IMPLICATIONS OF PEDAGOGICAL CONTEXT FOR ELICITING PEDAGOGICAL CONTENT KNOWLEDGE

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Since Shulman (1986) introduced the notion of pedagogical content knowledge (PCK), scholars have identified a specialized *mathematical* knowledge for teaching (MKT) that brings together purposes and reasoning from *both* mathematics and teaching. One approach advocated for teacher education is to develop MKT through tasks that situate teachers in the work of mathematics teaching, but it has proven difficult to design tasks to elicit particular aspects of MKT. We highlight in this article a simple, yet fundamental feature of MKT tasks that has received little explicit attention in previous analyses of MKT tasks and their enactment: *to what extent pedagogical warrants are necessitated by the pedagogical context.*

A *mathematical* warrant is based on mathematical values and concepts, such as precision or a particular theorem. A *pedagogical* warrant is based on knowledge about teaching, such as of particular teaching practices or common student ways of thinking. Our central argument is that it is productive to examine the question of the extent to which pedagogical warrants are necessitated by the pedagogical context and that this examination can be used to identify the potential of an MKT task for eliciting pedagogical content knowledge, as well as specialized content knowledge (SCK; Schilling, 2007).

We take SCK to refer to the mathematics of analyzing and elaborating underlying ideas of a specific domain. SCK tasks are important for highlighting how teaching mathematics can differ from learning to do mathematics for oneself; teaching often requires unpacking ideas where learning mathematics requires compressing ideas (Ball & Bass, 2003). Like Krauss *et al.* (2008), we take PCK to refer to "knowledge about teaching in a specific domain", which "includes how best to represent and formulate the subject" and "knowledge on students' subject-specific conceptions" (p. 717), and which may be culturally dependent (Depaepe, Verschaffel & Kelchtermans, 2013). The complex challenge of designing and enacting MKT tasks raises the question of how and when tasks provide opportunities for developing PCK and SCK.

To develop this argument, we use representative cases of interviews of teachers and mathematicians to analyze how the pedagogical context of the tasks shaped opportunities for eliciting SCK or PCK in tasks intended to develop MKT.

Mathematical and pedagogical contexts in MKT tasks

As a first step to unpacking how teaching scenarios described in MKT tasks may elicit different aspects of knowledge, we characterize mathematical and pedagogical contexts and purposes, as in Stylianides and Stylianides (2010)[1]. We exemplify these ideas with the tasks in Figure 1.

A task's *mathematical context* consists of the mathematical ideas and problems described explicitly in a task's text. In the first task, this context is ordering decimal numbers in general as well as ordering the specific decimals in the answer options. In the second task, this context is multiplying two-digit numbers, generally, as well as the specific products shown. The *mathematical purpose* of a mathematical context is the mathematics problem that underlies the task. In the second task, the mathematical purpose is to determine whether methods for multiplying two-digit numbers generalize. In the first task, the mathematical purpose is not as clear immediately, as ordering decimals for the purpose of evaluating student understanding fundamentally

Suppose you wanted to find out if your students could put decimal numbers in order. Which of the following lists of numbers would give you best evidence of students' understanding?

(a)	.5	7	.01	11.4
(b)	.60	2.53	3.14	.45
(c)	.6	4.25	.565	2.5

Ball & Bass (2003)

Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways. Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

(a)	(b)	(c)
35	35	35
× 25	× 25	× 25
125	175	25
+75	+700	150
875	875	100
		+ 600
		875

Hill, Schilling & Ball (2004)

Figure 1. MKT Tasks

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relies on assumptions about students' conceptions of ordering decimals. However, given a particular student conception the mathematical purpose follows. For instance, students may think it is possible to order decimals by ordering the natural numbers that result when the decimals points are removed. Then the mathematical purpose is to determine whether a set of decimals would have a different order as compared to the numbers with the decimal points removed.

A task's *pedagogical context* consists of the contextual elements of teaching practice contained in the task text or solutions (*e.g.,* student talk, curriculum materials). The mathematical and pedagogical contexts may overlap, as in the cases of problems that students in the scenario may solve or have solved, such as in the tasks in Figure 1. In the first task, the pedagogical context includes the three potential problems to be assigned to students, and the *pedagogical purpose* is to select a problem to evaluate student knowledge of comparing decimals. In the second task on multiplication, the pedagogical context includes the three pieces of sample student work, that the focus of work with students is multiplying large numbers, and that the purpose is to evaluate whether students' methods can generalize. Executing this purpose involves examining problems potentially given to students or examining work that students produced. The pedagogical purpose of a pedagogical context is the teacher's purpose in the teaching scenario, such as selecting a problem for evaluation or determining whether particular students' methods generalize.

The mathematical and pedagogical purpose and contexts shape the reasoning of a person doing the task. The mathematical context is necessary to the reasoning; it suggests which concepts are needed. The mathematical purpose describes the task to be done; it is the purpose to which concepts are applied. To solve either of the above tasks, one must do the mathematics problems posed, have a sense of the general mathematical idea cited and enact the purpose. This is impossible without understanding how decimal ordering and multi-digit arithmetic are defined.

The necessity of pedagogical purpose and context for reasoning is less clear. Pedagogical purpose and context may prime a person doing the task to identify as a teacher, thus prompting the person to draw on pedagogical knowledge s/he has available. But such a role is indirect, and may not actually be necessary for responding to a MKT task, such as the multiplication task. We now turn to scholars' previous analyses of pedagogical context in MKT tasks. We use these previous studies to make the case for attending to the extent to which pedagogical context necessitates pedagogical warrants.

Functions of pedagogical purpose and context

It is clear from previous analyses of uses of MKT tasks that pedagogical purpose and context can be, but are not always, critical for engaging teachers in pedagogical and mathematical inquiry. Towers and Martin (2009) adapted the decimal ordering task to ask groups of prospective teachers to "create their own list of four numbers that would assess student understanding of ordering decimals" (p. 45). After several possibilities were raised, the teachers converged toward one possibility based on identifying a variety of decimal concepts and debating how well proposed decimals would evaluate students' understanding. Collective discussion on this task, especially the pedagogical purpose of selecting student tasks, engaged teachers in improvisational and pedagogical practices that were "profitable for fostering student understanding" (p. 47). Stylianides and Stylianides (2010) described conversations between prospective teachers about two MKT tasks that described teaching situations. They suggested that the importance of pedagogical context in these tasks was engaging teachers in mathematical inquiry that could have been otherwise hard to motivate in a purely mathematical setting, but that is key to skillful teaching. From these authors' work we can conclude that pedagogical purpose and context offer opportunities to draw out teachers' MKT because it provides a mathematical setting that motivates teachers through engaging in teaching.

This conclusion highlights the potential for pedagogical context to impact the knowledge demands of an MKT task, but they leave open the identification of features of pedagogical context that were influential in this way. Indeed, pedagogical context can also function as a surface decoration with no real implications for the knowledge demanded. One of the first analyses of tasks assessing teachers' knowledge showed that historically, pedagogical context was merely "window dressing"—deleting it would make no difference to reasoning on the task (Hill, Sleep, Lewis & Ball, 2007). As an example, Hill *et al.* presented a task in which prospective primary teachers were shown a 5-by-8 rectangle and asked, "To find the perimeter of the rectangle above, a child can add 5 and 8 and multiply by 2 or multiply each number by 2 and then add. This example illustrates which of the following properties?" The answer options consisted of various arithmetic properties including the intended response, the "distributive property". One can solve this task knowing arithmetic properties; "the school context makes no further knowledge demands for determining the correct answer to the task" (p. 119).

Subsequent analyses of assessment tasks have probed into how exactly pedagogical context can change the knowledge demands. Lai, Jacobson, and Thames (2013) argued that pedagogical context may create authentic and nuanced complexity, such as negotiating competing purposes that are each pedagogically valuable but may be in tension with one another. The ordering decimal example in Figure 1 exemplifies this phenomenon. The primary pedagogical purpose in the ordering decimal example is evaluating student understanding. However, other pedagogical purposes arise in considering the answer options, such as providing students with the opportunity to interpret decimals with a zero digit or comparing whole numbers with other decimals. Only option (c) directly addresses the primary purpose of evaluating student understanding. With the other options, a student could order the decimals correctly by ordering the numbers obtained by removing decimal points. This critical inference has a pedagogical warrant, thus responses to this task necessitate the use of pedagogical context. As Phelps and Howell (2016) argued, one potential function of pedagogical context is to provide warrants for the inferences necessary to respond to the task.

Building on the above arguments, we propose that a defining characteristic of pedagogical context is *the extent to which it warrants inferences that are necessary for solving* *the task.* When pedagogical context warrants such inferences, the reasoning for a task must draw on specific pedagogical information, and hence such a task elicits not just mathematical knowledge for teaching, but PCK in particular. An example of a task with this characteristic is the decimal ordering task in Figure 1. In contrast, pedagogical warrants are beneficial but not necessary for the multiplication task in Figure 1, thus it does not have this characteristic. The pedagogical context may elicit helpful and culturally dependent pedagogical considerations, such as known student or curricular approaches to multi-digit multiplication, but the problem could be solved without these considerations.

Necessity of inferences warranted by pedagogical contexts

Our argument—that this defining characteristic can be used to identify the potential of an MKT task for eliciting PCK or SCK—is based on analysis of interviews of 60 experienced teachers and mathematicians in the US in which they were asked to think aloud.

Across the responses, when pedagogical warrants were necessary, *the responses either elicited knowledge about students and pedagogical practices or motivated such knowledge.* We suggest that these tasks present an opportunity to develop and examine PCK. When pedagogical warrants could be beneficial but were not necessitated by the context, *the responses exhibited SCK and PCK paths to the same conclusion.* We claim that these tasks also present an opportunity to develop links between SCK and PCK.

To illustrate, we use responses to two tasks, shown in Figure 2, called Selecting Examples and Evaluating Explanations. We chose these tasks for several reasons: they contrast in whether they necessitate pedagogical warrants; the tasks are among the shorter of interview tasks used; and task responses featured fewer response paths, so we can provide more complete descriptions.

Eliciting and motivating the need for pedagogical knowledge

The Selecting Examples task is similar to the Decimal Ordering task in Figure 1, in that selecting an example must be warranted by pedagogical knowledge.

Responses to the Selecting Examples task were typified by (1) a hedging choice of option (e) and (2) reasoning toward the intended (d). Key differences between responses similar to (1) or (2) came less from background and more from teaching experience. We discuss the solutions of a participant mathematician who chose '(b) and (e)' and two participants, a mathematician and a teacher, who each selected the intended '(d)'. Both the latter participants had taught the concepts of primes and composites with the mathematician's experience coming from a university course on "Appreciating Mathematics".

Throughout his response the mathematician who selected '(b) and (e)' stressed knowledge of students—with a sometimes self-conscious acknowledgement of lack of such knowledge. To eliminate (c), he said, "They are all squares, so this is a special case, and this would indeed confuse students, or at *least potentially confuse student*" (Here, and in the quotes that follow, the emphasis is ours). To eliminate (d) and (a), he hedged,

> (d) has 3^2, $2^3 \cdot 3$, $3^2 \cdot 5$. Well, each number is therefore divisible by a prime power, which is special, but *perhaps that's too technical for a student* just beginning to appreciate these ideas. *[…]* (a) has powers of 2, as in 8 and 32. But *would a student really appreciate that? Perhaps not.*

Selecting Examples Item

While planning an introductory lesson on primes and composites, Mr. Rubenstein is considering what numbers to use as initial examples. He is concerned because he knows that choosing poor examples can mislead students about these important ideas. Of the choices below, which set of numbers would be <u>best</u> for introducing primes and composites?

	Primes	Composites
(a)	2, 5, 17	8, 14, 32
(b)	3, 5, 11	6, 30, 44
(c)	3, 7, 11	4, 16, 25
(d)	2, 7, 13	9, 24, 40

(e) All of these would work equally well to introduce prime and composite numbers

Evaluating Explanation Item

Which of the following is the best explanation for why the conventional long division algorithm works, as in the example?

```
      111 R 29
37)4136
   37
    43
    37
     66
     37
     29
```

(a) It works because you divide 37 into parts of 4136 (the dividend) to make the problem easier to solve.

(b) It works because you subtract multiples of powers of ten times 37 (the divisor) from 4136 (the dividend) until you have less than 37 left.

(c) It works because if you multiply 111 (the quotient) by 37 (the divisor), and add in 29, you get 4136 (the dividend).

(d) It works because you subtract 37's (the divisor) from 4136 (the dividend) until you have less than 37 left.

Figure 2. Tasks whose pedagogical context function differently

Finally, he reasoned:

> Anyway, I'll say that (b) wins by just a whisper. But *I may be relying on what I see about them.* And *a student, who is learning these things for the very first time, may not appreciate* the level of subtlety that I'm seeing in them. And *if that's really the case,* then in that case, maybe (e) is the best case, because the primes are primes and the composites are composites, so all four serve to provide examples of primes and composites.

Potential cultural knowledge of students and whether they may be misled figured prominently in his reasoning. While his reasoning may not accurately reflect the thinking of many students, it does use or at least attempt to use envisioned student thinking. But after many uncertain statements, he fell back on the mathematics, reasoning that (e) may be the best response because each set labeled 'primes' or 'composites' indeed contained only prime or composite numbers, respectively. Had this participant had more pedagogical content knowledge about teaching primes and composites, he would perhaps have reasoned toward (d).

In contrast, the responses that led to (d) often confidently referenced teaching and students. To explain why the number 2 should be included in a list, the mathematician participant said:

> Now in an introduction to primes, I think it is very important to include 2 as an example of a prime. Because 2 is the 'oddest prime'. It's even. To be honest, because people forget.

Even when expressing initial ambivalence, typically between (b) and (d), participants clearly identified knowledge of teaching and learners. The mathematician participant explained:

> *Participant* The flaw in (b) is that you don't have anybody that's just a power of a prime, a pure power. In your definition of composites, you have a product of 2 primes or more.
>
> *Interviewer* Are you worried that students will *[pause]*
>
> *Participant* That they might not see that composites don't include powers of primes. *[…]* I like (d) better.

The teacher participant explained:

> (b) I like having one of the factors higher up there, higher than 7 or something *[…]* (d), I kind of like letting 2 be in the running, because it's the first prime. *[…]* So I'm leaning toward (d) *[…]* (d) does have a square number. So I'll give up my factor of 11 in (b) for this one important case where there is just this *one* prime factor of a composite number. Otherwise you have to add that on later, and that's trouble. *[emphasis by the participant]*

As Hewitt (2001) argued, the teaching of new terminology entails helping students link a name to current and future mathematical activity and developing awareness of key properties and relationships. The pedagogical context—that the teacher is introducing primes and composites—warrants the inference to include 2 as a prime and the square of a prime. Students may 'forget' that 2 is a prime because it is unusual as a prime; the reason to include a pure power of a prime is that otherwise, learner may 'not see' that composites can be powers of a prime, and in teaching, one may "have to add that on later, and that's trouble".

The potential for eliciting SCK and PCK solution paths

Evaluating Explanations differs from Selecting Examples in that the pedagogical context is beneficial but not necessary. We suggest that as a consequence, the intended response can be based on pedagogical *or* mathematical knowledge. To illustrate, we use the two main paths for concluding that (b) is the best explanation among those given. One path relies more on mathematics; the other relies more on knowledge of teaching.

The first path uses knowledge of the definition of the long division algorithm, and it concludes the best explanation among those given is (b), because (b) addresses the specifics of the long division algorithm. Options (a) and (d) hint at the fact that long division decomposes the dividend but does not say how the decomposition used in long division is distinctive from other ways of finding a quotient through repeated subtraction; option (c) verifies why the output of the algorithm is the quotient, but does not explain the algorithm itself. For instance, a participant who used this path explained:

> (c) is really the steps you go through to check that it's correct, but it doesn't really explain why it works. A monkey could have pulled 111 out of a hat, and 29 out of another hat, and (c) checks that it's correct. *[…]* I think (b) and (c) are both reasonable interpretations. *[…]* Yeah, I'll choose (b). They're both reasonable. (b) is at least directed at the actual question. (c) has the virtue of being accurate. But it's not really directed at the question being asked. It's simply what you do to check that you've done it correctly, which is a slightly different question.

Here, (c) does not work as well as (b) because it "answers a slightly different question." This is a mathematical warrant in the sense that (c) responds to a different mathematical question (whether one has "done it correctly"). The warrants against (a) and (d) are also mathematical in that they point out imprecision in language; precision is a mathematical value.

The second path to (b) is more sensitive to students—while also bringing in mathematics. For instance:

> *Participant* I mean, that's *true [pointing to (d)],* but *that's* why it works *[pointing to (b)] […]* Because, when you ask kids, are you really putting 37 into 41? How many 37's go into 41-hundred, they need to understand that *[…]* in lining things up, the reason you have to put the 1 over 41 is because it's not a 1, it's a 100. So kids

initially have no clue why they're lining up the numbers the way they are, until you point out that it's really 4000, and this is really 3700. So it helps them in lining up the numbers, or at least understanding why they have to line them up the way they do.

Interviewer Okay, and the others that are here, are there any other that you thought did explain, but not as well, or *[…]*

Participant Right. Well, (c).

Interviewer So why did you choose (b) over (c)?

Participant Because I think place value is a better way of explaining why long division works, than just *[…]* because when you explain *[…]* when (c) is your explanation, they believe that all they can do is guess and check. *[…]* I still have kids who still do this *[…]* so it's a guess and check method, it's not a good *[…]* rationale.

The complementarity of these paths is noteworthy. In the pedagogical path, the reason that (b) is the best response is mathematical (place value), but it also attends to consequences for students' learning ("guess and check, it's not a good *[…]* rationale") and what students are likely to be confused about ("lining things up"). The mathematical path looks strictly at the whether the explanation is directed at "the actual question" and whether the explanation is precise. Each step of the mathematical path has a pedagogical consequence for what and how students learn, and observations of students' thinking can be interpreted in terms of mathematics.

PCK and SCK implications of pedagogical context in teacher education

When MKT tasks with pedagogical contexts that require pedagogical warrants are used in teacher education, they represent special opportunities to provoke "intellectual need" for PCK. These tasks present situations that are unsolvable without knowledge of pedagogical practices and thus motivate the search for new knowledge in ways that connect to personal need and prior knowledge (*cf.* Harel, 1998). When MKT tasks elicit MKT without requiring pedagogical warrants, they can highlight how SCK and PCK are complementary facets of MKT. In teacher education that makes use of these tasks, reasoning about the situation can be mathematical or pedagogical. Such tasks are therefore an analog of "open-middled" mathematics tasks (Bush & Greer, 1999) that invite multiple solution strategies toward similar conclusions. Comparing solutions can demonstrate how mathematical and pedagogical reasoning, which appear different on the surface, have similar implications for teaching. Just as comparing multiple solutions to a mathematical problem can promote conceptual understanding, in the same way MKT tasks that can be solved using either purely mathematical- or pedagogically-warranted reasoning have potential to deepen understanding of the MKT domain.

The distinction between these functions of pedagogical context has significance for the design and use of MKT tasks for teacher education. MKT tasks are used in teacher education discussions to draw out and develop teachers' knowledge. When working on MKT tasks, whether intended to elicit pedagogical or specialized content knowledge, teachers' attention can emphasize pedagogical concerns at the expense of 'eclipsing' the intended specialized knowledge (Creager, Jacobson, & Aydeniz, 2016, p. 3; Suzuka *et al.,* 2009); and at other times teachers may attend to mathematical ideas in a way that sidelines pedagogical reasoning (*e.g.,* Schilling, 2007; Suzuka *et al.,* 2009). In enactments of these tasks, whether teachers' attention slides away from mathematics or away from pedagogy, the opportunity for using mathematics and pedagogy to inform each other is lost. Our work suggests that the necessity of the pedagogical warrants has consequences for the kind of MKT that has the potential to develop through discussions and ways to detect cultural variation in pedagogical knowledge.

The implication for teacher educators is that they should be alert to how task design influences the role of mathematical and pedagogical knowledge. If the goal is to develop pedagogical as opposed to specialized content knowledge, pedagogical contexts that require pedagogical warrants may serve the purpose better. Such pedagogical context can call attention to why knowledge of pedagogical practices is needed, as well as how pedagogical knowledge in particular may be culturally dependent; for example, how choice of example or interpretation of student thinking may depend on standards and curricula as well as norms for how these are enacted. This function suggests a potential conceptualization of 'pure' pedagogical content knowledge tasks as those with pedagogical context that necessitate pedagogical warrants; this conceptualization can support teacher educators in adapting and selecting tasks for use in teacher education.

On the other hand if the goal is to engage teachers in tasks with multiple reasoning paths that converge to the same conclusion, a profitable context may be one where pedagogical warrants are beneficial-but-not-necessary. In working on such an MKT task, some teachers' reasoning may leverage their personal pedagogical resources and others' reasoning offered will rely primarily on mathematics; this is especially likely if the teachers have mixed experience in teaching. The work on the task can be used to bring out more explicitly the elements of PCK and SCK used, as well as how they connect to each other. In so doing, a teacher educator may support teachers' development of pedagogical content knowledge and specialized content knowledge in ways that cohere to teachers' prior complementary knowledge.

In this analysis, we have focused on pedagogical context of MKT tasks, but there is good reason to believe that the mathematical context may play an equally important role in specifying the kind of reasoning required by an MKT task. Hewitt (1999) classified mathematics topics in the curriculum as *necessary* (deriving logically from mathematics students would have already learned) or *arbitrary* (students can learn only by being informed by an external source). He argued persuasively that good instruction would depend on which kind of topic was at stake. An insightful reviewer pointed out that the problems in Figure 2 differ in precisely this way: one deals with the arbitrary definition of primes and composites whereas the other involves the necessity of the division algorithm. The influence of differences in the mathematical context of an MKT task, including whether it is necessary or arbitrary in Hewitt's sense, likely influences the kind of reasoning involved in each task, but is beyond the scope of our analysis. We anticipate there is more to be understood about how MKT tasks engage teachers' reasoning, and exploring the mathematical context is an important next step.

Teaching features many instances where multiple resources can be used to reason through a situation. Ideally, teachers would be able to draw on a variety of forms of knowledge. And teacher educators would be able to engage teachers in drawing on multiple forms of knowledge and cultivating discourse where both pedagogical content knowledge and specialized content knowledge can develop. The pedagogical context of MKT items can play different roles in developing teachers' knowledge and teacher educators' knowledge. Understanding when pedagogical context necessitates pedagogical warrants has the potential to improve the use, nuance, and design of MKT tasks for teacher education, especially for the goals of developing and understanding PCK and SCK.

Note

[1] Stylianides and Stylianides (2010) discuss mathematical context, pedagogical context, and pedagogical purpose, but do not explicitly discuss mathematical purpose. We find it useful to analyze the context and purpose of both mathematics and pedagogy.

References

Ball, D.L. & Bass, H. (2003) Toward a practice-based theory of mathematical knowledge for teaching. In Davis, B. & Simmt, E. (Eds.) *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group,* pp. 3-14. Edmonton, AB: CMESG/GCEDM.

Bush, W.S. & Greer, A.S. (Eds.) (1999) *Mathematics Assessment: A Practical Handbook for Grades 9–12.* Reston, VA: National Council of Teachers of Mathematics.

Creager, M., Jacobson, E. & Aydeniz, F. (2016) Can pedagogical concerns eclipse mathematical knowledge? *For the Learning of Mathematics* **36**(2), 2-7.

Depaepe, F., Verschaffel, L. & Kelchtermans, G. (2013) Pedagogical content knowledge: a systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education* **34**, 12-25.

Harel, G. (1998) Two dual assertions: the first on learning and the second on teaching (or vice versa). *American Mathematical Monthly* **105**(6), 497-507.

Hewitt, D. (1999) Arbitrary and necessary: a way of viewing the mathematics curriculum. *For the Learning of Mathematics* **19**(3), 2-9.

Hewitt, D. (2001) Arbitrary and necessary: part 2 assisting memory. *For the Learning of Mathematics* **21**(1), 44-51.

Hill, H.C., Schilling, S.G. & Ball, D.L. (2004) Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal* **105**(1), 11-30.

Hill, H.C., Sleep, L., Lewis, J.M. & Ball, D.L. (2007) Assessing teachers' mathematical knowledge: what knowledge matters and what evidence counts? In Lester, F. (Ed.) *Handbook for Research On Mathematics Education* (2nd ed.), pp. 111-155. Charlotte, NC: Information Age Publishing.

Krauss, S., Brunner, M., Kunter, M., Baumert, J., Blum, W., Neubrand, M., Jordan, A. (2008) Pedagogical content knowledge and content knowledge of secondary mathematics teacher. *Journal of Educational Psychology* **100**(3), 716–725.

Lai, Y., Jacobson, E. & Thames, M. (2013) The role of pedagogical context in measures of specialized and pedagogical content knowledge. Paper presented at the 2013 annual meeting of the American Educational Research Association. Retrieved from the AERA Online Paper Repository.

Phelps, G. & Howell, H. (2016) Assessing mathematical knowledge for teaching: The role of teaching context. *The Mathematics Enthusiast* **13**(1-2), 52-70.

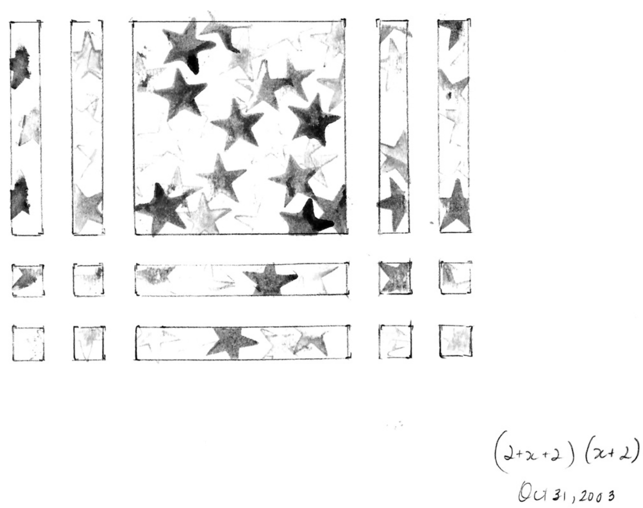
Schilling, S.G. (2007) The role of psychometric modeling in test validation: an application of multidimensional task response theory. *Measurement* **5**(2-3), 93-106.

Shulman, L.S. (1986) Those who understand: knowledge growth in teaching. *Educational Researcher* **15**(2), 4-14.

Stylianides, G.J. & Stylianides, A.J. (2010) Mathematics for teaching: a form of applied mathematics. *Teaching and Teacher Education* **26**(2), 161-172.

Suzuka, K., Sleep, L., Ball, D.L., Bass, H., Lewis, J.M. & Thames, M.H. (2009) Designing and using tasks to teach mathematical knowledge for teaching. In Mewborn, D.S. & Lee, H.S. (Eds.) *AMTE Monograph 6: Scholarly Practices and Inquiry in the Preparation of Mathematics Teachers,* pp. 7-23. San Diego, CA: Association of Mathematics Teacher Educators.

Towers, J. & Martin, L. (2009) The emergence of a "better" idea: preservice teachers' growing understanding of mathematics for teaching. *For the Learning of Mathematics* **29**(3), 44-48.



Polynomial Study #18, by Elaine Simmt of the University of Alberta.