TAKING ACTION - MATHEMATICS CURRICULAR ORGANIZATION FOR EFFECTIVE TEACHING AND LEARNING

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Taking Action is a form of curricular organization that I have worked with for the past six years as part of a collaboration between Arcadia University and Philadelphia public schools. This article describes this way of teaching and learning mathematics, analyzing it as a form of culturally responsive, and culturally specific, pedagogy (Averill *et al*, 2009; Leonard 2008) Future teachers are trained in the use of this form of organizing the teaching and learning of mathematics. They begin in a semester course and, later, lead groups of elementary school students (Grades 1-8, approximately ages 6-14) through six-week units of study

Taking Action is part of a larger curriculum-design structure that promotes children taking responsibility for their own learning In this structure, the teacher introduces a set of topics and potential investigations and leads the children through the processes of investigation design and implementation. During investigations, the teacher assesses student progress on specific mathematics objectives while holding individual and group conferences with students, teaching mini-lessons on specific skills and concepts, and facilitating dialogue on the progress of the mathematical investigations. After the investigations are well underway, the teacher helps students to develop "taking action" projects After the action projects, the unit culminates in an archaeology period, during which students consolidate and extend the skills and concepts that have been learned.

Although a careful study of experiences like *Taking Action* might reveal important insights into teaching techniques, my own reflections point more to characteristics of the curriculum design. As students develop mathematical understandings, they often recognize the importance of acting on their beliefs and the connections they see to issues in their lives and communities. Even if they do not see these at first, the process of figuring how to make an impact and following through with an action often helps them to see that they have been doing something that matters Mathematics is in this sense not just stuff that has to be done to get through it, but the stuff of meaning and action.

The most important element of what we have been doing, I believe, is the focus on this action work. At first it seems like it is not even part of the mathematics curriculum; it is in some sense an add-on to the "real" content. Yet, for many students, the action phase turns out to be a place where the mathematical understanding is developed or extended As Gutstein and Peterson (2005) point out, "once learners are engaged in a project, like finding the concentration of liquor stores in their neighbourhood and comparing it to the concentration of liquor stores in a different community, they recognize the necessity and value of understanding concepts of area, density, and ratio" (p. 4). Indeed, we have found that action work often prompts demands for advanced instruction in skills and concepts. While there are many reasons to pursue meaningful and engaging pedagogical techniques and practices, taking action might make it possible for almost any form of pedagogy to be highly effective.

The hardest part of innovation, envisioning what it might look like, is especially challenging for mathematics It seems we have a narrower set of images for math classrooms than for other subjects. How can we create when we do not know what we are creating? This work highlights the idea that curriculum design structures might be an interesting research direction for mathematics education Rather than social theory, cognitive psychology, subject sequencing, or other areas that have dominated mathematics education in the past, there may be some ways of organizing activity independent of content or pedagogy that can lead to effective mathematics teaching and learning *Taking Action* is, in this sense, a case study This reflective cycle of creative curriculum design followed by analysis of its characteristics based on field experiences, then followed by theoretical connections with other forms of pedagogy that share its characteristics, is an example of a type of research in mathematics education that may serve our work more richly.

Opening	Investigations			Archaeology
	Develop Projects	Do Projects	Take Action	

Figure 1. The five-part structure of Taking Action

Taking Action as curriculum design

The unique aspect of *Taking Action* is that, in the middle of their investigation work, students are required to reconsider what they have done and identify key aspects of their experience Based on this reflection, students are helped to design a way of interacting with people outside of their class in order to make an impact on the world using the mathematics they have been studying. This "impact" can be one of two types: either they must identify a person or group of people upon whom they believe they can make a worthy impact, or they must identify a person or group of people who they believe can make a considerable impact on their own mathematical work. This is done either individually or in small groups, based on investigations that have been carried out so far within the unit.

For example, in a first grade group working on a number of investigations involving a zoo - sizes of habitats, amounts of food, costs per year, and so on - the class came together around the issue of transporting animals They wondered, for example, how zoo architects design the sizes of doorways between rooms and cages. So they undertook a group investigation around whether a mature, female giraffe could fit through their classroom doorway. Their action project entailed eliciting the help of a docent at the city zoo They sent the docent descriptions of their research along with questions about giraffes that could not be answered through library and Internet research, showing that they had "done their homework" and "knew their stuff" Within one week, they received a response In this case, students felt like they were on the right track with their mathematics, but they needed confirmation and further support. The action work was designed to enlist an outsider who could make an impact. Their attempts to understand and apply the information in the letter became the initial experience of their archaeology work that followed. The docent suggested several questions that the zookeepers were considering related to transport of animals, having to do with minimal space requirements in transit. The students were thrilled that their work was relevant and not just a silly school exercise

Many students do not feel ready to move on to the next segment. In the midst of mathematical investigations, they may feel like they are only getting started Nevertheless, most choose some aspect of what they have done as standing out Either they have a result, or they have identified a way of working that has served them well as mathematicians or they have found a direction of research lacking in results, which in and of itself is an interesting result. For example, a fifth grade class working on a video to introduce a new videogame based on their neighborhood did not think that their work with scale models of buildings was the basis for an action project When they discussed what was critical, however, they kept coming back to disagreements about which standard units of measurement would be most worthwhile, both in their estimates of the heights of the buildings and in the creation of their set backgrounds.

Once students have identified key aspects of their work, they need to consider an audience - people upon whom they can have an impact or who can make an impact on them These might be other children, younger or older, that they want to share their results with. They may be professionals that use the mathematics who could serve as consultants. Or they may be people in the larger community who would be well chosen for such "action." The first graders chose an outside expert In the case of the fifth graders, the disagreements over units of measure became the main theme of a puppet show they performed for second graders who were studying measurement.

The "real audience" for their work makes it important that the students prepare well. For this reason, it is not always during the investigations that the actual mathematical learning takes place. The crafting of the letter to the zoo docent demanded strong understanding of the mathematics of a giraffe walking through doorways. The students could estimate an average height of a mature giraffe, the range the space required given a giraffe's limitations for bending down and folding their legs to move, and so on They could also calculate the typical size of a zoo doorway, and so on, but in order to elicit from the workers at the zoo the details they needed about what one could reasonably demand of a giraffe, they also needed to describe their work in a way that would not simply be "cute"; they wanted a professional response, and this required careful reflection on what they already knew. In the case of the fifth graders, the need to design an action forced them to reconsider their disagreements as meaningful mathematical content themselves Rather than being opinions, the choices of measurement unit highlighted for them the notion of a standard unit, as well as the role of the unit in the accuracy of calculations

Note that this process and the larger five-part structure do not depend on any specific mathematical content. They can be used at any grade level for any set of mathematical skills and concepts. Critical to the taking action phase of the work is that students look back over what they have done and figure out a way to share what is important to them. Their plans must be serious and they must prepare well in order to take advantage of the opportunity that this action project presents. Since the students choose the specific aspects of the mathematics that form the basis of their actions, and since they identify the best members of the outside community for their actions, they have a greater stake in the success of these actions than in a typical classroom activity. While the teacher can help with arrangements and logistics, we find that it is best if the students themselves contact the community members and make arrangements, as much as possible This is the case even for first grade students. If first graders contact an architect or zoologist or mathematician, or visit sixth graders to invite them to visit with their class, it is more likely to be received well and more likely to be taken seriously than if the teacher asks If the invitation is articulated in a way that demonstrates serious prior involvement with the mathematics, it is even more likely to get a strong, positive response

Students must rehearse and prepare materials for the action, anticipating how their action will make its impact If they want their audience to learn something or to make life choices that matter, their presentation of their work must be done in an entertaining, understandable and compelling way. It cannot be routine or boring. Thus, as part of the design of their project, the students themselves must identify and actualize what is compelling about the mathematics that they have been using. The curriculum unfolds under the assumption that pupils are capable of doing this

Students must also consider how to communicate the mathematics If the audience is less sophisticated mathematically, they need to select appropriate representations for the ideas, using them in ways that effectively communicate important points If the audience is more sophisticated, students want to communicate in ways that are based on the importance of what they have learned. If they want their audience to help, they must communicate what they have done so far and their current questions. If the presentation of the work is trivial or communicated without enthusiasm and evidence of commitment, then the consultant will likely not invest time and effort in serious help and guidance.

As students prepare for their action projects, they practice with others in their class, and use feedback to make improvements. They rehearse what they want to say, role-playing encounters and learning about one another's projects. In this way, students not only carry out an action project, but benefit from the action projects of others. The project preparation takes on a character decidedly different from typical classroom presentations, because the students are preparing for real-life audiences with scheduled deadlines. The multiple representations of the mathematics that are involved in the different projects help all students become fluent with a range of concepts, whether or not they were part of those projects

Examples of taking action

Here are some examples of *Taking Action* projects that students have carried out:

> As already mentioned, **1st grade** students undertook a group investigation around whether a mature, female giraffe could fit through their classroom doorway.
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> Five **3rd grade** students, who had been recording data on the use of various shapes in paper airplane design, held a design challenge contest to test their ability to teach others the lessons they had learned about shape and weight in the design of paper airplanes. They invited teams of school parents to participate and charged an entry fee that raised money for school supplies. The contestants were required to attend two training workshops during which the students taught key findings from their data on plane experiments before they were allowed to enter the contest.
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> A pair of **5th grade** students designing their own shopping arcade mall shared their model with another class of fifth graders - sharing not just the model but the design process, which involved extensive survey research, discussions with potential fifth grade shoppers, and visits to local contractors who build malls. The presentation provoked suggestions for how to improve not the mall design but the design process itself, which led to new ideas for the use of mathematics in architectural design and scale models
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> **7th grade** students arranged a meeting with city council members to advocate that the city officially declare non-support for the US war in Iraq. They shared their analysis of data on the local costs of the war, and demonstrated that the city could channel money away from war-based efforts toward programs for the homeless and for improved park services. The same seventh graders later worked on the council-sponsored city referendum that was eventually passed by the voters.
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> **8th grade** students wrote children's books that explained concepts of place value and ratio and proportion. They wrote multiple drafts, sharing them for advice with local adults who write professionally, and finally several times as reading buddies with first graders. Final versions of the books were donated to the school library.

Students have created puppet shows, met with carpenters and local political activists, convinced local communities to take a stand on blood diamonds and historical sites of slavery, written letters to editors of newspapers, contacted NGOs and enlisted businesses in local community projects. Each taking action project requires the students use the mathematics they have been learning and applying in ways that make a difference in real life. Thus the potential for mathematics to make an impact on the world is not just taught in these classrooms: students learn that such an impact only happens if you plan for it to happen, and if you plan well

Highly effective teaching

My claim is *Taking Action* as a mathematics curriculum structure shares the six habits of highly effective teachers that were identified by Ladson-Billings (1995, 1997, 2001):

- students are intellectual leaders in the classroom,
- students are apprenticed in a learning community rather than taught in an isolated way,
- a wide range of mathematical literacy is embraced,
- students' real life experiences are legitimized as part of the curriculum,
- students and teachers challenge the status quo, and
- teachers see themselves as political beings.

(Indeed, the work I report could be seen as a replication of Ladson-Billings' research, in different educational contexts.)

When teachers design activities that foster the exchange of mathematical thinking in order to decode and invent mathematical objects, they employ the metaphor of apprenticeship, in which students come to the classroom already as mathematicians developing their craft in a mathematical studio (Appelbaum, 2008). Students in such a classroom become intellectual leaders. Their ideas and suggestions to others directly influence the direction of mathematical exchange; their contributions become part of the ongoing construction of the mathematical canon within their community. Students who read, write, listen and speak mathematics in such a community are working with a broad conception of mathematical literacy. They employ funds of knowledge (González *et al.*, 2005) in ways that legitimize the mathematics of their everyday lives.

Students and teachers in these projects challenge the status quo by questioning most of the basic assumptions of mathematics education. Instead of classrooms where teachers provide instructions obtaining answers to types of mathematical questions in isolated contexts separate from everyday life, students design their own ways of interacting with people outside of their classes. The students provide feedback and constructive criticism to each other as they rehearse and perfect their action preparations. And, by working in these ways with students, teachers facilitate all members of the community to enact a politics of mathematics education that promises social transformation of the institution of schooling.

Project-based learning and making connections with

Recipe for Taking Action

INGREDIENTS:

Mathematician's notebook
Big mathematical ideas, strategies, and models
A sense of what's significant about your investigation
A willingness to imagine the impossible and the improbable
Patience and persistence

PREPARATION:

Step One: Take your investigation seriously Know/believe why it is interesting, to you and to other people Look back over the work you have done on this investigation. Answer these questions in a letter to yourself:

- What surprised me as I did this project?
- What does not stand out for me, but is nevertheless important to my work?
- What one thing that I did or that I discovered or that I invented is critical to my project in the sense that if I had not done, discovered, or invented it, my project would not be the same project?
- Do I want to make an impact on others or meet with a person or group that would do more to help me in my project work?

Use class meetings to help you with this part of your investigation, and to help others Post your letter to yourself, and respond to others' posts about their own work in taking action

Step Two: Imagine how the world could be different if somebody else knew about or used the results of your investigation or imagine how amazingly different your investigation could be if only you had help in working through some mathematical idea in it or imagine how your life could be different if you shared your findings in some personally significant way Class meetings will be helpful here as well

Step Three: What would you need to do if you were a genius and could make any of your ideas in Step Two happen? Well, you are a genius so get started What's the first step? Do you have to email someone? How about call? How about call and email? Do you need to rehearse the phone call with a friend first so that it is polished? How are you going to entice the other person to meet? Think big, but be specific. For example, when you go and talk to someone, you don't talk about how you used Polya's phases. That's not what is significant about your work. You have developed an idea coming out of your work that others need to see/hear/feel.

Step Four: Identify what you are fascinated about in your work so that you can make sure that this is shared.

Step Five: Make concrete appointments, with back-up plans in case something goes wrong. Allow time to prepare! If at all possible, leave at least a day in-between so you have time for further preparations

Step Six: Based on the initial conversation or email interchange you had in setting up appointments or follow-ups, review your mathematician's notebook for excerpts you will share as part of the up-coming plans.

Step Seven: After taking action, write up responses to the following three questions:

- What did you learn from this experience?
- What new directions for your action research grew out of the experience?
- How should your project efforts continue and/or change for the remainder of this semester, and into the future beyond this semester?

NOTES:

- When taking action on an investigation, you are no longer doing a project for a class. You are doing a project for yourself! This change in focus will open doors.
- Have a plan A, B, C, D, E. Don't just try to contact one person. Contact many, many people! It increases the odds of making your idea happen.
- Hone your message. What's the content? How can you sound like the mathematician you are? If you are talking emergency structures, what work can you do to make sure you sound like you know what you are talking about?
- Contact people for advice on who you could contact Who might open the door for you?

Figure 2. A recipe for taking action

everyday life are not new ideas (Skovsmose, 1994; Tichá & Kubínová, 1999). *Taking Action* is There are two key differences. First, in usual project-based learning, a teacher designs a project to ensure specific skills and concepts are learned. Projects are designed and carried out by students based on their own conceptions of what they have already learned. Second, even in learning environments where problem-posing is the focus, there has been little emphasis on the translation of the mathematics into an impact on the world. While it is often the case that much of the learning takes place in this action process phase, this learning is organized according to prescribed objectives. The purposes of the projects are not to help students master skills or develop understanding, but to help others appreciate what the students know and understand in order to make an impact. The learning is ancillary to the impact, even though working toward the impact incites greater learning.

Keitel, Klotzman and Skovsmose (1993) anticipated this by describing mathematics as a technology with the potential to work for democratic goals. They distinguished among types of knowledge based on the object of the knowledge. Their first level of mathematical work presumes a true-false ideology, corresponding to much of current school curricula. The second level directs students and teachers to ask about right method: Are there other algorithms? Which are valued for our need? The third level emphasizes the appropriateness and reliability of the mathematics for its context. This level investigates the relationship between means and ends. The fourth level requires participants to interrogate the appropriateness of formalizing the problem for solution; a mathematical/technological approach is not always wise and participants would consider this issue as a form of reflective mathematics. On the fifth level, a critical mathematics education studies the implications of pursuing special formal means; it asks how particular algorithms affect perceptions of (a part of) reality. Thus the role of mathematics in society becomes a component of reflective mathematical knowledge. Finally, the sixth level examines reflective thinking itself as an evaluative process, comparing levels 1 and 2 as essential mathematical tools, levels 3 and 4 as the relationship between means and ends, and level 5 as the global impact of using formal techniques. On this final level, reflective evaluation as a process is noted as a tool itself and as such becomes an object of reflection. When teachers and students plan their classroom experiences by making sure that all of these levels are represented in the group's activities, it is more likely that students, and teachers, can be attributed the critical competence that we envision as a more general goal of mathematics education.

In *Taking Action* students are attributed the critical competence (Skovsmose, 1994) to judge how and where their mathematical ideas can either make an impact on the world, or how the world can make an impact on their further development of ideas. Such a presumption is rare in the usual mathematics classroom, and especially unusual in primary education. What our work shows is that it is reasonable and simple to do this with young children. We arranged for groups of three university students training to be teachers to meet with groups of 8 to 10 children in the children's schools twice per week for periods of one to two hours. They met for six weeks in available spaces – dining rooms, corners of hallways, the back of classrooms, and so on. The student teachers were required to document how they knew that the children were meeting the school's objectives for mathematics and, at the same time, to document how they were helping the children to design their own investigations, carry out action projects, and experience an archaeology of their work.

At first, the university students were skeptical: they did not feel like they were teaching the children anything. They did not feel like they were teachers. In fact, they were not acting like the usual teacher in the classroom. Instead, they were creating learning environments where children were learning mathematics and using it to make a difference in the lives of real people. They were astounded at how excited parents and school personnel were at pupils' accomplishments, at the media interest, and at pupils' enthusiasm.

Implications

What are the implications of this work? First, we might look at the six habits identified by Ladson-Billings and ask if they are universal. If we find a curricular organization that appears successful to widely varying constituencies, will the habits of the teachers in these situations tend to display Ladson-Billings' six characteristics? Second, we might think logically in the other direction: If we design a curricular structure that has these six characteristics, can we further test the efficacy of these habits of "highly effective teachers" to see if they are truly those habits that all teachers should strive to integrate into their practices?

Whether or not the six habits reach across all cultures and nationalities deserves additional attention. I can report anecdotally that another use of the habits is as a reflective tool for teacher self-assessment. I have led in-service teacher discussion groups and university courses where the participants are asked to document how they have been integrating these six habits into their own practice. These reflections and the subsequent small group discussions have been a source of excitement and professional growth every time I have used them. These have all taken place in the United States. This suggests as well that teachers who strive to enact these habits in the ways they structure learning environments will be more satisfied with the results, both in terms of how they feel about their work and in terms of the performance of their students. Perhaps the question of universality is not all that important, and instead that striving to integrate these habits into one's own teaching is a valuable personal project.

There is potential for extending this kind of taking action work into new schools and new learning environments. The requirement that someone or some group think about what they have been learning, identify what is interesting or significant, and then find a way to connect with others in order to generate an impact – on those people, on the community, or on themselves – is the crucial catalyst for new relationships with curricular content. It makes it easier for the learner to identify what is significant for them if they were also part of the design process. But it is also possible to try out the recipe for action independent of the five-part structure. I have been experimenting with this as part of a graduate course for current teachers. Not all have been able to design five-part replacement units based on investigations for their current school-mandated curriculum, but each has been able to squeeze the taking action into their work as a parallel curriculum that takes at most ten-fifteen minutes out of a class meeting once, twice, or three times per week These teachers introduce the idea of action and facilitate discussions of what their students remember from a previous unit, discussing whether it was the mathematics, the way they learned it, or the applications that was most significant for them. This might take several short class meetings. The next step is to help students think of people in the world, or social justice concerns, connected to what they have highlighted for themselves They form groups based on overlapping or similar interests, and design action projects. Most of the work happens outside of class time, as the short class meetings are used to check in on progress, support students with identifying resources, and establish action plans and calendar deadlines. Family and community support often materializes as students are encouraged to ask community leaders and friends for advice on how to ensure their actions are successful.

I have found examples like those provided in this article can serve as the pictures some of us need for what is possible before we can begin to take on such work Another excellent resource is our youth: asking them what they would like to do with what they have been learning often opens up exciting conversations. It is interesting that the best action projects are those that are least "set up" by a teacher - that is, if the teacher does not have any preconceived ideas for what could be an action project, the students themselves often think in bigger terms than the teacher might have Schultz (2008) describes one new teacher's experience of this, as well as the ways that learners can still be proud of what they have done even if they do not reach their initial goals. Schultz thought small before testing out a new "civics curriculum," and his students responded with the idea that they needed a new school. This became a year-long action project that took over his entire curriculum. In Philadelphia, we worked with young people twice a week for six weeks, and the action project component of the curriculum took place in the fourth or fifth week of our time in the schools, leaving more time for the archaeology work that followed. Whether the action becomes the entire curriculum or an important piece of a larger curriculum, it is a useful form of culturally relevant pedagogy

Rather than celebrate or promote *Taking Action* as a good idea, this curricular structure exhibits critical characteristics of an organizational form that enables students to experience mathematics as "the stuff of meaning and action." It does this in three ways: First, it requires students to look back on their work as something of meaning and then to use that work to take action. In that light, one can take a critical stance and label it a crass and simple attempt; it is an extreme and trivial case of a structure that promotes meaning and action with mathematics at the core; we would hope for more nuanced alternatives. Second, it shares with other pedagogical approaches the six habits of highly effective teachers noted by Ladson-Billings. In this second sense, the field work with taking action is an affirmation of her research, demonstrating that working in ways that share those six characteristics will lead to positive responses from those who judge teaching and learning: We are consistently invited back to work with the children from year to year based on the results of the students' performance on district benchmark examinations. Third, *Taking Action* is not meant here as an answer to problems with mathematics education. It is, rather, an example of a type of curriculum structure that has the potential to provide meaning for both teachers and students, and that can provide pictures of possibility for curriculum reform. In this latter sense, I am hoping that my discussion of it will lead my colleagues to "take action" in curriculum re-design Once we collect a number of curricular structures, we can perform a more nuanced analysis of what these curricular structures have in common, and how they differ in terms, for example, of the Ladson-Billings' habits or Skovsmose's critical competence This would be a new form of theoretical action in our field - a sort of natural science of curricular design, with its own typography, concepts, and dynamic relations *Taking Action* is a call for a "curriculum studies" of mathematics education

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