first identifying attributes.



Their guesses focus only on the number of faces the object has and not on whether or not it rolls. Perhaps the movement of objects seems very transient and difficult to keep in mind compared to a static quality which can be measured or counted. To control or slow down possible change, boundaries must be clarified. Conversely, when boundaries become unclear and the function of mediating between two separate things (self and other) is lost, confrontation also seems to be avoided in one's imagination. In reality, capabilities are also lost along with memory about the past, so that the idea of progress (or regression) loses any significance.

Final reflections

Compared to direct and spontaneous behaviour in groups, our symbolic world of language (verbal and non-verbal) is less direct and more reflective about past and future as well as present. Reflection can be idealized as a more objective view but it acquires reality only as we become lost within emotional experiences. True reflection stays in touch with the negative and painful as well as the positive aspects of experience. Loss of a sense of time or history may be evidence of a loss of boundaries, disorientation of the self and loss of capacities. In contrast, history can be treated as some absolute determiner of events where no one can be held responsible for anything. The more we

understand about the complexity of the spaces we live in, the more we will be able to enable learning.

Teachers help learners to see questions such as "it is enough?" or "is it right?" not as end points of judgements but pointers on a journey. Without sensitive and aware support children may find it difficult to think about ideas, such as are found in algebra, which are potentially powerful in their implications for change.

The ability to keep in mind what may be in conflict as well as interdependent depends on being able to contain ambivalent, conflicting feelings. Containment requires a belief in one's own capacity for creativity, power or authority. The spontaneous creation of ideas requires commitment to the unknown and risky rather than rigid conformity to a plan. In contrast, denial of ambivalent feelings, or keeping relationships fragmented, blocks the possibility of renewal of energy and hope.

Teachers know that it is good practice to clarify objectives, plan activities and share evaluation of achievement with learners. Ideals, authority and disappointment are all bound up in this. There are also unplanned outcomes, that out of the tensions of a learning situation, experiences must be modified and new meanings emerge for everyone. Allowing time for meanings to evolve, for experiencing the frustrations of learning and sharing new directions are aspects of the two way interaction between learner and teacher. The mystery and power embedded in this authority-dependence relationship will always provoke a number of feelings e.g. unquestioning acceptance, rejection, conflict or passive covert resistance.

Shape, movement and position define fundamental dimensions of physical reality. We may also begin with their equivalents in human relationships - ideas about one's self, about transformation and creativity. When we take some action, abstract symbol systems such as algebra provide a structure for prediction and testing out, measurement provides points for beginning and ending, and shapes a metaphor for the significance of the story unfolding.

References

- [1] R. Nicodemus, "Psychoanalytic perspectives" in C. Shiu, *Final unit: drawing to a conclusion*, Course EM236: Learning and Teaching mathematics, Open University, 1992
- [2] M. Klein, "Our adult world and its roots in infancy", in *Envy and gratitude*, Hogarth Press, 1980 (Virago pbk, 1988)
I. Saltzberger-Wittenberg, *The emotional experience of learning and teaching*, Routledge and Kegan Paul, 1983
M. Barrett & J. Trevitt, *Attachment behaviour and the schoolchild*, Tavistock, 1991
- [3] Course EM 236 video cassettes, parts 1 and 2, Open University Educational Enterprises, 1992. The selected excerpts start at the following visible video "page" numbers. Part 1: *Teddies* - p1, *Accommodation* - p194, *Divide...* - p249, *Linesman* - p300, *Graphical* - p318. Part 2: *Dividing* - p23, *Tak-tiles* - p268, *Symmetries* - p294, *Back into shape* - p342, *Transformations* - p362, *Rolling* - p424.
- [4] R. Nicodemus, *Teamwork in open and distance education*, Kogan Page, 1993

When Two Becomes Three...



René Magritte, *L'esprit de géométrie*, 1937 (Tate Gallery, London): In the beginning there is apparently only one – myself. But there will be two in the primitive narcissistic state in which the other is also myself. And there will also be two when I mirror myself in my mother – who is expe-

rienced as a counterpart of myself. Everyone has known this dual relationship, some from both sides. It can also be experienced vicariously in literature and art; notably in paintings of the Madonna and Child. Magritte inverted the usual representation and chose a challenging title.



Michaelangelo, *The Holy Family*, 1556 (Uffizi Gallery, Florence): How does two become three? ...The intrusion - for such is the way it is experienced - of the father has been associated with the introduction of language. For the psychoanalyst, Jaques Lacan, the infant encounters lan-

guage through a pre-established symbolic agency called the Name-of-the-Father. The intrusive third - whether real or imagined father - is associated by Lacan with this transition to what he called the Symbolic order. Other representations of a threesome are shown on pp5, 54.