- What curriculum can support students' future learning in partner disciplines?
- What methods help mathematics teachers respond to student work that incorporates ideas from a partner discipline?

Interdisciplinary curriculum is different from a mathematical application. It's different from mathematics in context. A mathematical perspective alone is not sufficient to design curriculum or evaluate student work, because the curriculum fulfills obligations to the partner discipline as well as to mathematics. Greater awareness of the scholarship of interdisciplinary pedagogy can make teaching across disciplinary boundaries manageable for a wider range of mathematics teachers.

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Intentional play-spaces for teaching|learning mathematics with young children

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Preschool pedagogy is now calling for educators to be intentional with regards to mathematics teaching and learning. In this communication, I provide an alternative conception of *intentionality* in which both children and adults act as teachers|learners within a play-space. I begin with an excerpt from an early childhood setting involving Martin, a 4-year old preschooler, and myself, as a teacher-researcher [1].

Learning and teaching: reciprocal responsibilities

During centre-time, I sat down at a table and poured out a bucket of coloured polygons. Martin (4 years, 10 months) sat down next to me, and Kiera and Taryn took the other two chairs. At first, we all independently began sifting through the shapes and putting them together. I looked at Martin's design and asked, "What have you got going there?"

"It's a traffic light. Now, I'm adding colours" (see Figure 1). I copied Martin's design so that I also had a red triangle, yellow hexagon, and green triangle traffic light. We agreed they looked the same.

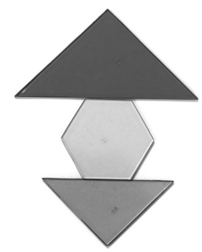

Figure 1. Martin's initial image of a traffic light.

I suggested we play a game where one person makes a design and the other person tries to copy it (see Figure 2). We each had a turn, but Martin was not particularly interested in playing. Instead, he continued putting two or three shapes together to see what they made. Martin then pushed two red trapezoids together: "This is a stop sign!"

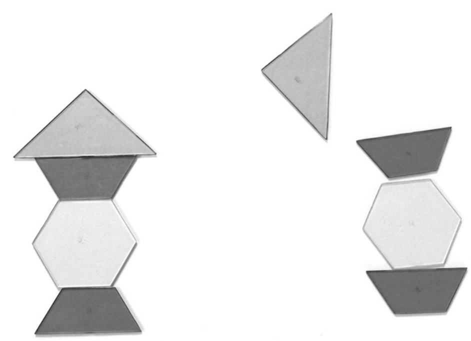

Figure 2. Game of copying designs.

"Hmmm … well … not quite. You've made a hexagon. Like this." I covered the two trapezoids with a yellow hexagon. "A hexagon has six sides. See ... one, two, three, four, five, six. A stop sign has eight sides" (see Figure 3).

"Where's a stop sign?"

"I don't think there is a stop sign here." We searched briefly through the polygons. I was well aware of Martin's fascination with traffic lights, stop signs, yield signs, and all other forms of traffic signs.

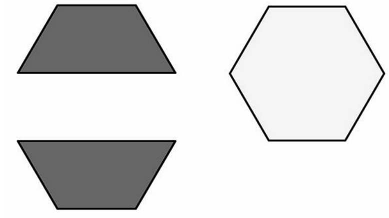

Figure 3. Two trapezoids congruent to a hexagon.

"We could try making one out of paper," I said tentatively. Martin nodded in agreement. I left the table and returned with scraps of construction paper, including several red squares, scissors, and a pencil. I looked at the square paper, not sure how to create an octagon in a way that was accessible to a 4-year old. Martin watched somewhat impatiently. After a few seconds, I cut off the corners creating a reasonably regular eight-sided figure.

"Here, Martin! This is an octagon ... a stop sign!" I handed a red square and scissors to him and said, "Here, you try one. You just need to cut off the corners."

Martin eagerly cut off the four corners and then looked at his work. "That's not a stop sign," he said with frustration (see Figure 4, left).

"Oh, I guess you have to be careful how you cut them." I marked two dots on each side of the paper equidistance from each vertex. "Here, try again. Cut dot to dot" gesturing the imagined lines on the red square.

Martin was successful on this attempt and smiled with the result (see Figure 4, right).

"Great job! That's terrific, Martin". We counted the eight sides together.

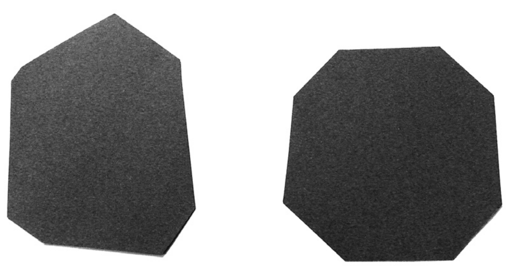

Figure 4. Martin's first attempt (left), followed by his second attempt (right) to make an octagon.

Martin took the pencil and a piece of white construction paper. Using his red stop sign as a model, he tried to draw one (see Figure 5, left). Once completed, he looked at his drawing in frustration, "That's no good."

"Oh gosh, Martin. Wow, you almost had it. Hmmm …. You could do it the same way as the square."

Independently, he sketched a square, placed dots on the sides, and drew the outline saying, "Connect the dots."

He did not like the top right corner, so I helped him redraw it and erased off the four corners leaving the octagon. He coloured it in with red felt pen (see Figure 5, right).

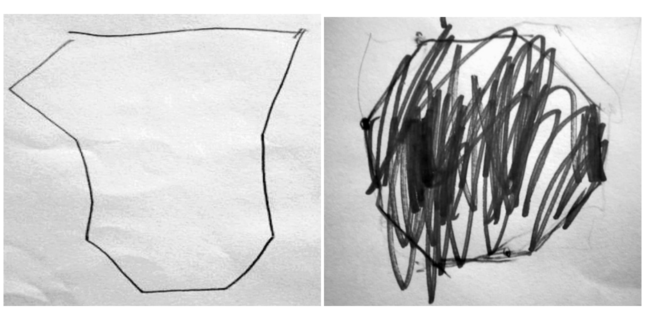

Figure 5. Martin's first attempt (left) and second attempt (right) at drawing an octagon.

Martin and I continued for several more minutes creating road signs with the polygons and drawing signs on paper (*e.g.*, yield, no parking, one-way sign).

What is intended by *intentionality*

The previous episode is not assumed to be ideal or even an exemplar of preschool pedagogy, but offers an entry-point into a conversation about intentionality. "Intentional teaching" has surfaced in the early childhood literature as a way to emphasize the teacher's role in purposefully planning for specific outcomes, key skills, and program goals while keeping the children's interests in mind. It attempts to strike a "balance" between child-initiated play and teacher-directed activities to minimize the long-standing tension between these pedagogical perspectives. Yet, the metaphor of balance assumes that child-initiated play and teacher-directed skill development exist on opposite sides of a metaphorical scale.

A definition of *intentional* means something "done on purpose, resulting from intention" [2]. The question remains: what is "it" that is done on purpose? In the early childhood literature, "intentional teaching" is one response to balancing child-centred and teacher-directed pedagogy in the preschool years. The purpose or intention is to help students achieve a pre-determined set of concepts, skills and goals. Yet, something that is intentional need not specify an outcome as a purpose. Intention is the effortful or *inten*se act of directing one's *attention*, stretching or ex*tend*ing outwards, and "being sensuously affected and solicited by the world through the medium of our living body" (Thompson, 2007, p. 30).

The brief episode with Martin is an example of intentional teaching, but what was intended was not narrowly prescribed as a set of "outcomes". The intention was to offer an experience of mathematical thinking and learning that was prepared for in advance and where "incoming" skills may be necessarily addressed within the experience.

Preparing the activity did not mean planning a prescribed agenda for it (Davis, Sumara & Luce-Kapler, 2008). To begin, the selection of the materials, in this case polygon shapes, was purposeful in that I assumed that forms of mathematical thinking and discussion would emerge by having that set of materials available. The materials themselves and the children's and teacher's interactions with them, did not guarantee that learning would occur (Davis, 2004; Kieren, 1995). The polygons created a play-space that was not perceptually bounded; that is, I did not and could not specify in advance the modes of action in which children would engage. However, the modes of action are not infinite, nor are they random. Based on my previous experience, I assumed that the mathematics that might be brought forth through our interactions could include topics in geometry, number, pattern, and measurement. In particular, a range of two-dimensional geometric topics such as labelling and comparing shapes, composition of two of more shapes to form new shapes, congruence of simple shapes with composition, symmetric designs, tessellations, and transformations (*e.g.*, translation, rotation, and reflection). The challenge for an educator is to be aware of the potential mathematical objects that could arise with and through objects, understand children's learning in relation to those objects, and also be attuned to new possibilities for acting in the space.

In the episode with Martin, we initially engaged in actions that were familiar (putting shapes together), although not predictable. Interestingly, our initial efforts—building a traffic light and playing a matching game—had directedness, but they did little to stretch us towards ideas beyond ourselves. However, through the interaction, Martin's interest in traffic signs continued to arise; first, with constructing a traffic light using coloured shapes (attending to colour) and next, to constructing a stop sign (attending to colour and shape). I hesitated in correcting Martin's shape identification. That is, his composition of two trapezoids formed a hexagon, not a stop sign (or an octagon). Many other children (and even adults) make a similar mistake. Perhaps it might be seen as unnecessary to correct an error of a four year old who is not expected to consistently count the sides of the shape, let alone properly label it. However, attending to Martin's labelling of the shape and his strong interest in road signs prompted a search for octagons. As we searched through the polygon shapes, I was actively thinking mathematically whether it was possible to create a standard octagon with the polygons that were available. When I could not think of a possibility, I suggested making stop signs on paper. The incidental act of making a distinction between a hexagon and an octagon triggered many new actions.

Although I had a square piece of paper in my hand, I had not until that moment imagined how to create an octagon from a square. A further constraint was to create an octagon that could be replicated by a four-year-old child. In fact, it prompted even further thinking and new action after the episode was over, as I wanted to consider approximately where the dots on the sides of a square should be placed if a regular octagon were to be drawn. I had to recall trigonometry objectives that had not been applied in a number of years, but it brought forth new awareness for me as I learned that the side length of a square needed to be segmented in a ratio of approximately 30:40:30 (see Figure 6).

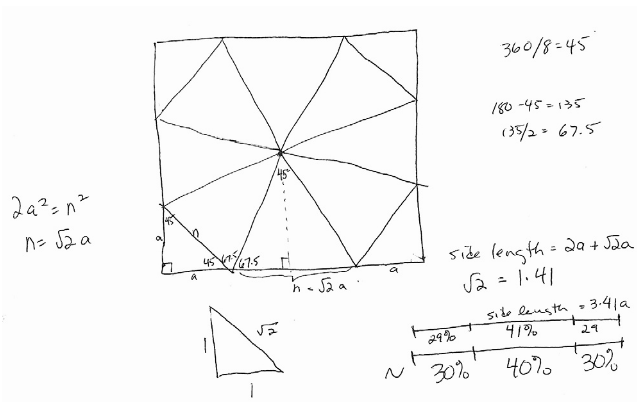

Figure 6. Teacher-researcher inquiry into ratio of side lengths for regular octagon.

Certainly, having a four year old learn to create and draw an octagon would never be an intended outcome in any preschool curriculum. Yet, the intentional object and act expanded both Martin's and my ways of acting mathematically. Martin made a distinction between hexagons and octagons, was able to make and draw octagons, and he independently engaged in repeated practice as octagons began appearing frequently for several weeks as his fascination with road signs continued. Similarly, reviewing trigonometry for a preschool educator's professional training is unlikely to be an intended objective; yet, the interaction with Martin became an occasion for me to utilize past experience and bring forth new mathematical awareness when I pull up to a stop sign and imagine it superimposed on an imaginary square.

This episode is an example of an intentional experience for both teachers and learners, not because it is an interaction with pre-specified mathematical content, but because it is an act of intention—an effortful action to direct our attention and stretch us towards something beyond our current ways of knowing and doing. Rather than solving a problem, we engaged in "skilful know-how" in which the inextricable interaction of Martin-Teacher-Environment "both poses the problems and specifies what actions need to be taken for their solution" (Thompson, 2007, p. 11). The outcomes of our intentional activities were not pre-specified, nor were they random events; instead, the experience expanded our ways of acting mathematically within a play-space of possibilities.

Concluding remarks

This communication offers an alternative to intentional teaching as a balancing act between child-centred exploration and teacher-directed practices. Intentionality within teaching and learning are effortful acts of directing attention to mathematical objects and ideas, and shifting patterns of engagement that extend teachers|learners ways of thinking and acting mathematically.

Notes

[1] Pseudonyms are used for the children's names. Dialogue was reconstructed with field notes recorded during and after the interaction. The focus on the dialogue with Martin is illustrative of multiple overlapping and independent interactions.

[2] From *OED Online,* March 2014.

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