Conceptually-Oriented Mathematics Teacher Development: Improvisation as a Metaphor

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Mathematics educators agree that a teacher's conceptions of mathematics and mathematics teaching affect classroom practice (Thompson, 1984, 1992) One way of categorizing teachers' conceptions of mathematics is by examining their orientation toward mathematics instruction. Thompson *et al.* (1994) contrast two seventh-grade mathematics teachers' behavior in similar circumstances, specifically focusing on the nature of the discourse in the classroom. They posit that the differences in the teachers' behavior were based upon differences in the teachers' orientations towards teaching mathematics.

A calculationally-oriented mathematics teacher, while not necessarily focused on computations, is someone who is:

> driven by a fundamental image of mathematics as the application of calculations and procedures for deriving numerical results. (p. 86)

By contrast, conceptually-oriented teachers:

> focus students' attention away from the thoughtless application of procedures and toward a rich conception of situations, ideas, and relationships among ideas. (p. 86)

These differing orientations have an impact upon classroom discourse, as the teacher sets the tone for and facilitates the discourse of the classroom (see also NCTM, 1991).

The purpose of this article is to introduce a metaphor for conceptually-oriented mathematics teachers and to discuss its possible implications for mathematics teacher preparation and professional development. The metaphor is that of the conceptually-oriented mathematics teacher as proficient jazz improviser, in contrast to an inexpert or learning player, or one who does not improvise. In the next section, I elaborate on the metaphor itself. Following this, I provide a rationale for the ways in which this metaphor provides insight into the ways in which mathematics teacher educators might prepare and support the continuous development of conceptually-oriented mathematics teachers.

The significance of this analysis lies in the importance of conceptual metaphors. Lakoff (1994) states that metaphors:

> permit us to reason about [the target domain] using the knowledge we use to reason about [the source domain] (p. 210)

While metaphors have been employed to begin to understand students' learning and reasoning about mathematics (Lakoff, 1987, 1994; Lakoff and Núñez, 1997; Presmeg, 1997; Sfard, 1997), the use of metaphorical reasoning in this article does not focus on students' mappings of source domains, often concrete and familiar, to the target domain of abstract mathematical ideas. In this case, the source domain is jazz improvisation and the target domain is conceptually-oriented mathematics teaching. The metaphor can present ways to reason about the preservice and inservice professional development of mathematics teachers by examining ways in which jazz musicians begin their education and continue their development.

The background of the development of this metaphor lies in empirical work done for my doctoral dissertation (King, 1997). The focus of that study was on understanding the reasoning employed in making moment-to-moment instructional decisions by two conceptually-oriented mathematics teachers in a university course. I developed an earlier version of this metaphor to begin to explain the reasoning of these teachers, as opposed to information-processing or other models of teacher decision-making in classrooms. (See Clark and Peterson (1986) for other models of teacher decision-making.) This article does not draw directly on my empirical work: its purpose is to outline the metaphor as a means of provoking thinking about preservice and inservice teacher education, in light of what we might learn about teaching by making metaphorical mappings to improvisation.

The metaphor

A metaphor is:

> a mapping (in the mathematical sense) from a source domain [...] to a target domain. [...] There are ontological correspondences, according to which entities in the [source] domain [...] correspond systematically to entities in the [target] domain (Lakoff, 1994, p. 209)

In the following sections, I outline the correspondences between a jazz improviser and a conceptually-oriented mathematics teacher. For this article, I define teaching as interacting with students for the purpose of creating opportunities for students to learn. In this way, teaching is necessarily linked to student learning.

Improvisation and conceptually-oriented teaching

> Improvisation, like composition, is the product of everything heard in past experience, plus the originality of the moment. The contents of even a very accomplished improviser's solos are not all fresh and original, but are a collection of clichés, established patterns, and products of the memory, rearranged in new sequences, along with *a few* new ideas. (Coker, 1964, p. 36; *emphasis in original*)

The power of a metaphor is in bringing the knowledge of the source domain to the target domain (Holyoak and

Thagard, 1995; Lakoff, 1994). Thus, in order to gain an understanding of the metaphor as I outline it in this article, one needs to have some understanding of the source domain (proficient jazz improvisation) and the target domain (conceptually-oriented teaching). As expressed in the previous quotation from noted jazz musician, educator, and theorist Jerry Coker, improvisation is the unique enactment of the song in the particular moment

Thus, the performer makes use of forethought, remembrance, previous knowledge and new ideas in the enactment of this moment While improvisation is found in many art forms and types of music, I focus this discussion on jazz because, for many, improvisation is an integral part of performing jazz, and there has been a movement toward theorizing about jazz improvisation, the pedagogy of jazz improvisation specifically, and jazz performance generally (Baker, 1969, 1988; Coker, 1964, 1978).

A conceptually-oriented teacher, as described by Thompson *et al.* (1994), is a teacher who bases her actions [1] on:

- an image of a system of ideas and ways of thinking that she intends the students to develop;
- an image of how these ideas and ways of thinking can develop;
- ideas about features of materials, activities and expositions and the students' engagement with them that can orient the students' attention in productive ways (a productive way of thinking generates a 'method' that generalizes to other situations);
- an expectation and insistence that students be intellectually engaged in tasks and activities. (p. 86)

This view of teaching is in contrast to what they refer to calculationally-oriented teachers, ones who focus primarily on numbers and operations, solving problems with numerical solutions by means of defined procedures, de-emphasizing the context of the problem or the students' learning or previous knowledge. These researchers describe the difference in terms of the teacher's expectations: a calculationally-oriented teacher expects "students to explain their procedures", while a conceptually-oriented teacher expects "students to explain their reasoning" (p. 87). A focus on students' reasoning leads to a responsiveness to the teacher's interpretation of student thinking

Who is/are the performer(s)?

The performer, the jazz improviser, is the teacher. She is the improviser because she is the person with some intent to perform. This correspondence can be seen in expressions many teachers, teacher educators and researchers use to discuss the practice of teaching. Thompson (1985) refers to teachers as 'choreographers' because:

> they must have a structure that they can 'dance' through as they confront the obstacles that are inevitable in the flow of classroom interactions. (p. 231)

In another example of teachers and researchers discussing teaching in ways consistent with the teaching-as-improvisation metaphor, Chazan and Ball (1999) challenge "the anti-telling rhetoric prominent in mathematics education reform" (p. 7). They contend that the teacher's role as a facilitator of discourse includes:

> teacher moves as the product of subtle *improvisation* in response to the dynamics and substance of student discussion (p. 7; *emphasis added*)

These responses, for them, include judicious telling. The act of teaching is the improvisational performance and to play or perform is to teach or interact with students. This performance might include several days of class, rather than a single lesson or class period

Remillard (1997) uses improvisation as a construct to describe teachers' behavior during instruction, seeing their implementing of plans as follows:

> enacting tasks included two types of activities: (a) assessing what was taking place in the classroom by *reading* students and the tasks they were engaged in and (b) *improvising* by selecting among a range of possible pedagogical alternatives for proceeding (p. 5; *emphasis in original*)

Sassi and her colleagues (Sassi and Goldsmith, 1995, 1996; Sassi *et al.*, 1997) describe the work of mathematics teacher educators as requiring 'improvisational perception' in the context of teaching mathematics.

While researchers, teachers and teacher educators have used improvisation to describe the act of teaching, particularly for teachers who intend to be responsive to their students' understanding, no one has explored the metaphor in detail to justify the use of improvisation as a construct to help the mathematics education community think about teaching. Nor has a well-developed metaphor been used to help the field think about mathematics teacher education, both inservice and preservice, in different ways.

What is the instrument?

In this formulation, I take the discourse of the classroom as the instrument, because it is what the teacher guides (plays) In the *Professional Teaching Standards* (NCTM, 1991), the focus of the teacher's actions in the classroom is on facilitating a learning community and its discourse. Here, I interpret discourse broadly, including gestures, non-verbal communication and all written work. These interactions are interpreted by the participants based upon their understandings of the content of the discourse as situated within the participants' sociocultural understanding of the context in which the discourse takes place (Potter and Wetherell, 1987).

In a conceptually-oriented teacher's classroom, part of this context is the importance of meaning-making in addressing mathematics and a focus on the understanding of another, whether student or teacher. The rapid decisions a teacher makes in these situations have an impact on the actions of the teacher in ways that drastically change the discourse in the classroom (Thompson *et al*, 1994). As an improviser in the band, the soloist modulates her

performance to the song, to what the other instruments are doing and to the sensibilities of the audience. One can view the students as other participants in this improvisation, the other instruments that the soloist is responsive to, following the direction of the lead improviser, the teacher

This formulation of the metaphor acknowledges the students' role as co-creators of the classroom discourse, thus as participants (as performers) in the improvisation. While the power relationships in the class lead the teacher to be the leader or soloist, in a classroom where conceptually-oriented mathematics teaching is being enacted the students are a part of the improvisation, not passive receivers of knowledge.

This view of students as participants in the creation of the classroom environment can be seen in the work of Cobb and his colleagues' discussion of the mutual constitution of classroom norms and the teacher's and students' roles in this negotiation (Cobb and McClain, 2001; Cobb, Yackel and Wood, 1991) This formulation is in contrast to placing the students in the audience, where they are observers not participants in creating or sustaining instruction.

This formulation also places the activity in the classroom as central, as opposed to the mathematical content or the instructional materials used there. While the content and materials have an impact on the teacher's activity, the focus is on the teacher's efforts to manipulate the classroom discourse for the purposes of student learning, not to change the content or the activity

What is the song?

While a jazz musician is improvising, there is still a song, an artifact of composition that exists prior to notes being played, which provides the basic features (the key, the vehicle, the style, etc.) of what the improviser will play. The song may be communicated in what are known as 'fake books' that provide the basic structure of the song, including the key, main melody and general arrangement. Or, with a well-known song, these elements might be part of the institutional memory of jazz musicians and are communicated verbally by the band's leader To emphasize, the playing is not random and the improvisation builds upon this structure, rather than operates against it

For the mathematics teacher, the 'song' could be considered as analogous to what Simon (1995, 1997) calls the 'hypothetical learning trajectory' (HLT). This includes the teacher's mathematical learning goal for her students, her hypothesis about the process of learning and a plan for learning tasks consistent with this hypothesis. This HLT may be embodied in a lesson plan, but is more than simply curriculum materials and teachers' notes It includes the teacher's thoughts about the process by which students are to move toward the intended learning goals and the tasks that the teacher hypothesizes will elicit this process.

While curriculum materials may provide tasks and curriculum frameworks may provide goals, the agency of the teacher should not be diminished in recognizing her role as a decision maker in devising and implementing this plan. As previously stated, this plan may be carried out over the course of several days, as opposed to just one class period

The fact that the learning trajectory is hypothetical acknowledges that this learning path is flexible and the teacher will be responsive while teaching/interacting with students (the improvisational play or performance).

Reform curricula may provide the basis of an HLT in their elaborated teachers' manuals, with information on what students' might do and explanations of the way in which a lesson might build students' mathematical knowledge (e.g. *Connected Mathematics Teachers Guides*, CMP, 1998). However, the lessons in the curricula and the information in these manuals will be modified by teachers based upon their own goals and interactions with their students to fit the local teaching context, even before enacting the lesson. In similar ways, jazz music's fake books provide the general outline and structure of the song: yet, unlike a classical music score, they are not intended to be played as written, but to be improvised upon

What is the music?

Once the actual notes are played, one has music The students' taken-as-shared knowledge, exhibited through discourse, is the music Again, the students are participants in the performance, with their knowledge reflected through their contributions to the discourse. Also, the music (students' knowledge), as an enactment of the song (the HLT), influences the ways in which the performer (mathematics teacher) will improvise (teach). In the same way, the students' knowledge (music), in the enactment of the HLT (song), influences the ways in which the teacher (performer) will co-ordinate classroom interactions (improvise).

This formulation makes clear that the song and the music are different. There is only one live performance of the song during which the music comes to life However, many performers may play the same or a similar song, with the music being different each time. In the same way, the lesson and the enactment of the lesson in a classroom are different A lesson may be taught several times, but each time the classroom discourse will be different and students' mathematical understandings exhibited in this discourse will not be the same Each enactment of the lesson is unique and thus what should be judged in an effort to examine students' opportunities to learn is not the lesson (the composition or song), but the enactment of the lesson (the music).

This is not to say that the organization of a particular lesson (composition of a song) does not matter During the bebop era of jazz, there were songs that many musicians could not play well, but for those who could the music was terrific. In the same way, there may be HLTs or lessons that some teachers are not able to enact well in classrooms. This inability may be a reflection of the lesson, but also may be a reflection upon the proficiency of the teacher. In music, there are also songs that virtuoso musicians are able to improvise upon and make exceptional or quite different music. As examples, the rock music improvisation upon the 'Star-Spangled Banner' by Jimi Hendrix or the rhythm and blues improvisation upon the same song by Marvin Gaye produced quite different music from the typical enactments of this song Both rock and rhythm and blues grew out of the improvisational tradition of jazz

Who is the audience

While participants, such as the teacher and students, also have a perspective on a performance, for the purposes of this metaphor, the audience is outside observers of the classroom environment (principal, mathematics supervisor, researcher, fellow teacher, parents) These are the people who may view the performance of the teacher and students as full observers. The students, as participants in creating the environment for a conceptually-oriented classroom, are not in a position to view the performance from the perspective of one who was not a participant.

Often, unknowledgeable audiences do not enjoy jazz performances, because they do not know the song being improvised, do not appreciate the way the performer improvises the song or do not appreciate the art of improvisation at all (Coker, 1978). Similarly, it may be that outside observers may not appreciate conceptually-oriented teaching as they do not know the mathematics of the learning goal and how it fits into a larger scheme of mathematics, do not appreciate the ways in which the teacher is responsive to students learning in the classroom or do not appreciate the act of being responsive in teacher-student interactions This lack of appreciation may lead to principals or other supervisors observing for evaluation of classroom teaching being unable to evaluate the conceptually-oriented teaching as a different model of teaching Parents might be concerned that the teaching does not fit the image of teaching that they expect

Facility and knowledge

Jazz improvisers must have certain types of knowledge and facility that can be viewed as corresponding to the knowledge and facility that conceptually-oriented teachers need. For example, a jazz improviser needs to be able to execute the playing, in the sense of being able to play the notes on her instrument. In the same way, a teacher needs to be able to elicit from her students both the willingness to participate in the discourse and productive discursive interactions Such eliciting might include asking questions, posing problems and creating a classroom environment that fosters discourse, including questioning from other students. These practices are not easy and in the next section I will discuss further how the mathematics education community might work to develop such a facility in teachers.

A second crucial element that the jazz improviser must have is a deep knowledge of music, in general, including chord progressions and keys In the same way, a conceptually-oriented teacher needs to have a deep knowledge of the mathematics she teaches. As discussed by many (e g. Ma, 1999; CBMS, 2000), this knowledge needs to be deep and flexible, and to relate mathematics and student learning.

Jazz pedagogy and teaching pedagogy

As previously mentioned, the importance of examining any metaphor is to provide ways to reason about and gain insight into the target domain by reasoning from the source domain Hence, this section will share information about jazz pedagogy and relate it to teacher development. Jazz pedagogy is a fairly recent development in formal schooling. Previously, jazz musicians were developed in an apprenticeship model, and only since the late 1960s has jazz pedagogy moved into the academy as an area of emphasis in music schools During this shift toward academia, the idea that only 'special' people were able to improvise, or that improvisation could not be taught, began to fade - although it continues to exist today.

Instead, the development of jazz musicians in academic settings has now become the norm, displacing the apprenticeship model of the past (Baker, 1988) In this transition, the number of proficient jazz musicians has increased. As the need for mathematics teachers able to teach with a conceptual orientation grows in the U.S., the development of jazz pedagogy in the academy can provide insights into the continuing improvement of mathematics teacher education. In general, a jazz music education program is composed of coursework on theory, history, business, pedagogy and performance experiences in bands and ensembles. This structure is much like preservice teacher education programs for prospective mathematics teachers, with academic coursework in mathematics (as well as other subjects for the elementary teacher), foundations of education and pedagogy followed by fieldwork components. In most jazz education programs, however, participation in various jazz bands and ensembles forms part of the students' experiences throughout the degree program.

The requirements in these programs include early and constant expectations to perform in jazz settings. These expectations are fulfilled in different locales, in which students perform according to their skill and level of knowledge about jazz improvisation. These settings range from dance bands to studio jazz bands, in which the performers are expected to be able to play with minimal rehearsal time and perform on multiple instruments.

Preservice teacher education generally does not allow for students to participate in instruction in elementary and secondary schools in any significant way until the third or fourth year of a four- or five-year program With many separate fifth-year certification programs for intending teachers with no undergraduate education experience, they do not have these opportunities until this fifth year. As a result of reasoning within this metaphor, programs might seek to move beyond observation to multiple forms of interaction with students for the purposes of learning (teaching/performing) earlier in the academic careers of preservice teachers. This recommendation is consistent with others who argue for earlier, substantial field-based activity early in the preservice program (McIntyre *et al.*, 1996)

An important point to note about jazz education is that this education never ends. The process of continuous, gradual improvement is built into the study of jazz and the continued professional development of jazz musicians. Even the greatest musicians continue to study, practice, receive and review critiques of their playing throughout their careers. Unlike teacher education in the US, in which a teacher's education is focused during preservice education, with a smaller emphasis on co-ordinated inservice education (Stigler and Hiebert, 1999), the education of a jazz musician is seen as a continual process that began prior to entering college and will continue far beyond the collegiate experience A recommendation for coordinated and focused inservice

development of teachers, as seen in many documents on improving mathematics education (Kilpatrick *et al*, 2001; NCTM, 2000), follows from the metaphor.

Learning to play music

There is a difference between learning to play music as notes written on a page and learning to improvise - to enact the music in a performance, with the expectation of moving beyond what is written on the page. In the same way, there can be a difference between prospective and current teachers learning mathematics for the purposes of taking a traditional mathematics test in a course or a standardized examination for certification and enacting the same mathematics in an instructional setting

> The playing of jazz presupposes a certain skill with scales and chords in any key (Baker, 1988, p 4)

Following the metaphor, the flexible teaching of mathematics analogously presupposes a certain skill with the components of mathematical discourse, at a variety of grade levels: mathematics and discourse around mathematics.

This elaboration is quite different from simply 'knowing mathematics'. In the development of this metaphor, knowing mathematics is a necessary, but not sufficient, condition for being able to teach mathematics conceptually. In addition to knowing mathematics, in order to elicit students' participation in discourse in ways that promote meaningful mathematics learning, preservice and inservice teachers should begin to learn how to write and ask questions that allow students to provide answers that allow their mathematical knowledge, the music, to be accessed or heard.

Writing and asking questions includes modifying questions found in texts and other teacher materials. Teachers should also begin to learn to respond to students' questions in ways that open possibilities for discussion, and begin to orchestrate classroom discourse so that students participate and actively engage with the mathematics.

Learning to improvise

In contrast to learning to play notes written on a page, learning to improvise requires special preparation, given in jazz education programs. While a typical jazz education program includes learning Western classical music and theory, as well as music from other genres, it also requires learning the art of improvisation. Major themes in learning to improvise include learning to listen, playing in different circumstances and with different purposes and learning songs

Listening

Of major importance for the jazz musician is to be able to hear the intricacies of the music, while keeping in mind the general structure of the song. In jazz improvisation courses, students listen to music, carefully transcribing different improvisations of the same song. In these transcriptions, the students examine key phrases and the ways in which different performers improvise through these phrases. The invention of the long-playing (LP) record and its evolution to compact discs (CD) and other digital media have allowed greater access to jazz pedagogy (Berliner, 1994).

In the same way, the availability of videotape allows mathematics teachers to create records of practice that preservice and inservice teachers can examine with one purpose as learning to listen (Ball and Lampert, 1998). Learning to listen is important for orchestrating productive dialogue, as Schifter and Fosnot (1992) stated:

> But in order to know what to ask, teachers must learn how to listen - what to listen for - in what their students are saying (p. 31)

Thus, as learners of conceptually-oriented teaching practices, preservice and inservice teachers need to learn how to listen to their students as well as what to listen for, those key phrases. While much of the work with cases, both video and written, begins to address these issues, mathematics teacher educators need to become more systematic in the choice of these cases and the field needs to develop more cases of the same lesson being taught in different circumstances. These multi-case lessons would become the *standards* of this field, in the same way that the song *Giant Steps* (Coltrane, 1960) became a standard in bebop

Learning songs and practice/jam sessions

In conjunction with careful listening, jazz musicians must learn the song for the purposes of playing. This type of learning is different from learning the song to identify characteristics of the song being played (key, tempo). Learning the song moves beyond the recognition of the song, in *Name-That-Tune* fashion [2]. Students dissect the song and learn all of its crucial elements including the melody, chord patterns and phrasing. The students then play the tune in the original key and in several other keys.

Students often play with play-along recordings that provide the necessary background, over which students can improvise. They also play with each other, as different students play at being the main improviser. In this way, students become both the background musicians, an art in its own right, and the improvisers. These jam sessions may be recorded for later critique, or more experienced players or observers may critique the performance immediately following it.

In the same way, the preservice or inservice teacher must learn the hypothetical learning trajectory for the purposes of teaching. This type of learning extends beyond knowledge of the mathematics in the teaching goal itself, to include learning about how students gain knowledge of the mathematics, the anticipated actions of a student with a particular task and the mathematical profitability of that task to achieve particular educational goals.

Preservice and inservice teachers then need to play, to enact the lesson in classrooms with students. This enactment can be in peer teaching opportunities in which preservice teachers pose as students. A possible strength of the preservice or inservice teachers posing as students is to have these teachers further justify their choice of action as a student in the situation. These justifications should be in terms of their anticipation of what students might do.

Preservice and inservice teachers also need to enact lessons with school-aged students as a means of continuing to learn about conceptually-oriented teaching. However, in order for teachers to learn about conceptually-oriented

teaching, these lessons need to be enacted with students who are prepared to participate in a conceptually-oriented lesson, an improvisation. Students who will not 'swing' with those prospective teachers, who have not negotiated compatible classroom norms (Yackel and Cobb, 1996), may not therefore be able to provide the right type of discourse opportunities with which to practice improvisation. Thus, opportunities to work in classrooms with children who are able to be full participants in a conceptually-oriented classroom for both preservice and inservice teachers should be a part of professional development.

Public critique and commentary

A key component of these practice sessions is the opening of one's practice to outside scrutiny and critique. Teaching is seen as a personal act and the reluctance of U.S. preservice and inservice teachers to criticize one another could lead to vacuous critique. On the other hand, learning to critique one's own and one another's work in sensitive ways is crucial to further growth and development as teachers (Mewborn, 1999)

Even the most accomplished jazz musicians' performances and recordings are subject to critique in the leading journals Musicians themselves, and observers of the recordings, provide liner notes with recordings. These liner notes provide insights into the performances and provide rationales and criticism for the way in which the performers played the music. These criticisms become ways in which future jazz musicians learn to critique their peers and themselves

In the same way, substantive critiques of teachers' practices and/or records of practice that can be made public and shared with teacher-learners could be valuable tools to begin to help teachers see examples of and learn how to provide constructive criticism to their peers. This work can be part of the cases that the field develops (e.g. Stein *et al.*, 2000; Wilcox and Lanier, 2000) to help enhance preservice and inservice teacher education.

Lifelong learning

Finally, a jazz musician is a lifelong learner. Musicians such as Miles Davis, John Coltrane and Dizzy Gillespie sought to improve constantly and developed new forms of jazz types: bebop, hard bop, free jazz, cool jazz. They continued to learn and experiment, working with younger musicians and incorporating their new ideas, throughout these greats' careers. In the same way, teacher education should not be seen as complete upon receiving certification. Instead, the teaching profession needs to develop a culture of continuous learning for practitioners. Today, many are using the Internet as an opportunity to continue their development as jazz musicians and the field of mathematics teacher education should examine the ways in which jazz musicians are using this tool to continue their development (http://www.JazzCorner.com).

Cross-cultural parallels

This metaphor provides an example of looking across to a different culture, that of jazz education and its key component of improvisation, to provide insights into mathematics teacher education. Another comparison that has been made is to the culture of teaching in Japan

Some suggestions derived from the metaphor are similar to aspects of the work of the Japanese 'lesson study', in which the participants spend a focused year working on a single lesson. They visit one another teaching the lesson, so that there are multiple visions of practice upon which to draw, to examine multiple examples of the performance by different players. The focus of the teachers' work is around a 'research lesson' or *kenkyuu jugyou* (Stigler and Hiebert, 1999).

The teachers spend a significant amount of time working on developing these research lessons, the hypothetical learning trajectories or songs, with which students will engage They then teach the lessons, with observations: they play with an audience That audience consists of peers who serve as both composers of the song and critics of the performance.

The observers examine the teaching critically, focusing on the implementation of the lesson, not the teacher. The lesson is revised and repeated before a larger audience of observers, many of whom are not composers. However, this new audience can and does serve as a critic of the instruction/performance and helps to provide insight. The teachers then create a final artifact, the song with liner notes or research lesson, for publication with local or national dissemination.

While the 'lesson study' focus is on inservice teachers, schools of education and departments of mathematics can begin to think critically about how to incorporate some of these aspects into preservice teacher education and university-based inservice experiences In conclusion, the metaphor of conceptually-oriented mathematics teaching as proficient jazz improvisation provides some clues to how the mathematics education community responsible for the preparation and continuing education of teachers might think about appropriate forms of curriculum and instruction. New types of experiences as well as rethinking how the community orchestrates traditional experiences are key in preparing and supporting conceptually-oriented mathematics teachers.

Notes

[1] While both mathematics and jazz have historically been male-dominated fields, I have chosen to discuss the mathematics teacher and jazz musician as female

[2] *Name that Tune* was a game show in the U.S. in the 1950s and again in the 1970s during which contestants would try to guess the name of a song, given some general characteristics and the first few notes of the song

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