Communications

Towards subjective truths in mathematics education

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In many predominantly English-speaking countries we have problems understanding the notion of *didactics* as it is used in many European and other countries. The term appears almost exclusively in the UK as a description of a form of teaching which in recent decades has been seen pejoratively. Didactic teaching is taken to mean teaching with little regard for student understanding, the passing over of knowledge from the possessor, the teacher, to s/he who needs to possess that knowledge, the student. Clearly there is no point in engaging with the discussion opened up by the Gascón and Nicolás (2017) unless one uses the sense of 'didactic' as on the continent of Europe and elsewhere around the world.

I understand from their piece, as well as from elsewhere, that *didactics is the science of learning and teaching in general,* meaning that it is not tied to any specific content. "Its central task consists in planning and organizing successful processes of students' learning" (Zierer & Seel, 2012). The didactics of mathematics is then *the science of learning and teaching mathematics.* In considering the question posed by Gascón and Nicolás to academics in the research field of mathematics education, therefore, we must consider the nature of science and whether didactics can be subsumed within the fields of sciences. They also set us the challenge of whether didactics can set value judgements and normative prescriptions that can inform teachers of mathematics on the provision of study processes.

Mathematics education research (didactics of mathematics) as a science

The image of science is one of the accumulation of knowledge about aspects of the physical world. Since Kuhn, Popper and Lakatos we see that accumulation as a series of conjecture, proof and refutation in a never-ending process of development. Science aims at replicable and predictable results, with an intention towards understanding or at least identification of cause and effect. Science aims towards truth, though it never knows if/when it is reached. I have just been reading about the developing cosmology of the late Stephen Hawking in which he and colleagues discuss the multiverse hypothesis and the notion of fine-tuning. His ideas developed and change in the final years of his life but that work will continue.

Social 'science' is an unfortunate label since it suggests that social phenomena can be determined in the same way as physical sciences and that we can progress towards universal truths. On the contrary, social studies or social theories develop hermeneutically; that is, its activity is inherently interpretive. Indeed, since Giddens' important work in 1977, *Studies in Social and Political Theory,* we must consider the double hermeneutic. "On the level of methodology, a hermeneutically informed social theory would require, above all, a double hermeneutics, taking note of the essential identity of the linguistic medium from which both the social reality and its sociological account are woven" (Bauman, 1989, p. 212). That is to say, in regard to social theory, and here I am of course referring to education in particular, our theories are built from interpretations of what are already interpretations. We analyse questionnaires, for example, and draw our own conclusions of what those completing questionnaires are saying; a double layer of interpretation. Furthermore, interpretations are embedded in and dependent on language. Hawking's universes are about interpretations too, of course, but cosmologists aim to test scientific theories in ways that social theories cannot. Critical experiments can be designed that may support one theory as against another, though it may take decades for technology to reach a state in which the thought experiments can become actual. We must also note Kuhn's discussion of hard-core and peripheral theories in potential scientific revolutions. Science is a single hermeneutic activity. The scientist interprets findings to construct and then test theories. The objects of study and the products of science, largely mathematical equations, do not answer back. Teachers and students do.

Imagine aiming to develop a similar critical experiment in education. To take one example, some years ago I engaged in a dialogue with Marty Simon in which we both analysed a transcript from one of his research studies, of a single student working with the researcher to solve problems in the division of fractions. Marty's analysis was based on identifying reflective abstraction in the learning process of the student, in which the researcher's questioning facilitated the development but did not affect it. The student might or might not have made that intellectual leap. I analysed the transcript from the perspective of the emergence of a zone of proximal development in the exchange between researcher and student. That zone might not have emerged, though I argued that it did. Rather than being a critical experiment, which might have led to the rejection of Piagetian constructivism in favour of Vygotskian sociocultural theory, or the opposite, we were left with two coherent but parallel accounts. We were stuck in terms of producing a publishable article. We could neither present a decisive acceptance of one and rejection of the other, nor find a way to merge the competing theories into a new one. This is not surprising in our field of mathematics education research. As has been argued (see *e.g.,* Lerman, 2010), following Bernstein (*e.g.,* 2000), in social studies theories develop either within themselves or as new languages alongside existing ones. Constructivism develops with new studies and so too does sociocultural theory. In my view the sociopolitical turn proposed by Gutiérrez (2013) has developed a new language that has entered the field of mathematics education research. It will not replace existing theories but will sit alongside them as alternative discourses.

To come to the ideas of Gascón and Nicolás I think that they are probably right at one level. I think we can make

general statements on didactics that are testable empirically, though I would not suggest that they are necessarily universal; statements such as:

- Children will not learn well if they are hungry.
- Teaching a class of 90 children requires different strategies to those needed for a class of 30 and different again for a class of 15.
- Some daily physical exercise improves the attention span of children.

Making testable statements about teaching and learning at the classroom and individual level is much more problematic, precisely because such tests must be local, dependent on what children, or teachers, or whoever are the subjects of the study, say, and dependent again on how the researcher interprets what is collected as data. To take results from such research and try to replicate them in another context may find similarities but often does not result in a positive outcome. This is hardly surprising when the children are different, the teachers are different, perhaps the school and the social context of the school are different, and certainly there are differences across cultures.

Even if we believe that large-scale randomised experiments reveal truth, with hypotheses concerning textbook choices, ways of teaching specific mathematics content or whatever, we face the filters of interpretation and recontextualisation. In terms of impact in classrooms the results of the experiment first have to filter down to classroom teachers, being moderated by governments on the way, but then will be taken up or not by teachers depending whether they think it will support their particular students in their particular context.

Can didactics set value judgements and normative prescriptions?

The article by Gascón and Nicolás does a good job, it seems to me, in the discussion clarifying what might be meant when "a value judgement or a normative assertion" might be turned "into a statement susceptible of expressing an objective truth". The authors identify teaching ends, research ends, and principles, which seem to include theories of learning, regarding means and ends.

But rather than help in determining when a statement might be susceptible of expressing an objective truth, the separation, for analysis purposes, of teaching ends, researching ends, and learning theories or other principles may indeed help to show how education in general and mathematics education in particular might not be able to express objective truths at all. Being a double-layered hermeneutic enterprise, from the researcher constructing her or his theories from students' or teachers' interpretations of their experiences, to the classroom teacher interpreting what her or his students need at a specific moment as drawn from her or his evaluation of the current state of their learning, educational statements/decisions cannot be objective. To be objective involves taking the individual out of judgements, an impossibility in our enterprise.

Value judgements are 'ought' statements. Moving from a description of what 'is', the proper task of research, to what 'ought to be' seems to me to be introducing ethics, or aesthetics, not our job at all. Of course as educators, concerned in particular for those children failed by schooling, which correlates with those most disadvantaged in society, at least in the UK, we must care about quality teaching. The word 'quality' exemplifies the problem: I suspect we cannot agree on what that looks like, nor what we are looking for.

Now this does not mean research in mathematics education is useless. The idea of recognising the importance of listening to students as the only way of deciding what next activity they should encounter has come into common language in teaching through research. That does not mean that in my classroom I can replicate what happened in the article I read about listening. I may judge that my classroom, my students and my school cannot work that way, or I may not be up to it myself as a teacher. But I will have learnt something from that research and will interpret it and act on it in my own way. The research becomes a subjective truth for me, at that time, in that place.

I am arguing that didactics can only propose value judgements and normative prescriptions that can inform teachers of mathematics on the provision of study processes, and teachers will take up, or not, those judgements and prescriptions and interpret and apply them as they see fit. And they are right to do so. To expect to produce objective truths, which set correct ways of providing study processes, is to misunderstand social studies as a research field.

Why seek to view didactics as a science?

I think there is a final question that should be addressed here: why is it desired by so many that mathematics educational studies, or the didactics of mathematics should be seen to be scientific? It could be that there is a great deal of poor learning taking place in schools, as a result of poor teaching. If we could arrive at prescriptions that we are sure should work we might be able to teacher-proof pedagogy, contradictory as that may sound. It could be that, in the competition for status and resources in our universities, science wins and so if we can subsume didactics of mathematics into the sciences we can achieve higher status and consequent resourcing.

I fear that may do a disservice to our field. Our work is richer and more complex when recognised as a social science. I am satisfied that even though what is produced by research in the didactics of mathematics is only susceptible to what I have called subjective truths, in that the individual teacher ultimately interprets research findings, government prescriptions or whatever, for the classroom, raising questions and doing research, and that includes teacher research, are essential for our field and for all those engaged in teaching and learning.

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Prescriptions and proscriptions on mathematics teaching: interesting cases of *lost in translation*

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When I was invited, *en français,* by Jean-François to engage with Gascón and Nicolás's article and offer a response to it, my first reaction was that I had already addressed this issue, in this very journal (Proulx, 2015). Dropping the article, I said to myself, jokingly: *tout est dit!*

But I took it up from my desk and as I read it further, as a *francophone* educated in both Anglophone and Francophone traditions, I felt I was facing, on many accounts and levels, significant and interesting cases of 'lost in translation' [1]. Thus, I offer in this communication a meta-reaction to Gascón and Nicolás's text, that is, not in terms of its content about prescriptions, but mainly concerning our field of research itself. The following issues are not as developed as one might want, due to space constraints, but also because they are intended to be triggers to remind us of what can be gained from being sensitized to these translation issues. These issues are some I am constantly reminded of through being actively engaged in both the Anglophone and *francophone* worlds, as well as in various worlds of research, schooling and science. I offer these issues in no order of importance.

Issue 1: 'didactics' in English is prescriptive

Even after more than 50 or so years of research in *didactique des mathématiques,* 'didactics' still remains, for everyday persons, a very pejorative term that calls to mind prescriptions, moralizations, instructions, *etc*. Hence, from a non-specific point of view, the question of Gascón and Nicolás's survey is a truism. But that is not what they meant, I assume.

Issue 2: is *didactique* really "didactics"?

I also assume that Gascón and Nicolás refer to the French *didactique des mathématiques* when they use the English word 'didactics'. A genuine question is whether that translation is adequate. One can wonder if researchers who are using 'didactics' in English really mean the same thing as *didactique* in French. Were the usages of 'didactics' by Bauersfeld at the Bielefeld Institute the same as *didactique* (see his interview in Karp, 2014)? This is far from sure. In fact, even for francophone users of *didactique*, from one country to the next and even within a country, the meaning varies significantly. As Bednarz (2001) stated, there are many *didactiqueS des mathématiques,* insisting on its plural form (see also Sierpinska, 2000, as well as the interviews I conducted in Proulx, 2013, where all interviewees had to commit and offer their definition of *didactique des mathématiques*). The variety of *didactiqueS* might call into question the direct translation of *didactique* to 'didactics' in English; perhaps the S aims at translating that very idea.

Issue 3: is 'didactics' used to refer to mathematics education research?

As Sierpinska (2000) illustrated in her review of various research perspectives used in our field, the relation between *didactique des mathématiques* and mathematics education research is far from being a direct one. Some might say that mathematics education research is a part of *didactique*, but others might claim the containment goes the other way. Hence, when reading 'didactics', is one translating it into 'mathematics education research' or into *didactique des mathématiques*? (I myself cannot decide.) To take a specific example, we do not know if Ed Dubinsky answered the questions posed by Gascón and Nicolás with mathematics education research in mind, or by reflecting on what he thought *didactique des mathématiques* means. Would these varied interpretations matter? Would they produce different answers?

Issue 4: is didactics or *didactique des mathématiques* a science?

This is a huge theme and I am not venturing far into it. I only raise the question, in order to address the translation of 'didactics'. *La didactique des mathématiques,* when it became a field of its own (thanks chiefly to Brousseau), was conceived of as a science with all the associated ambitious intentions, as Brousseau has said many times (*e.g.,* predicting, controlling, explaining, describing, *etc.;* see Stengers & Schlanger, 1991). It is still translated today as a science by a number of French *didacticiens*, but it is no longer as widespread as it once was. At first, the nascent discipline was often lodged in faculties of science and mathematics departments in many universities around the world. But, with the development and strengthening of *sciences de l'éducation* (see, *e.g.,* Charlot, 1995), this is less and less true. *Didactique* is now finding its place in education faculties and departments (see again the interviews in Proulx, 2013). Does that mean that *didactique* is less and less translated as a scientific discipline? What will it be 10 years from now? If, as Issue 3 signaled, the expression *didactics* is a translation of 'mathematics education research' (here a translation from English to English!), then it is mostly an education discipline, related to the social or human sciences. In fact, mathematics education is lodged, in many Anglophone universities, in education faculties and departments.

Issue 5: who responded?

Far from questioning the legitimacy of the respondents in Gascón and Nicolás's question about prescriptions, one might wonder why all the responses in the paper appear in English. Apart from Dubinsky, all the other respondents are not Anglophones. I assume that either they answered in their

preferred language and were translated in English by Gascón and Nicolás or they decided to answer in English, which did not represent their first language. In all cases, I do not need to make the case that the words used and their intended meanings are filled with potential for being lost in translation!

Issue 6: who are we?

As Artigue wisely reminded us, we (whoever we are in the field) are not only researchers but also teachers, teacher educators (and not trainers, which has a completely different meaning in English, and clearly does not translate the francophone expression *formateur* which means 'shaper'), and many other roles we like to see ourselves in: administrators, supervisors, evaluators, consultants, experimenters, applied mathematicians, policymakers, *etc*. So, when we are asserting something, who is doing the talking? Are there constant translations in our discourse? Am I using the same language and discourse when I talk to a teacher? Am I taking the position of a teacher when talking with teachers? Am I using my position as a researcher? Do I consider which role serves me, or my audience, best? When communicating findings whom do I embody? And how do I convey it to my audience? There are significant ethical issues here, as a well-chosen role might grant me a better ear from my audience. We all have opinions on a myriad of issues. Is my opinion better received if I speak as a researcher, even if it is not necessarily well supported by my findings? Will this be clear to the audience? Instead of seeing something lost in translation, here it appears that some dimensions are being added. Is this a case of 'addition in translation', where we bolster what we say by using strategic positioning?

Issue 7: prescriptions are translations

If one takes issues of translation to another level, one might say that issues of prescriptions can themselves be seen in terms of translations. Indeed, when talking about prescriptions, one is talking of translation of research results into those prescriptions (or proscriptions, depending). I know of no researcher who has explicitly launched a research project to investigate a question like "What are the prescriptions of…?". Prescriptions follow research findings and can be grounded in them, but they are not the first level findings that we dig for. In the translation of findings into prescriptions, many aspects are lost: context, generalities, idiosyncrasies, *etc.,* that are for many the essence of research, especially in a qualitative research orientation (see Geertz, 1973).

I will stop here. Obviously, there are other issues to raise in relation to this topic and one might want to engage in outlining them. It appears to me that the fruitful dialogue that Gascón and Nicolás aimed at might also need to happen at a more meta-level, as I have sketched parts of here. Sensitivity to the issues that I have raised, issues I believe to be helpful for our field in general as a way of opening up to a variety of viewpoints and language-points, seems essential for this dialogue to be fruitful.

As a closure, let me be *bon joueur* and address briefly Gascón and Nicolás's question about prescriptions. For a while, I considered (and was pressed by the editors) directly drawing out aspects from my 2015 communication in FLM. But, since it is already published, I might as well simply point to it and encourage readers to read it. That is done. I have decided instead to address their question by quoting something the educational anthropologist Harry Wolcott said almost 30 years ago. There is, however, a need to translate his words into our own field, since Wolcott addresses issues of qualitative studies in general, a field that he sees more as descriptive than analytical. Here is the quote, for you to translate:

> Qualitative researchers seem particularly vulnerable to the tendency—and urge—to go beyond reporting *what is* and to use their studies as platforms for making pronouncements of *what ought to be*. A critical divide separates the realm of the observable from the realm of values about good and better. This is not simply the matter of a big leap: You cannot bridge the chasm between the descriptive and the prescriptive without imposing someone's judgment, whether originating from the people in the setting ("What we really need around here…"), from expert opinion ("If these people knew what was good for them…"), or from the researcher's own assessment ("I cannot help wondering whether…"). True, there is an evaluative dimension to all description, but the antidote is restraint. The urge to lend personal opinion and judgment seems to become strongest when we start searching for the capstone with which to conclude our studies. You can recognize it creeping into your work (or, if you prefer, into mine) with the appearance of words like "should," "must," "need," or "ought." There is nothing wrong with offering your personal opinions or professional judgments—but be sure to label them as such, and to search out and acknowledge their origins in your thinking. (1990, pp. 55-56)

Again, *tout est dit!*

Note

[1] I here use interchangeably *italics* and single quotes for words in the languages that I am writing in this communication. This said, it is not clear, even to me, in which language I am writing as I go on. How coherent, some might say, in relation to the topic at stake on being lost in translations.

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