BRIDGING INTUITIVE AND ANALYTICAL THINKING: FOUR LOOKS AT THE 2-GLASS PUZZLE

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> *If one is truly to succeed in leading a person to a specific place, one must first and foremost take care to find him where he is and begin there. This is the secret in the entire art of helping.* (Kierkegaard, 1998, p. 45)

The observation that the human mind operates in two distinct thinking modes—intuitive and analytical—has occupied psychological and educational researchers for several decades now. Cognitive and social psychologists have done extensive experimental and theoretical work on the two modes of thinking, much of it under the umbrella of dual-process theory, where the intuitive and analytical modes are called System 1 and System 2, respectively (Evans & Frankish, 2009; Gilovich, Griffin & Kahneman, 2002; Kahneman, 2002, 2011). Much of the relevant research in psychology and in mathematics education has focused on the explanatory power of intuitive thinking as a source of errors and misconceptions in human behavior, decision making, reasoning, and problem solving (*e.g.,* Fischbein, 1987; Leron & Hazzan, 2006, 2009; Stavy & Tirosh, 2000). In this article, however, our emphasis is rather on the power and usefulness of intuitive thinking (Gigerenzer, 2005).

We use the dual-process terminology mainly to distinguish between responses that come to mind quickly and automatically (referred to as *intuitive* or *System 1*) from those that are more deliberate and effortful (referred to as *analytical* or *System 2*) and which are mostly in line with accepted mathematical norms. To identify intuitive responses, we follow Kahneman (2002) in looking for those features of a situation that are most salient (or most accessible):

> Absent a system that reliably generates appropriate canonical representations, intuitive decisions will be shaped by the factors that determine the accessibility of different features of the situation. Highly accessible features will influence decisions, while features of low accessibility will be largely ignored. Unfortunately, there is no reason to believe that the most accessible features are also the most relevant to a good decision. (p. 459) [1]

According to dual-process theory, the intuitive mode is fast, automatic, subconscious and effortless; the analytical mode is slow, conscious, controlled, and effortful, requiring the limited resources of working memory (Evans & Frankish, 2009; Gilovich *et al.,* 2002; Kahneman, 2011; Leron & Hazzan, 2006, 2009). For students of mathematics, these two modes of thinking may represent (subconsciously) two powerful but different "authorities": the intuitive mode operates under the authority of common sense and of what comes naturally to the students' mind, while the analytical mode operates under the authority of normative mathematics, as represented by textbooks, teachers and the professional mathematical community. Recognizing that students of mathematics will inevitable find themselves at times struggling with the gap between these two authorities, we look in this article into ways of helping students *bridge* this gap, thus, we hope, creating a peaceful and productive co-existence in their minds between the two powers.

Beside the extensive theoretical background cited above, the main thrust for writing this article comes from our experience at several "bridging" workshops for mixed audiences of students, teachers and researchers (a 4-hour discussion group at the 2011 PME conference in Ankara, and at several universities and high schools in both Israel and Denmark). In an attempt to capture the lively experience in those workshops, we have chosen a story-telling style of writing: the bridging issues are presented as informal reflections on a virtual workshop (an abstracted version of several real ones), organized around the 2-glass puzzle. Thus, rather than presenting our story in the traditional research format, we have chosen the view of the "reflective practitioner" (Schön, 1983).

The classroom bridging workshop lends itself to several ways of sequencing the activities and the resulting mathematical, educational, and psychological ideas and concepts. Here we have chosen one possible way of sequencing, starting with what we perceive to be the most common and natural level, and moving gradually to the more nuanced and sophisticated. The workshop itself is presented as a series of four graded classroom activities [2].

The intuition trap

The 2-glass puzzle, like many other good puzzles, initially leads the typical solver into what might be termed an *intuition trap:* it evokes an immediate, intuitive answer which

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on further reflection, turns out to be wrong. The goal of this first activity is to introduce the students to the 2-glass puzzle, have them playfully "fall into the intuition trap", then introduce the correct solution "analytically", and use this experience to discuss in the classroom the double-faced relationship—sometimes supporting, sometimes conflicting—between the intuitive and analytical modes of thinking.

The activity begins with the teacher presenting the puzzle:

> We have two glasses: a wine glass and a water glass. We draw a spoonful from the wine glass and transfer it to the water glass and mix. Then we draw a spoonful of the mixture from the water glass and transfer it back to the wine glass. Now which is more: the wine in the water glass or the water in the wine glass?

(Readers who are not yet familiar with this puzzle might want to think a bit about the solution before reading on.)

It has been our experience in our many workshops that most people's immediate reaction is that there is more wine in the water glass, and that the reason they give is something like, "because we have transferred a full spoon of wine from the wine to the water, but only a mixed spoon on the way back". That this is indeed people's first intuition is further supported by the following easy experiment (which we have repeatedly carried out in our workshops): ask anyone who is already familiar with the puzzle and with its correct solution what they think would be most people's first immediate answer, and they will answer: "More wine in the water glass" [3].

Having discussed with the class their solution for some time, the teacher now presents the correct solution (which, alluding to the dual process terminology, we will henceforth refer to as the *analytical solution*):

> Well, actually the amounts (wine in the water glass and water in the wine glass) are equal! Why? Because each glass now has the same amount of liquid as it had in the beginning. Let's look at the wine glass, for example. It now has some water in it, but since the total amount of liquid in the glass remained the same, this water must have displaced exactly the same amount of wine, and this must be exactly the wine that we have in the water glass.

At this point some students say "Wow!!!" and are clearly surprised and excited at the beauty and elegance of this "conservation principle". Many other students, however, seem mystified and bewildered. The teacher explains again, and even asks some of the students who have understood the solution to explain it to their peers, but still many students remain mystified, and others say they have understood and accepted the solution but seem unconvinced and unhappy.

In conclusion, the teacher summarizes the experience by discussing with the class the two modes of thinking—the intuitive and the analytical—and the clash that may sometimes exist between them.

Reflection on the intuition gap

System 1 thinking works by immediately and automatically reacting to the salient features of the problem, often ignoring less prominent (but possibly no less important) features that may be hidden under the surface. This behavior has good evolutionary roots: briefly stated, it gives good adaptive results on average, under typical conditions of the ancient environment of our hunter-gatherer ancestors. It is typical of an intelligent system (Gigerenzer, 2005), and should be applauded as the part of human nature where all humans perform extremely complicated tasks with great expertise and success. In many modern situations, however, the conditions as well as the norms for judging correctness of solutions have changed radically from the conditions in these ancestral environments, and under such a-typical conditions the intuition may lead to non-normative responses. This effect is similar to the way our (intelligent) optical system generates optical illusions and, for this reason, System 1 errors are sometimes called cognitive illusions (*e.g.,* Pohl, 2004). In research and sometimes in teaching, these atypical conditions may be created deliberately in order to lead the subject into what we have called the intuition trap. Such situations include some famous puzzles and jokes, such as the 2-glass puzzle discussed here, as well as much experimentation by research psychologists, who use such traps to reveal basic mechanisms of the human mind (*e.g.,* the bat-and-ball task; see Kahneman, 2002).

In mathematics education, the dual-faced relationship between intuitive and analytical thinking is a central concern, and intuition traps are an excellent way to bring it to the surface and engage students in related activities and discussions. Finding intuition traps and engaging in them—even deliberately falling into them—can be both illuminating and fun, provided special care is taken to nurture a suitable classroom culture that encourages exploration and playfulness, and celebrates errors and their analysis as opportunities for new learning and new insights about mathematics and about one's own mind.

The scenario depicted in our previous classroom activity is an example of a teacher skillfully using the 2-glass intuition trap to have the class experience the clash between the intuitive solution (more wine in the water glass) and the analytical solution (they are equal). The teacher has chosen to have the students experience this clash by pulling the analytical solution out of her sleeve. Like all good drama, the teacher has built up conflict and tension, to be resolved and alleviated later. Other teachers might have chosen differently, for example working with the students on "debugging" their intuitions, as we elaborate in the section on bridging up.

Bridging down: making the formal intuitive

In the previous activity, the teacher has done a good job in leading the students to experience and discuss the intuition gap, and she is justified in feeling good about that. She is feeling less happy, however, about the way her analytical solution was received by the students. Though undoubtedly clever and elegant, this solution has left many students confused and unhappy because they could not "connect" with it. They could follow the verbal explanation but they could not "see" the idea behind it. In the terminology of the present article, they could not form an intuitive understanding of the analytical solution. To help them bridge this gap, the teacher is now going to *bridge down* the analytical solution to the students' intuition. She does this by devising and presenting a *bridging task* which would be logically (roughly) equivalent to the original analytical task, but psychologically much easier (*i.e.,* closer to the students' intuition):

> Let me give you another puzzle, and maybe you can see some connection to our 2-glass puzzle. Imagine a house painted white, with a row of pigeon-holes under its roof, and a number of white pigeons sitting under the roof, exactly one pigeon in each hole. We call this house "the white house". Now imagine a house painted black with the same number of black pigeons, again sitting one in each hole under its roof. We call this "the black house". Next, imagine that suddenly there is a loud noise and some of the pigeons from each house fly in the air, remain there for some time, and eventually settle back randomly in one of the houses (not necessarily their original house), again exactly one in each hole. Now which house has more pigeons of the opposite color?

In our experience, many students will discover the correct solution to the pigeons task, based on the conservation principle: because the number of pigeons in each house in the end remains the same, the black pigeons in the white house must have replaced exactly the same number of white pigeons, which are now sitting in the black house. With the vivid image of the house and the pigeons sitting one in each hole, they may have developed an intuitive feel for this solution. Furthermore, even those who did not discover the solution on their own would easily understand the solution once it is explained to them. The pigeons bridging task could also help them (perhaps with some additional help from the teacher) to gain an intuitive understanding of the analytical solution to the original 2-glass puzzle, or perhaps even to now discover it on their own. In this sense, we might say that the intuition gap has been bridged or, more specifically, that the analytical solution has been bridged down to the students' intuition.

To summarize, the teacher has introduced a bridging task, which helped the students to bridge down the analytical solution to their intuition.

Reflection on bridging down

After the teacher has presented the analytical solution (utilizing the conservation principle), many students might feel tense and uneasy, because they find the teacher's solution tricky and mysterious and hard to understand. The idea of bridging down is to use variations on the original task in an attempt to make the analytical intuitive (or to make the alien familiar). The idea is to design a bridging task, that is, a variation of the original task (such as the pigeon houses), which will be logically more-or-less equivalent to the original task, but at the same time psychologically significantly easier and more intuitive. We have termed this didactical device bridging down, reflecting the image of the analytical solution sitting high up in the air, and the role of the bridging task as bringing it down to the level of the students' intuition. As we elaborate in the next section, the pigeon version is more intuitive not because white and black pigeons *per se* are more intuitive than wine and water, but because, being discrete rather than continuous, it is easier to construct a mental model for it.

Ejersbo and Leron (2014) have operationalized and empirically tested the concept of bridging task in the context of the infamous Medical Diagnosis Problem. They note that a good bridging task is subject to two conflicting constraints. On the one hand, the task should be close enough to the original task that the student can see the connection and make the transition. On the other hand, it should be close enough to the students' intuition to enable the students to solve it with relative ease. We have not carried out the actual experiment for the 2-glass task, except informally at our workshops, but it is likely that the pigeons task satisfies both these constraints: on the one hand, significantly more students would solve it correctly (as compared with the 2-glass puzzle) and, on the other hand, having solved the pigeons task, significantly more students would successfully solve the original 2-glass puzzle.

As we elaborate in the next section, this helpful method turns out to be only one side of the story of bridging the gap between intuitive and analytical thinking.

Bridging up: debugging intuition

In the previous activity, the teacher has skillfully addressed the problem of bridging the clever analytical solution down to the students' intuition and, again, she is entitled to feel happy about this achievement. But again, reflecting more on the previous activity, she may discover that, good as it was, this bridging activity has left another important gap wide open: what about the students' original (incorrect) solution? Indeed, understanding the teacher's solution did nothing to help the students see what was wrong with their original intuition on the 2-glass puzzle. Knowing that this kind of universal intuition is not easily eliminated, the teacher decides now to start with the students' intuition and help them "fix" it a little, until it yields a correct solution to the puzzle. An analogy with computer programming is useful here. When you discover a "bug" in your program (which you always do), you do not discard the program; you "debug" it. In this sense, the teacher now wishes to help the students "debug" their intuition regarding the 2-glass puzzle, and she does this via something like the following dialog.

Teacher: Ok, let's look at your intuitive answer a bit more closely. Let's assume that instead of wine and water glasses, we have two jars: one jar is filled with lots of tiny red marbles and the other with lots of tiny white marbles (we'll call them the red and the white jars for short). Now we take as before a spoonful of marbles from the red jar and transfer it to the white jar and mix. Next we take a spoonful of the mixture from the white jar and transfer it back to the red jar. Now which is more: the red marbles in the white jar or the white marbles in the red jar? What's your intuitive feeling?

Student: I'd say the same as before, because we first transferred a full spoon of red, but on the way back we transferred a mixture.

Teacher: Ok, let's look at the situation more closely. Let's assume for example that the spoon contains 100 marbles. Now when we transfer the mixture from the white jar back to the red, how many of the marbles in the spoon would you say are white and how many red?

Student: I'd say about 80 white and 20 red.

Teacher: So how many whites are in the red jar in the end?

Student: 80 of course.

Teacher: And how many reds in the white jar?

Student: 100 first, but then less 20, 80 too! They are the same!

With the teacher's help, the student has debugged their intuition so that it now yields the correct answer. We say that the teacher has helped the student *bridge up* her intuitive solution to a correct (analytical) solution. Note that the new analytical solution is quite different from the previous solution, which was based on the conservation argument. The new solution may appear less elegant (and is certainly less general) than the previous one, but it has the great advantage that it was born out of the student's original intuition, and the student can feel (even after the debugging) that it is *her own* solution. It also carries the important educational message, that intuitions are a precious resource and should be cherished and nurtured: they might need to be debugged but not discarded.

We note that the teacher has, on the way, introduced two minor bridging-down tasks: replacing the wine and water by red and white marbles; and working with a "generic example" (100 marbles in the spoon and an 80-20 mixture in the mixed spoon).

Reflection on bridging up

The scenario depicted in the previous section brings up an extremely important educational issue. We maintain that in case of a strong and recurrent intuition that leads to a non-normative solution, this intuition ought to be considered a precious and valuable resource, despite the incorrect answer. As several authors have elaborated (*e.g.,* Smith *et al.,* 1994), ubiquitous deep intuitions should not (and cannot) be replaced, even when they lead to non-normative answers. Rather, we should help the student, with a well-designed didactical intervention, to debug and refine the same intuitions until they yield the normative answer.

How do we go about debugging a student's intuition? We may start by analyzing the presumed source of the bug; that is, taking into account Kahneman's (2002) analysis quoted above, we investigate the salient features of the problem that make System 1 jump with the buggy solution, as well as the hidden features that are ignored by it. Specifically, in the 2-glass puzzle, we start from the student's answer and explanation: "There is more red in the white glass, because we transferred a full spoon of wine in one direction, but only a mixture in the other". On reflection, it seems clear that the hidden (and crucial) quantity ignored by System 1 is the small amount of red marbles going back to the red glass in the mixed spoon. We can thus use various devices and transformations (in effect "waking up" the analytical System 2) for making this small amount more salient in the student's thinking. The transformations we have taken in the dialog above are *discretization* (changing the wine and water into red and white marbles) and *concretization* (working with a generic example: assuming that the spoon contains 100 marbles, and that in the mixed spoon the mixture is 80 white and 20 red marbles.) This example is generic in the sense that we never use any specific properties of the numbers 100, 80 or 20. Except for notational complications, these could have just as well been replaced by variables. It then becomes more or less transparent that the minority amount in both glasses is 80 marbles. The hidden feature of the minority amount is made conspicuous by introducing the marbles and the concrete numbers. Evans (2006) supplies a theoretical foundation to the didactical transformations we have used: according to Evans, if a problem cannot be solved by System 1, we need to transform it in such a way that the solver will be able to represent the problem situation (specifically, its nested-subsets relationships) *in a single mental model*. The transformations of discretization and concretization that we have introduced above (as well as the pigeon version in the previous section) clearly help the student to construct exactly this kind of mental model.

Bridging the bridges

In classes where the teacher has chosen to carry out the bridging up activity first, as described in the previous section, her final goal now is to help the students arrive on their own at the sophisticated and elegant solution based on the conservation principle. The current workshop takes up from where we left off in the previous section:

> Now suppose someone accidentally knocked down the two jars and all the red and white marbles have scattered all over the floor. Suppose further that this same person (having duly apologized) has collected the marbles from the floor and put them randomly back into the original jars, taking care however that each jar should in the end contain the same amount of marbles as it originally had. Now which is more—the red marbles in the white jar, or the white marbles in the red jar?

Here the teacher may proceed by introducing the pigeons bridging task, this time not in order to explain the already-given solution, but to help the students discover the solution for themselves.

Reflections on bridging the bridges

Let us pause here to take stock. We started in the first activity with a puzzle that is (like many good puzzles) an "intuition trap". Similarly to optical illusions and the famous tasks by Kahneman and Tversky, it had most probably been *designed* to trap people's intuition and lead them away from the correct solution. Having thus elicited the students' mistaken intuition, we have proceeded in the second activity to more-or-less ignore it, and have presented them with an elegant solution, based on a clever conservation principle, which, however, was far removed from the original intuition and, for that matter, also from the original puzzle. When it appeared that for many students understanding this solution is hard, we have resorted to bridging down, in which we have replaced the original task with a similar one, whose solution, however, was much more intuitive.

Now what do we have on our hands? We have four solutions: one, the students' own, which they feel at home with, but which has now been shown to be incorrect; two, the elegant and clever solution which the teacher has pulled out of her sleeve, and which some of them admire but many find alienating and hard to understand; three, our bridging down pigeon variation, which is hoped to have made our elegant-but-alien solution more accessible and intuitive; and four, a new bridging up way of looking at the student's own solution and of turning it into a correct solution via some small fixes ("debugging"), which shows that their initial intuition was not so bad after all.

Having bridged the intuitive and the analytical solutions from all sides, and having narrowed many gaps, there is still one gap remaining wide open: the gap between the original puzzle and its original (debugged) solution on the one hand, and the new puzzle and its solution on the other hand. For the time has come to admit that our elegant conservation-based solution actually solves another, much more general and abstract puzzle; indeed, this solution works also for multiple spoonful transfers or, for that matter, any kind of mixing with no spoons at all, as long as the quantities in each glass return in the end to its original level. (The two glasses, incidentally, need not even have identical quantities to begin with.) Our last activity, with the spilled jars, has come to bridge this last gap, between the two versions of the puzzle and its solution. It has led the students to abandon the spoon transfer, and hopefully arrive instead at the conservation principle [4].

Conclusion

We have used the 2-glass task as a "generic example" for developing the idea of bridging the intuitive and analytical modes of thinking, and for highlighting the distinction between bridging up and bridging down. We believe, however, that these ideas and methods are quite general, and are applicable in many curricular topics where "intuition traps" are frequently encountered and, as such, ought to be included in the toolbox of mathematics education researchers and practitioners. A good example is the students-and-professors task in high school algebra (*e.g.,* Clement *et al.,* 1981). Another example from decimals is students' frequent belief that 3.14 is greater than 3.5, "because 14 is greater than 5". In both cases, one can think of good bridging tasks.

In this article, we have described classroom scenarios designed around the 2-glass puzzle to highlight the gap between the intuitive and the analytical modes of thinking, and to discuss various ways of bridging this gap. Experienced teachers have at their disposal a bagful of useful tricks, which may be classified into three types: mathematical tricks, didactical tricks, and cognitive tricks. By "tricks" we do not mean to convey any magical or negative connotation; rather, we mean clever devices for performing a difficult task.

Mathematical tricks utilize deep properties of numbers (or other mathematical objects) for achieving spectacular results. For example: how to perform the operation 22 x 18 in your head? Answer: $22 \times 18 = (20 + 2) \times (20 - 2) = 20^2 - 2^2 = 396$. (For a more spectacular trick, the reader is invited to work out the final digit sum of 2^{100}.)

Didactical tricks aim to teach students practical methods of getting the right answer on exam questions. For example: to divide one fraction by another, invert the second one and multiply by the first.

Cognitive tricks, which are our focus here, are meant to bring about cognitive change. In our case, cognitive tricks can lead to the peaceful co-existence of intuitive and analytical views of the same situation. Examples include the introduction of the pigeon-house puzzle to bridge the 2-glass puzzle, and involving students in a conversation or an activity to help them "debug" their intuition. Superficially, these cognitive tricks may look like didactical tricks. The difference is that didactical tricks aim to change behavior, while cognitive tricks aim to change thinking and understanding; the former usually have performance (exams) in mind; the latter, conceptual development. As in the opening Kierkegaard quotation, our point here is not mainly to get the correct answer, but to work with and remain with the student's intuition, while trying to stretch it to yield a correct answer. Clearly, teachers have an important role in helping students build bridges and it is important in the future to study this role in more detail.

Notes

[1] For a more comprehensive discussion of Dual Process Theory and its application in mathematics education, see Leron and Hazzan (2006, 2009).

[2] The focus of this article is bridging the gap between the intuitive and analytical modes of thinking. The 2-glass puzzle serves here as a "generic example" for illustrating and discussing the various kinds of bridging in theory and in practice and related pedagogical issues, and it has not been our intention to offer new solutions to this classical puzzle *per se.* For related treatment of the puzzle and its solutions see, for example, www.donaldsauter.com/wine.htm.

[3] Case (1975, pp. 78-82) administered this puzzle to 150 students at the high school, undergraduate and graduate level and has made similar observations, both on how people answer and on how they justify their answer. His report is also in agreement with our observation that many students find the correct solution, once it is explained to them in a logical way, quite hard to understand.

[4] Having gone through this entire journey, we may speculate on the process the mythical inventor of the 2-glass puzzle went through and admire her ingenuity. Having likely started from the conservation principle (which is well known in higher mathematics and physics), she worked up the story of two glasses and the spoon to mask this principle and led the solver into the intuition trap. This story has become plausible once we have noticed that the spoon and the whole transferring procedure are irrelevant for the correct solution.

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Emma Castelnuovo Award

The Emma Castelnuovo Award Committee is pleased to announce that nominations are being accepted for a new ICMI award named after Emma Castelnuovo, an Italian mathematics educator born in 1913, in celebration of her 100th birthday and honoring her pioneering work. She died in April 2014.

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