The Social Construction of Meaning — a Significant Development for Mathematics Education?*

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I would like today to direct your attention to what I consider to be a significant new research area for us in mathematics education, and the best way I can do this is to explain not only what it is about, but also how I came to see its value. This talk will therefore be a kind of journey through some ideas and will, I hope, convey something of the flavour, and also the substance, of the new area.

However, in order to help you comprehend and evaluate what I have to say you should know that my own working context is in teacher education, at a University Department of Education, and you must also remember that it is within the U.K. system. One consequence is that I start my research from the assumption that the teacher is the most important agent in the whole educational enterprise. Much of the practice of teaching and of teacher education in the U.K. is based on the idea of the "autonomous teacher." This idea is a myth, of course, in the sense that every teacher is subject to all kinds of pressures but it is a myth that we value and preserve. I am not concerned today with whether or not this is a good or a bad myth, but I will be happy to agree for now that it has its dangers as well as its blessings!

My research interests have always been concerned with the mysteries and the complexities of the mathematics classroom—the context in which teachers try to acculturate pupils into the mathematician's ways of understanding the world. My research philosophy is that of "constructive alternativism" [Kelly, 1955] which means that I look for alternative ways of construing and interpreting classroom phenomena in order that the acculturation process can be achieved more successfully than it is at present. One of the first strands of this research to get developed concerned my work on teachers' decision-making. "The teacher as a decision-maker" was a conception designed to catch the process whereby the teacher deals with the many choices occurring both before and during teaching. I was particularly interested in the decisions made during the classroom interactions, now referred to in the research literature as "interactive decision-making" [Shavelson, 1976]. It is a very powerful construct in that it links the work on teachers' knowledge, ideology, attitudes, etc. with the work on teachers' classroom behaviour, methods, language, etc. Various aspects of mathematics teachers' decision-making were learnt [Bishop, 1976a] and many more are waiting to be explored. For example, dealing with pupils' misunderstandings and errors constitutes a large part of a teacher's activity but the decision-making construct forced me to attend to the fact that, in the classroom situation, what is significant is the teacher's perception of the errors and misunderstandings. This is sometimes forgotten by those researchers who study children's errors in a laboratory-like atmosphere away from the interactive classroom. I therefore looked at perrs (teacher perceived errors) and was particularly interested in the teachers' strategies for dealing with these [Bishop, 1976b]. This research developed some very useful activities for teacher education; for example, "freezing" a moment of decision in a video-tape of a lesson and analysing the choices and criteria open to the teacher. Into such discussion it is possible to inject many constructs from psychological research which would otherwise seem very remote from the classroom.

It is also satisfying to see that this construct has been taken up in a very serious and large-scale manner by the Institute for Research on Teaching at Michigan State University. The whole work of the Institute is based on the "teacher as thinker" model and the decision-making construct is well embedded in that model. This conception recognises the fact that the tasks, constraints and problems of teaching develop certain characteristic ways of thinking in teachers, which clearly has enormous implications for both initial and in-service teacher education [Clark and Yinger, 1979].

The second research strand developed from a long-standing interest in visualisation, and once again was concerned with the classroom situation. My first attempts were with different teaching methods and their interactions with various aspects of spatial ability, but I found both of these constructs (T.M. and S.A.) to be rather remote from the real classroom. I therefore reworked both constructs, and changed "teaching methods" to "spatial activities," while "spatial ability" became "visual processing."

Firstly the move away from "methods" to "activities" is highly significant. The idea of "teaching method" creates a distinction between it and mathematical content which I became increasingly uneasy about. "Teaching method" is also a researchers' and not a teachers' construct in that no teacher can possibly see the necessary range of teaching

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that a researcher can, and the teachers I worked with were not happy either about the method/content dichotomy. "Spatial activities" on the other hand links much better with content and seems to fit more with teachers' ideas of teaching although it is also capable of considerable extension beyond those ideas [Bishop, 1974]. It can be embedded in the more general construct of "mathematical activities" and this is a notion which several researchers are currently exploring. For me, the notion of a mathematical activity relates to both topic and process, and is a unit of both method and curriculum. I particularly value its focus on what the pupils are (supposedly) engaged in and it also enables us to analyse activities by such things as type (open, closed, practice, exploration, analysis, etc.) and group size (whole class, small group, individual). I can concern myself with devising relevant, meaningful spatial activities [Bishop, 1982] and I can focus my student teachers' attention on the initiation, organisation and control of those activities. "Spatial activities," as a sub-set of mathematical activities, is I think a very rich and important construct.

The reworking of "spatial ability" was made possible by analysing the distinction between the ability to Interpret Figural Information, i.e. the knowledge, conventions and "vocabulary" of the many figural forms we use in mathematics, and the ability for Visual Processing [Bishop, 1983]. Much spatial ability testing only really tests what I call IFI and although that is important in mathematics, I wanted to see what VP could offer in the classroom context. For example, we know that individuals differ markedly in their ability for visual processing, some preferring to do it a lot and some not at all. Krutetskii's [1976] "geometers" certainly showed extreme preference for it. We know also that there exist differences between individual teacher preferences as well as between those of individual pupils and we can explore how this ability can be developed or how a person can be encouraged not to rely on it. It links with ideas of intuition and imagery, and can also relate to the use of analogy and metaphor.

Of particular interest is how imagery can be shared between teacher and pupil, and this is where the use of diagrams and figures can be so important. Spatial activities can also be studied in terms of their value in helping the externalisation of imagery and the sharing of visual interpretations. Language has a strong part to play here of course because much imagery can be evoked by appropriate language and examples [Kent and Hedger, 1980].

While these two research areas were developing I had become increasingly aware of the gap between much research in mathematics education and the actual classroom situation. In one paper [Bishop, 1980] I concluded that, from the point of view of most theories of learning, the mathematics classroom with its noisy atmosphere, with its multiple objectives, with its fixed-time lessons and with its atmosphere of mutual evaluation, was not a very good place in which to learn mathematics! The problem I could see as a teacher educator was that research on the learning of mathematics was becoming more and more sophisticated while classrooms were becoming more and more of a challenge to most teachers. As a consequence many people were feeling that the quality of learning was declining.

Now I was not the only person to notice this, of course, and I could see different developments which were designed to make the classroom sitation more controllable and more "appropriate" for learning as it was thought it should be done. In the USA, and to some extent elsewhere, one development put more effort into the production of the "ideal" textbook. Much time, money and effort is invested in what some people unfairly call "teacher-proof" texts. These are carefully designed to avoid sex and racial bias, and to build in motivators, reviews, examples, historical quotations, check-tests, spaced-practice exercises, etc. The student's text and the teacher's text interweave precisely and the teacher is told exactly what must be done. She thereby loses her authority to the text's authors. One can detect in research also a search for lesson components which can be put together to produce the "ideal" lesson [Good and Grouws, 1979]. In the U.K. too we can find our ideas of teacher training dominated by the notion of the "mathematics lesson." Lesson planning is stressed, lesson components are analysed, and exercises are given in "lessonising" the curriculum.

A second move to control classroom learning was also developing in the U.K. and elsewhere. This was the move towards individualised schemes (like SMILE, and KMP) which built to some extent on the earlier research on programmed instruction. However we have well-documented evidence of the ways in which such schemes totally change the teacher's role from those of teacher, authority, helper, to those of administrator, marker, paper distributor [Morgan, 1977]. The danger here is that the more sophisticated the individual materials become, the more they intervene between the teacher and the pupil. The teacher once again loses her authority to the anonymous pieces of paper.

My own response to the challenge of the complexity of the classroom is not to seek salvation in the textbook full of ideal lessons, nor in the loneliness of the individualised material, but to seek better ways to understand the classroom. It is only complex because of our ignorance and if we could understand it better, if we could interpret it more richly, then perhaps we could learn how to handle it better. This brings me to the third research effort which has occupied my mind over recent years.

I refer to it as the "social construction" frame and like to distinguish it from our more traditional "mathematics lesson" frame which, as I have already indicated, has tended to dominate our thinking about mathematics education. This "social construction" conception has grown out of the wider range of research perspectives which have been brought to bear on the phenomenon of life in classroom. Classrooms ethnographers, sociologists, those who study verbal interactions, teachers' decisions and pupil/teacher perceptions have opened our eyes to a rich tapestry of classroom phenomena. We are now, for example, much more aware of aspects like teacher stress, pupils' fear of mathematics, of the effects of interpersonal perceptions, of pupil-pupil interactions, of the powerful position of the teacher in the classroom and of pupils' strategies for coping with their relative powerlessness.

What I have been attempting to do is to pull out from these researchers what I feel are the more significant aspects for us in mathematics education.

Fundamental to our understanding of mathematics classrooms is the fact that one is dealing with people. It may seem trivial to say this but the fact can easily be overlooked when discussing details of lesson components, for example, or pupil ability, or motivation, or any other psychological or mathematical construct. It is true, of course, that the classroom, being part of an institution, institutionalises the participants. But each classroom group is still a unique combination of people—it has its own identity, its own atmosphere, its own significant events, its own pleasures and its own crises. As a result, it has its own history created by, shared between, and remembered by the people in the group.

A corollary which is of significance to the teacher is that each individual person in the classroom group creates her own unique construction of the rest of the participants, of their goals, of the interactions between herself and the others and of all the events, tasks, mathematical contents which occur in the classroom. Such "objects" as children's abilities, mathematical meaning, teacher's knowledge, rules of behaviour, do not exist as objective facts but are the individual products of each person's construction

Recognition of this social construction of phenomena leads me to propose a new orientation for mathematics education. This orientation views mathematics classroom teaching as *controlling the organisation and dynamics of the classroom for the purposes of sharing and developing mathematical meaning* This orientation has the following features:

1. it puts the teacher in relation to the whole classroom group,
2. it emphasises the dynamic and interactive nature of teaching,
3. it assumes the interpersonal nature of teaching, i e that the teacher is working with learn*ers* not merely encouraging learn*ing*,
4. it recognises the "shared" idea of knowing and knowledge, reflecting the importance of both content and context,
5. it takes into account the pupil's existing knowledge, abilities and feelings, emphasing a developmental rather than a learning theoretical approach,
6. it emphasises developing mathematical meaning as the general aim of mathematics teaching, including both cognitive and affective goals,
7. it recognises the existence of many methods and classroom organisations, i.e. it does not by definition exclude any methodological techniques already established,
8. it is a conception which permits development of the teacher through initial teacher training and beyond.

Central to this view of classroom teaching is the idea of mathematical meaning — a notion which should perhaps be clarified at this point. What I am seeking to emphasize is the personal nature of the meaning of any new mathematical idea. A new idea is meaningful to the extent that it makes connections with the individual's present knowledge. It can connect with the individual's knowledge of other topics and ideas in mathematics but it can also connect with knowledge of other subjects outside mathematics It may well relate to imagery, analogy and metaphor, but these connections will be of a different type. The idea can be an example of another mathematical idea (because that is the nature of mathematics) and may well generate examples of its own Finally, and arguably most importantly, it can connect with the individual's knowledge of real world situations. It is obvious therefore that no two people will have the same sets of connections and meanings, and in particular teacher and learner will have very different meanings associated with mathematics. The teacher will "know" the ideas she is teaching in terms of the connections they make with the rest of her mathematical knowledge The learner however is the "meaning maker" [Postman and Weingartner, 1971] in the educational enterprise and must establish the connections between the new idea and her existing knowledge, if the idea is to be learnt meaningfully As Thom [1973] says, "The real problem which confronts mathematics teaching is not that of rigour, but the problem of the development of meaning, of the existence of mathematical objects." The educational goal we are concerned with here then, is that of sharing, and developing, mathematical meaning

This conception has enabled me to focus my analysis on three fundamental aspects:

mathematical activities
— chosen to emphasise the learner's involvement with mathematics rather than the teacher's presentation of content,

communication
— chosen to emphasise the process and product of shared meanings,

negotiation
— chosen to emphasise the non-symmetry of the teacher/pupil relationship in the development of shared meanings

I have already illustrated the importance of the idea of mathematical activities but a few more thoughts are necessary here. It is significant for the teacher's pre-class decisions in that the teacher no longer thinks of how she will present content during the class but rather she must make the didactical conversion from mathematical content and knowledge to mathematical activities suitable for the pupils. In the U.K. we still "think" too much about content, knowledge and mathematical topics and not enough about what the pupil's activity will be in class. A focus onto mathematical activities for the pupils can improve that situation and can put the pupil's activity at the centre of the teachers' concerns.

Not only does this affect pre-class but it also affects the teacher's interactive decision-making. Teaching is, as a result, more concerned with the initiation, control, organisation and exploitation of the pupils' activity. There is more of a dynamic, organic-growth, feeling in the classroom than of a compartmentalised list of specific knowledge or skills to be taught from nothing, and to be finished at a set time.

Another aspect which "mathematical activity" makes us attend to is collaborative working. Pupils value collaborative working but mathematics teachers in the U.K. have an ambivalent attitude towards it — most seem to prefer pupils to work on their own but will say things such as "you can work with your friend if you don't make too much noise"! In fact in U.K. mathematics classrooms there will exist much collaborative learning but most of it will be covert and often "illegal" instead of being deliberately planned and encouraged by teachers. If only we could develop more small group mathematical activities for pupils the teachers could be encouraged to take a more positive attitude toward collaborative and interdependent working than they do at present.

Communication is not a new construct in education but in my view it has never been well analysed or activated within mathematics education. In general, in the U.K., mathematics classrooms are places where you do mathematics not where you communicate or discuss mathematical meanings. Meanings and understanding are about the connections one has between ideas — a new idea will be meaningful for a pupil to the extent to which it connects well with the pupil's existing ideas and meanings. Communication in a mathematics classroom is therefore concerned with sharing mathematical meanings and connections. We can only share ideas by exposing them, and "talk" is clearly a most important vehicle for exposing connections. Also important are symbolism, uses of diagrams for conveying images, examples from different contexts, analogies and metaphors, and written accounts and descriptions. Some of these we know relatively more about (symbols, definitions...) others we know relatively less about (analogies, metaphors, contexts). Moreover if we add into the construct of communication the dimension of sharing, then the three-way process, from the pupil to teacher as well as teacher to pupil and pupil to pupil, shows us how ignorant we are about pupils' analogies, metaphors, contexts, examples, etc. and about ways of enabling these to be exposed and shared. For example, activities can be developed which encourage and legitimise this exposition and sharing, such as investigations which involve creating symbolism, or projects which draw on knowledge of the pupils' environment, or discussions of mathematical ideas and their diagrammed analogies (number lines, etc.) Several research studies have shown us that in classrooms it is the teacher who does most talking. What I should like to see are developments which show teachers how pupils can be encouraged to take more part in the sharing of mathematical meaning. I think that exploiting the ideas of two-way and three-way communication could be a profitable way forward.

If communication is about sharing meanings then negotiation is about developing meanings. Without wishing to suggest that the teacher is the authority for mathematics in the classroom, it is the case that the teacher is given authority and power by the society for the specific education of her pupils. This authority means that the teacher has certain goals and intentions for the pupils and these will be different from the pupils' goals and intentions in the classroom. Negotiation is goal-directed interaction, in which the participants seek to attain their respective goals. We can include in this idea the working out of a "modus vivendi" in the classroom, i.e. the rules of procedure, discipline and behaviour which teachers already know much about. What the construct of negotiation also offers is an idea of "modus sciendi", a way of knowing, which is what the teacher is trying to develop by the use of her own, necessarily richer, mathematical knowledge and understanding. This construct then specifically catches the necessary power imbalance implicit in the teaching/learning relationship but it describes it in such a way that we can see alternatives to the mere imposition of knowledge from the powerful teacher.

What it therefore forces us to do is to consider how to encourage teachers to use their power *not* to impose their knowledge on the pupils. It makes us think more about how teachers can encourage the negotiation process, how teachers can encourage pupils to play a greater part in the development of their own mathematical meanings, how teachers can recognise more positively the pupils' context and goal structure, and how teachers might evaluate better the development of meanings.

In conclusion, then, may I suggest that this "social construction" conception and the three constructs, "activities," "communication" and "negotiation," offer us many rich avenues to explore in research. Like any good construct, they recast what we know about and direct us to what we need to know about. I would, as a result, particularly urge more attention to the following:

- the development of activities, particularly those which exploit the pupils' context and those suitable for small group work,
- the analysis of the relationships between *activities* and mathematical topics,
- studies of teachers' interactive decisions with pupils engaged in activities of different types,
- studies of teachers' techniques to encourage sharing of mathematical meanings,
- the analysis, from the "sharing" perspective, of pupil-pupil discussions,
- studies of the process whereby visual imagery can be shared,
- the analysis of teachers' decision-making concerning mathematical authority,
- the analysis of teachers' strategies which permit negotiation,
- the development of methods of evaluating the development of meaning.

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In 1891 I have been able to solve a few problems in mathematics and physics, including some that the great mathematicians puzzled over in vain from Euler onwards: e.g., the question of vortex motion, and the discontinuity of motions in fluids, that of the motions of sound at the open ends of organ pipes, etc. But any pride I might have felt in my conclusions was perceptibly lessened by that fact that I knew the solution of these problems has always come to me as the gradual generalization of favorable examples, by a series of fortunate conjectures, after many errors. I am fain to compare myself with a wanderer on the mountains, who, not knowing the path, climbs slowly and painfully upwards, and often has to retrace his steps because he can go no farther—then, whether by thought or from luck, discovers a new track that leads him on a little, till at length when he reaches the summit he finds to his shame that there is a royal way, by which he might have ascended, he had only the wits to find the right approach to it. In my works I naturally said nothing about my mistakes to the reader, but only described the made track by which he may now reach the same heights without difficulty

von Helmholtz