Ethnomathematics and Education

MARCELO C. BORBA

In this paper I will discuss the notion of ethnomathematics, which can be seen as an epistemological approach to mathematics, and will relate ethnomathematics to education. This discussion will lead to a view of how mathematics should be incorporated into school curricula and to suggestions regarding how mathematics should be pedagogically practised.

1 A philosophical background to ethnomathematics

In this section I will summarize a view of human beings on which the idea of ethnomathematics is based, a view of how people relate to other human beings and to the world. I will then focus on two particularly important ideas: "dialogue" and "problem". Finally I will establish connections between the idea of ethnomathematics and this view of human beings.

1.1 Human beings and their dialogical relations

This view of humans is based on a phenomenological approach in which a person is seen as a "being-in-the-world-with-others". [1] S/he is a "be-ing" since her/his essence is manifested in her/his daily ways of existing in the world. S/he is "in-the-world", not in the sense of water "in" a glass, but in the sense of being in a relationship with the world which expands to fill a space without dimensions. This relationship expands further into the world as s/he comprehends new meanings about this relationship with the world. S/he is "with others" because s/he always works with something and/or talks with someone (even if only to her/himself).

In this phenomenological view of human beings, a human is only seen in connection with the world. S/he cannot be seen without the world; neither can the world be seen without her/him. Moreover, the concepts "human" and "world" themselves are intrinsically linked since both terms reflect meanings which have been constructed by humans. Each human's relation to other humans is based on certain comprehensions: understanding existing meanings and making new meanings. Each person is also always in a *place* in the world and living in a historical *moment*.

In her/his existence, a person experiences events in which s/he is also involved. These experiences can be seen as a "chain of consciousness" which is in continuous and indivisible flux, like a river, where thinking is both changeable and constantly flowing. However experience is lived in its own time, different from "official", chronological time. A reflection on an experience is no longer in the original flux of experience, but is in a new part of the ongoing flux, looking back at an earlier time. Thus consciousness is an endless, recurrent process which embodies, in a broad sense, reflecting, knowing, and thinking.

In the terminology used by Paulo Freire [1981], consciousness can be "intransitive" or "transitive". A person with intransitive consciousness doesn't link her/his experiences together; s/he always lives in the present moment and therefore cannot make important connections. S/he is likely to change only superficially, e.g. in response to fashions. A person with transitive consciousness develops a more reflective perspective which allows her/him to make connections between her/his different experiences and therefore to make significant changes in response to these experiences. Freire argues that reaching a "critical transitivity" is necessarily an active and dialogical (that is, in dialogue with other people) process. Therefore transitive consciousness and dialogue are both fundamental to the processes of personal and educational growth.

1.2 Dialogue

Dialogue can be seen as a horizontal relationship between two or more human beings, in which the "being" of each person opens her/himself to the other(s) in an authentic way. Dialogue is an intersubjective relationship in which human beings try to know each other and reveal their true selves to each other [Bicudo, 1979]. The subjects involved in the dialogue communicate using not only intentional signs (e.g., words), but also using unconscious signs such as pauses, ways of walking or breathing, gestures, and so on. In this context, the meaning of words cannot be limited to those stored in the dictionary. However just giving something a name shows the importance this thing has in a given culture. According to Alfred Schutz [in Wagner, 1979], words are bounded by past and future elements of someone's speech; words also have emotional and irrational values which are not explicit. Meanings of signs also change from one cultural [2] group to another, since each group "shapes" the meaning of words to their context. Finally, it is important to remark that dialogue cannot take place if the realms of concern of the human beings involved in the dialogue have no intersection. In other words, if the problems which involve them are completely different, the dialogue cannot occur.

The word "problem" has been used just above in a very different way than in most mathematics education literature. The next section will focus on the key idea of "problem".

1.3 Problem

What is a problem? [3] If I ask an adult who is standing in front of me, "What is the color of the pants I'm wearing?" is that a problem? In education it is important to distinguish a problem from a simple question to which the answer is known without any need for reflection. Another common misuse of the term problem is when it is associated with simply "not knowing". If I ask someone how many universities there are in the U.S.A., is it a problem for that person if s/he does not know the answer? This is probably not a problem for her/him because s/he isn't likely to care about the question. Whether or not the answer is already known, whether or not the answer can be easily obtained, if s/he doesn't care about the answer, it is not a problem for her/him. In this approach, what is of interest to someone is important to the idea of "problem". If an obstacle occurs in the course of someone's own existence *and* if s/he does not know how to overcome the obstacle, then s/he has a problem.

A problem can be authentic or it can be imposed. An imposed obstacle or puzzle would be a pseudo-problem, a situation which occurs frequently in mathematics teaching. Students are usually asked to solve problems which are not problems for them personally; they only attempt to solve these pseudo-problems in order to get a good grade.

Although the discussion so far may have implied that the definition of problem I have been developing is too subjective, Demerval Saviani [1985] is very clear when he argues that:

> A problem, as any other aspect of human experience, has a subjective side and an objective one, closely connected by a dialectical unity. ... The concept of problem implies a consciousness of a situation of necessity (subjective aspect) and a situation that puzzles his consciousness (objective aspect). [Saviani, 1985, p. 21, author's translation]

The objective and subjective aspects of the definition of problem are both culturally bounded since what is interesting for someone, the aspect of subjectivity, depends partially on the cultural traditions of that person. Obstacles (the objective aspect) are also culturally bounded, because what is an obstacle in a given culture might not be one in another culture.

A problem then can be seen as a situation which involves an impasse in the flow of life and which is important to that person's existence. When a problem results in a mathematical treatment, it can lead to the generation of mathematics by the person(s) who was (were) puzzled by this situation.

2 Ethnomathematics [4]

In the last section it was seen that a person is a cognizant being who functions within the language and interpretative codes of her or his socio-cultural group. A language is a code understandable only to people who have participated in common past experiences. Each language expresses a way of knowing developed by a group of human beings.

One way of knowing is mathematics. Mathematical knowledge expressed in the language code of a given socio-cultural group is called "ethnomathematics". In this context, "ethno" and "mathematics" should be taken in a broad sense. "Ethno" should be understood as referring to cultural groups, and not to the anachronistic concept of race; "mathematics" should be seen as a set of activities such as ciphering, measuring, classifying, ordering, inferring and modelling. As defined by Ubiratan D'Ambrosio [1985]: "ethnomathematics is the mathematics practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on." [D'Ambrosio, 1985, p 45] Even the mathematics produced by professional mathematicians can be seen as a form of ethnomathematics because it was produced by an identifiable cultural group and because it is not the only mathematics that has been produced.

This view of professional mathematics is consistent with George Joseph's statement that, because of the Eurocentric bias of most academicians, there is a "misrepresentation of the history and cultures of societies outside the European tradition". However, Joseph's statement that "mathematics can be looked at as an international language, with a particular kind of logical structure" is not consistent with an ethnomathematical view of professional mathematics. [Joseph, 1987, p 14] While Joseph recognizes that each mathematics has a particular kind of logical structure, he says that mathematics is international. In doing so, Joseph is assuming that mathematics is independent of culture, rather than being an historical construction which is socially and culturally bounded since the way it is organized and the way it is expressed represent the codes and understandings of professional mathematicians who are themselves culturally bounded. Therefore "academic mathematics" is not universal (in the sense of being independent of culture) any more than "Quipu mathematics" is, or "carpenter mathematics", or "Shantytown mathematics", [5] and so on, nor is it international in the way Esperanto was intended to become a language common to all people. Although academic mathematics may be international in that it is currently in use in many parts of the world, it is not international in that only a small percentage of the population of the world is likely to use academic mathematics.

However mathematics can be considered universal in the way that Alan Bishop uses the term. Based on his analysis of different cultures, Bishop argues that activities such as counting, locating, measuring, designing, playing and explaining "... are both universal, in that they appear to be carried out by every cultural group ever studied, and also necessary and sufficient for the development of mathematical knowledge". [Bishop, 1988, p 182] Bishop also believes that "... mathematics has a cultural history, but also that from different cultural histories have come what can only be described as different mathematics". [1988, p 180]

Even though Bishop does not use the terminology ethnomathematics, his view comes towards the approach developed in this paper in arguing that every culture does mathematics although the mathematics is expressed in ways unique to that culture. Thus "ethnomathematics can be seen as a field of knowledge intrinsically linked to a cultural group and to its interest, being in this way tightly linked to its reality [6] and being expressed by a language, usually different from the ones used by mathematics seen as science. This language is umbilically connected to its culture, to its ethnos." [Borba, 1987, p 38]

2.1 Efficiency of ethnomathematics

The ethnomathematics developed by different groups are likely to be more efficient at solving problems related to their cultures than academic mathematics is (unless, perhaps, the problem is in a school context) because the ethnomathematics developed by a given cultural group is linked to the obstacles which have emerged in this group An obstacle and the need to overcome it draws people's attention to a situation which can be described as a problem, as discussed in this paper. When the solution of this problem involves a mathematical treatment, the solution contributes to the development of ethnomathematics in this culture Over time, this ethnomathematics is probably going to be more efficient than the models stored in textbooks and written in codes not always accessible to a given cultural group, because it is connected to the culture where the problem was generated Hence ethnomathematics should not be misunderstood as "vulgar" or "second class" mathematics, but as *different* cultural expressions of mathematical ideas

3 Ethnomathematics, education and ideology

The notion of ethnomathematics has clear implications for education. If different people produce different kinds of mathematics, then it is not possible to think about education as being a uniform process to be developed in the same way for different groups Instead mathematics education should be thought of as a process in which the starting point would be the ethnomathematics of a given group and the goal would be for the student to develop a multi-cultural approach to mathematics

For educators to develop an educational approach based on ethnomathematics, it is important to consider the concept of problem discussed above. Problems could be found and developed which were based in ethnomathematics, thus avoiding the use of pseudo-problems Students should actively participate in the design of their pedagogical program, as proposed by Freire: "The content of an education for critical consciousness must be developed by searching with the students for experiences which give meaning to their lives." [Freire, 1970, p 28] Therefore problems to be solved would be chosen by both students and teachers in a dialogical relationship which fosters a critical consciousness (as discussed in Section 1.1) Knowledge can be seen as a product of this dialogical relationship. Each partner is going to be learning from the other in a dialectical way

Mechanical views of dialogical educational processes should be avoided; one should not expect eleven-year-old boys or girls to develop a sophisticated comprehension of the contradictions of the political-economic system. Children do develop a consciousness of relationships in their world out of their reflections on the ways they play: on the rules of a game, on the friendships among the partners of this game, and even on the mathematical relationships of this game

A pedagogy with students as partners with the teachers doesn't mean that the educational process is value-free The incorporation of socio-cultural aspects in mathematics education and the dialogical way of doing it each have a role to play A dialogue where the teacher speaks through her/his ethnomathematics (usually developed in college) and students speak with theirs, is not neutral. Such a dialogue can allow students to strengthen their socio-cultural roots, since their (ethno) knowledge is legitimized (recognized as valuable) in the educational process This pedagogy can also emphasize that mathematics is not a single, unique expression and cannot be seen as a "straight line". A forest might be a better image of the whole set of ethnomathematics, in which each tree would be considered as a different expression of ethnomathematics, socio-culturally produced.

Dialogue, which should be seen as a horizontal rather than vertical or hierarchical relationship, doesn't mean that the role played by the teacher is the same as the one played by the students An equal relationship doesn't mean a uniform one. The teacher is different from the student because, among other reasons, s/he has an explicit intention of educating. S/he has worked and studied towards various goals as an educator, one of which may be developing a democratic educational relationship between teacher and students which can facilitate the students' development of a critical consciousness. In order to foster this development, such a teacher believes s/he has to share power with the students in the educational process.

3.1 Ethnomathematics and education: are they really compatible?

The accepted mathematics in this educational proposal ranges from ones developed by students to the one accepted/developed/intended by the teacher In the classroom dialogue, the teacher can learn from the ethnomathematics "spoken" by the students, just as the students are learning from the academic ethnomathematics of the teacher. This dialogical process has no dichotomy between education and research, between teacher and researcher The one who educates is also the one who researches the ethnomathematics developed by students Therefore research influences educational praxis, and vice-versa

The ethnomathematics of a cultural group is part of the group's life; the mathematics is generated by the culture in an "umbilical" way. Ethnomathematics is developed by the cultural group's interest in its problematic situations, which then further develop the group's interest in its ethnomathematics. This interest in ethnomathematics is natural because it was generated by the members of the cultural group in response to their own situations However this awakened interest in ethnomathematics does not automatically transfer to an interest in learning/developing any other ethnomathematics, such as academic mathematics. Students may not have much interest in investigating deeply the concepts which underlie their ethnomathematics. If the teacher forces students to work on problems, even problems based on the underlying ideas of the students' own ethnomathematics, they will be pseudo-problems, just as so often happens in regular schools with academic mathematics.

This argument could lead us to a belief that there is no way out of the dilemma of the use of pseudo-problems in the classroom However the previously discussed idea of dialogue offers a potential solution, since dialogue in its authentic form implies a mutual speaking and hearing. Hopefully the people involved in a dialogue can find convergent points and intersections in their realms of meaning. The teacher/researcher has a particular ability and responsibility to help the students find the intersections between their realms of meaning and the teacher's.

4 Final comments: ethnomathematics in current school situations

Using this same framework, educators such as Borba, Frankenstein, Gerdes and Skovmose [7] have been developing pedagogical proposals along the lines supported by this paper However most of these pedagogies have been applied, with encouraging results, in "non-formal" schools and in adult education. Thus the question still remains whether this kind of proposal makes sense in current formal school situations Although there is still a long way to go in developing such a pedagogy for formal classrooms, it can be argued that such a framework can be tried in school situations and initial answers can be developed

The ideas developed in this paper indicate that curricula cannot easily be changed by simply substituting some content for others It is necessary to consider more fundamental kinds of change. In traditional curricula the use of pseudo-problems is unavoidable since students do not participate in choosing the themes which are going to be developed by the teacher during the school year.

"Thematization" and "project organization", to use Skovsmose's terminology [1985], are ways which many authors have found of both breaking the atomization of traditional curricula and building a new view of mathematics. In this approach the themes and/or the projects to be developed are decided by both students and teachers. The themes are not necessarily "mathematical", or "biological", or "artistic"; themes developed jointly with students will probably not closely match the academic disciplines. They are just researches to be undertaken by the group, where the role played by teachers is to help the students develop a critical view of the world, a "transitive consciousness" in Freire's words [1981]

In this educational proposal, the ethnomathematics, the ethnobiology, ethnochemistry, etc., practiced by different groups of students would be the starting point of the pedagogical process. This "ethnoknowledge" developed by groups of students should be compared with the (ethno) knowledge developed by the academic disciplines in a way that this academic knowledge can be also seen as culturally-bounded. The students and the teachers should discuss the efficiency and relevance of different kinds of knowledge in different contexts With this approach, mystification about science might be avoided and mathematics might no longer be seen as an oppressive and all-powerful realm of knowledge.

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Notes

[1] This view of man is based on Heidegger [1981], Schutz [in Wagner, 1979] and Borba [1987]

[2] Culture should be understood in this paper as what was added to the world by humans (as the result of human work, of human's creative and recreative struggles) and as meanings which are shared by a cultural group

[3] This view of problem is based on Saviani [1985] and Borba [1987].

[4] A discussion of the reason for using the term "ethnomathematics" instead of "oral", "informal", "non-standard", "cultural", "natural", or "every-day". etc, mathematics can be found in Borba [1988a] and Borba [1988b].

[5] I'm referring here to the papers written by Ascher [1986], Schliemann [1986], and Borba [1987b], respectively about the Quipu about carpenters, and about a Shantytown

[6] Reality should be understood in this paper as the human dimension of the world; where the natural components of the environment and those elaborated by humans are present. then reality doesn't end in empirical data

[7] For instance, see Borba [1987a 1987b], Frankenstein [1983], Gerdes [1985] and Skovsmose [1985]

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Nascent quantification — a simple urge to measure and count — appeared in Western culture in a variety of contexts and situations in the sixteenth and seventeenth centuries. On one level its appearance correlates with important changes in the political, economic, and religious life of those turbulent centuries and to well-known historical processes such as the growth of centralized government and the expansion of overseas trade. On a deeper level, quantification can be considered as a peculiar mental activity, one that satisfies a need for precision and finitude, and its appearance in a period of numerous crises suggests that more than practical needs for rationality and knowledge were being met.

On the practical level the distinguishing features of political life in the sixteenth and seventeenth centuries were the rise of powerful monarchies, the geographical consolidation of nations and the development of new concepts of public administration which, in the seventeenth century, entailed mercantilism and the government's claimed right to regulate economic activity. In theory this created a justification for evaluating national resources, including the population and the volume of trade, and increasingly such evaluations were quantitative, in varying degrees of accuracy.

In the economic history of the period three separate strands of development contributed to an elevation of the public consciousness of numbers. The rise of capitalism and of national economies drew greater numbers of people into the world of monetary exchange, where some elementary calculation and bookkeeping were necessary skills. Overseas trading and adventuring stimulated a mathematical approach to navigation and introduced ordinary seamen to the wonders of arithmetic. Finally, the great price rises of the sixteenth century and the concomitant population expansion loosed some men from their roots, setting them adrift in English society, and startled other men, like Bodin, into quantitative inquiries in an effort to create order.

Patricia Cline Cohen
