Graphic Calculators: an Interpretative Framework

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In recent years, a pedagogy which stresses the importance of multiple representations in the understanding and manipulation of mathematical concepts has become prevalent (Confrey, 1994). At the same time, advances in technology have allowed for the development of various graph-generating tools such as the computer and the graphic calculator. Since the graphic calculator is portable, cheap and does not require mains electricity, it represents a potentially important tool for the learning of mathematics, particularly in the developing world. Waits and Demana (1993), although referring to the American environment, expressed it as follows:

> Frankly, the invention of graphic calculators [...] is what made C^2PC [the technology-intensive Calculator and Computer Precalculus mathematics course at Ohio University] a viable and implementable course. [...] Students at typical high schools rarely have access to a computer lab during a precalculus class and a computer at home. [...] We believe that students must use computers on a regular basis for both in-class work and for homework outside of class if there are to be significant changes in the mathematics that students learn in the 1990's. (p. 92)

In the developing world, the access to computers labs is more limited than in the American environment, hence strengthening this argument in such contexts. Even in the Portuguese environment, Carvalho E Silva (1996) argues that, largely due to economic factors, computers are hardly used in the classroom and that, in reality, graphic calculators represent the direction of the pedagogical future.

Unfortunately, despite the potential and actual importance of this tool, there is not much literature dedicated to explaining or understanding how the graphic calculator, specifically, functions in relation to the learner. In fact, much of the literature relating to the graphic calculator is anecdotal or describes evaluative studies which fail to distinguish adequately the role of the tool from that of the instructional process (Penglase and Arnold, 1996, p. 58).

In this article, I wish to suggest that a Vygotskian approach to learning, with its emphasis on mediated activity within a particular socio-historical context, is appropriate to address the relationship between the mathematical learner and the different sign systems (multiple representations) afforded by the graphic calculator.

An overview of literature

I do not intend a comprehensive review of literature. Rather, I hope to show, via some representative examples, how current literature on the graphic calculator ultimately gave rise to questions and criticisms which necessitated and sometimes informed the development of my analytic framework.

For an extensive and detailed review of literature, the reader is advised to consult Penglase and Arnold (1996), who discuss much of the published research on the graphic calculator, and Dunham (1993), who provides a brief but useful summary of the results of research studies on the graphic calculator. Interestingly enough, these reviewers are critical of the current state of research: Penglase and Arnold focus their criticisms on the inability of many researchers to distinguish the tool from the context and instructional process within which it was used; Dunham and Dick (1994) argue that research on the graphic calculator has been descriptive rather than explanatory.

The nature of graphic calculator research

Many articles, such as those of Waits and Demana (1993, 1996), Dick (1992, 1996), and several of those found in the Proceedings of the International Conference on Technology in Collegiate Mathematics (ICTCM), discuss the implications of the introduction of the graphic calculator for the curriculum. Despite the fact that many of these discussions are important and informative, they do not attempt to explain psychologically *how* or *why* the graphic calculator functions as it does. There are also many articles such as these which give useful suggestions for the uses of the graphic calculator. These articles are certainly a very important resource for the teacher, but again they do not contribute directly to the critical literature.

There are also several articles and references to doctoral dissertations which describe experiments in which one group of students is given a graphic calculator, another is not. The two groups are then compared on several measures, such as achievement in tests (Quesada and Maxwell, 1994; Jones and Boers, 1992). Again, however, although many of these articles provide empirical material (sometimes problematic due to research design limitations), their explanatory and theoretical aspects are limited. Penglase and Arnold (1996) are particularly critical of much research in this category for its failure to:

> recognise that judgements of effectiveness [of the graphic calculator as a tool] result directly from existing assumptions regarding both assessment practice and student 'achievement'. (p. 58)

These observations support my argument that there is a scarcity of research directed towards an explication or interpretation of how the graphic calculator functions as a tool for learning.

Notwithstanding this general dearth of analytical material, an interesting interpretation (which both suggested and supported the development of my interpretative framework) derives from Jones (1993, 1996). Jones (1993) defines the graphic calculator as an 'intelligent technology' which is capable of significant cognitive processing on behalf of the user. Most importantly, he distinguishes between the two principal effects of working with the graphic calculator. There are the possible changes in performance which occur when the student is working with the technology; these are termed effects *with* the technology. There also may be changes that occur as a result of working with the tool; these are termed effects *of* the technology.

In this paper, Jones describes the process of working *with* the technology as one in which the user plans, implements and interprets the solution and the graphic calculator performs the techniques. He contends that this partnership gives the student the potential to work at a much higher level than may be possible without the graphic calculator.

According to Jones, the other possible effect occurs as a result *of* the use of the graphic calculator and is based on the idea that learning in a multi-representation environment automatically enhances understanding. However, he maintains that there is little evidence for this notion:

> The *assumption* that there will be cognitive residue from learning with graphic calculators is based on the *belief* that presenting the same mathematical material in both algebraic and graphical forms helps students invest meaning in and thereby helps promote an increased depth of learning of the material.
> (p. 215, my emphases)

Ironically, an area that has attracted a reasonable amount of detailed research relates to the difficulties that students may have when encountering graphs. Goldenberg (1988) argues that multi-representational environments in which graphs are accorded a high status are potential sources of much confusion:

> Students often made significant misinterpretations of what they saw in graphic representations of functions. [...] they could induce rules, that were misleading or downright wrong. (p. 137)

However, many important questions concerning the use of graphical information remain unexamined. For example:

- There are conventions attached to the reading of graphs (Goldenberg, 1988). Frequently, these are not addressed directly; rather the teacher hopes that the student will learn these codes indirectly through experience. Is this acceptable? In a Vygotskian sense, does the reading of graphs primarily require everyday or scientific concepts?

- Does a conscious acknowledgement of the problems inherent in visual data in the computer/calculator environment assist in the building of appropriate graph-reading and manipulating skills? On this point, Vygotsky regarded conscious reflection as essential for higher human thinking processes.

Graphic calculators and computers: different tools

Since there is already a fair amount of research which attempts to interpret how computers assist in the learning of mathematics, an important task is to consider whether an understanding of the interactions within the computer context can automatically be transferred to the graphic calculator context.

Superficially, one might hold that most pedagogic or cognitive effects of the use of computer graphics are mirrored in the graphic calculator environment. After all, both provide a means whereby the student can easily represent graphic and tabular and sometimes symbolic information. However, even if one assumes that the computer software can only plot graphs and is unable to deal with symbolic representations or manipulations (as is the case with a graphic calculator such as the TI-82), I will argue that the learning experience with the graphic calculator is sufficiently different from the learning experience in the computer environment that it warrants its own dedicated research and interpretation.

One crucial difference between graphic calculators and computers relates to the respective status of each technology as a cultural artifact. In particular, the ways that males and females use and perceive these technologies in the developed world differ. Boys are more likely to see the computer as a high-status object and thus tend to compete for its use. Graphic calculators do not enjoy the same high-status as computers and accordingly do not reflect the same male need to dominate (Smart, 1993). As a result, differences in attitudes or performance between boys and girls with respect to these different technological tools may be quite specific. Some interesting gender-related studies which do focus on the use of the graphic calculator in the developed world are those by Ruthven (1990), Shoaf-Grubbs (1993) and Smart (1995).

Conversely, it is plausible that in developing countries (where computers are relatively scarce) the graphic calculator may enjoy a relatively higher status than in developed countries (where alternate technological tools such as 'higher status' computers are relatively available); consequently, the differences between males and females with respect to technology may not vary so substantially as in the developed world.

An interpretation of the different effects of the graphic calculator with reference to gender and socio-economic conditions is beyond the scope of this article. Suffice it to comment that there is a need for extensive investigation in this area.

The graphic calculator does not have the same interactive capabilities as the computer, and so the type of relationship that a learner forms with a graphic calculator is qualitatively different from most probable or possible relationships with a computer. Unlike a lot of popular software, the graphic calculator acts merely as an algorithmic performing machine and cannot provide feedback to the user. In this important respect, the computer and the graphic calculator are fundamentally different.

Additionally, drawing in 3-D on the graphic calculator is particularly difficult and different perspectives of the same object cannot be automatically generated. In contrast to this, 3-D images are dealt with easily by computer software systems such as Maple or Matlab. These computer-generated images can be rotated and manipulated in a manner that is impossible on the graphic calculator.

Furthermore, the user is unable physically and directly to 'mess about' with the graph on the graphic calculator, by pulling or pushing it, as is often possible with computer software such as Function Probe (developed by Confrey and colleagues at Cornell University). Smith and Confrey (1992) implicitly refer to the significance of this feature of computer graphing software:

> Direct graphing actions [with Function Probe] allow the user to 'create' the action as a visual experience taking transformations 'back' to their geometric origins. We suggest that many of the difficulties students have with transformations are rooted in the heavy reliance on examining only how changes in algebraic parameters change the graph of the function. (p. 227)

In this fundamental way, the environment of the graphic calculator differs from that of the computer. The importance of physical action in context is recognised by Vygotsky. According to him, knowledge, thinking, feeling and action are integrated in the process of learning.

> The unity of perception, speech and action which ultimately produces internalisation of the visual field constitutes the central subject matter [...] of the origin of uniquely human forms of behaviour. (1978, p. 26)

In summary, the socio-cultural status of the graphic calculator and the computer differ significantly. In addition, although there are several points of correspondence between the two technologies, they differ with respect to several fundamental features: the interactive capability and also the possibility for the user to modify the graph directly through physical acts is very different with each tool. Hence, there is a need for research specifically devoted to the graphic calculator as opposed to the computer or technology in general.

Developing an interpretative framework

The Vygotskian paradigm

This article is concerned with interpreting how the graphic calculator (a contemporary technological tool) and its sign systems (graphs, tables) mediate in the learning process of a mathematics student. Although Vygotsky was writing in the early twentieth century, his foregrounding of the use of signs and tools as agents of mediation makes his theory (and its later refinements) an ideal candidate for application to the interpretation of the relationship between the learner and the graphic calculator.

In brief, Vygotsky posits that education is both a theory of development and a process of enculturation in which mediated action links the external social world to internal mental processes. All higher human mental functions are products of mediated activity in which the role of the mediator is played by a psychological tool or sign, such as language, graphs, algebra, or a technological tool, such as a computer or a graphic calculator. According to Vygotsky, these forms of mediation, which are themselves products of the socio-historical context, define and shape inner processes.

In interpreting the way in which the graphic calculator effects learning, it is helpful to regard the use of the calculator as an external activity (manipulating mathematical concepts via graphs or numbers) which is ultimately transformed into an internal activity (understanding maths). In this way, the use of the calculator may fundamentally transform the learning process.

> If one changes the tools of thinking available to a child, his mind will have a radically different structure. (Vygotsky, 1978, p. 126)

In this article, I distinguish between two primary ways in which the graphic calculator may function as mediator of the learning process. A useful description (implicit in the literature) of how the graphic calculator mediates in the learning of mathematics is provided by a re-interpretation of Pea's (cited in Salomon, 1990) distinction between the *amplification* effects of technology and the *cognitive re-organisation* effects of technology. This separation of effects is elaborated on, and discussed, below.

Furthermore, I argue that my study highlights the significance of the social and institutional context within which the graphic calculator is used. In particular, I maintain that the extent to which the graphic calculator is used as an add-on tool for verification and support rather than as an integral part of a mathematics course contributes to its social significance and hence to its efficacy as a tool for learning.

A useful distinction

> Of course, like all over-simple classifications of this type, the dichotomy becomes, if pressed, artificial, scholastic and ultimately absurd. But if it is not an aid to serious criticism, neither should it be rejected as being merely superficial or frivolous; like all distinctions which embody any degree of truth, it offers a point of view from which to look and compare, a starting-point for genuine investigation. (Berlin, 1996, p. 2)

Many of the ideas and categories in my elaboration of how a graphic calculator may mediate in the learning of mathematics were suggested by my reading of various accounts of instructional programs and/or experiments involving the graphic calculator. In particular, and as I previously mentioned, I found valuable Jones' (1993) differentiation between effects *while* working with the graphic calculator as compared with the effects *as a result* of working with the graphic calculator. Similarly, I found Pea's (1990, cited in Salomon) distinction between the amplification effects and/or the cognitive re-organisation effects of technology illuminating, particularly when reading the literature and setting up my study. Subsequently, I attempted to apply these notions to my own analysis of how a student used a graphic calculator (to be described shortly).

The amplification metaphor which is developed here is a more precise and specific version of the cultural amplification concept of Newman, Griffin and Cole (1989). This notion describes the existence of intellectual tools from the social world which assist in the learning of new material. According to the amplification metaphor, the graphic calculator amplifies the zone of proximal development (ZPD) by removing cumbersome and time-consuming tasks from this zone. The user thus has more space in his/her zone to perform conceptually demanding tasks with greater effectiveness and ease. Amplification is concerned with the speed and facility with which the learner operates *while* using the technology, rather than on qualitative changes which may happen as a result of using the technology. Thus, amplification can be observed in the short-term (the technology directly and immediately assists the student in solving the problem).

Those effects which may occur *as a consequence* of using the technology are termed cognitive re-organisation effects. I have interpreted these effects (within a Vygotskian framework) as a systemic change in the consciousness of the learner as a result of interaction with a new and alternate semiotic system. If a learner is able to use mathematical conceptions more meaningfully or even differently *as a result* of having used the graphic calculator, one can claim that the technology mediates by serving not only as a tool with which to think, but also as a tool which helps thinking to develop (Pea, cited in Salomon, 1990). Thus, cognitive reorganisation is concerned with long-term changes in the quality of learning.

I maintain that many of the benefits which practitioners and educational studies claim for the learner using a graphic calculator can be usefully categorised as amplification effects or as cognitive re-organisation effects. Unfortunately, a review of literature in these terms, although informative and illuminating, is beyond the scope of this article. Suffice it, then, to give a few examples of how these effects are *implicitly* claimed in the literature. (I am not claiming that there is rigid line between these two types of effects; in fact, it may well be that amplification is a precursor to cognitive re-organisation or that the two effects are inter-related and dependent on each other; all of this forebears further study).

Some implicit claims of cognitive re-organisation effects are that the graphic calculator assists in the learning of mathematics by providing visual representation of the basic calculus concepts (Hollister, 1993), and that the use of the graphic calculator empowers students to make connections between the algebraic and geometric representation of certain mathematical concepts (Ruthven, 1992; Waits and Demana, 1993).

Some examples of how the student may use the graphic calculator to amplify the ZPD are given by Dick (1996), who asserts that:

> there are many instances where we simply wish to generate a graph quickly for our inspection and consideration in the mathematics classroom. For these ends alone, the graphing calculator is an invaluable aid, allowing all the students to participate actively in producing their own examples and not just passively accepting those of the instructor.

Jones (1996) also implicitly refers to the amplifying effect of the graphic calculator when he discusses the ability of a graphics calculator to carry out a variety of algebraic processes, on behalf of the student, that would have required considerable mental effort if the learner had been working with pencil and paper only.

Both amplification and cognitive re-organisation effects are claimed by Dick (1992) when he asserts that students using graphic calculators perceived problem solving differently: they could now focus on the statement of the problem and analysis of solution rather than on the symbolic manipulation.

Grounding the theory

In 1995, I decided to set up a project within the first-year mathematics course at the South African University in which I teach, so as to ascertain how the graphic calculator might function in relation to a specific learner.

Before going into detail about the actual project, it would be useful to outline the process whereby the theory informed the type of data I collected and how this data, in turn, helped generate and fashion several theoretical notions. Thus, although I am presenting my research in a linear fashion, this does not actually mirror the to-and-fro process whereby I worked.

When I set up the project, I was excited by the possibility of using Vygotskian theory to interpret the relationship between the graphic calculator (a contemporary tool) and its sign systems (graphs, tables). Given the benefits generally claimed for the use of the graphic calculator in the literature, I naïvely expected that my interpretation would revolve around a distinction between amplifying and cognitive re-organisation effects (as explained above). Accordingly, I designed a task (to be described shortly) to highlight this distinction. Additionally, I expected students who had used a graphic calculator for the entire year to approach a mathematical problem noticeably differently from students who had not had access to this tool.

However, and as I will argue later, the limited context in which the graphic calculator was used (as a result of financial and practical considerations) may ultimately have mitigated against strong effects of the graphic calculator. Accordingly, I had to elaborate the theoretical explanation of my data by situating my study in its time, place and purpose, so that I could identify "ways in which mind reflects and constitutes a specific sociocultural setting" (Wertsch, 1990a, p. 71).

The empirical study

The project

As previously mentioned, I set up a graphic calculator project in a first year mathematics major course in 1995. Given that many of our students cannot afford to purchase a graphic calculator, and given that the mathematics department was in any case cautious about plunging headlong into the use of a new technology (in the South African context, the use of the graphic calculator is not commonplace), I was forced to design a graphic calculator project which limited the use of this tool. Twenty arbitrarily selected mathematics major students (out of about 400 students) were lent a graphic calculator for the year by the mathematics department. The rest of the mathematics major students did not have access to these calculators.

All mathematics major students wrote the same tests and examinations (in which the use of the graphic calculator was not allowed) and attended equivalent lectures on one of three different time slots. The course derived from a standard American first-year textbook (Larson *et al.*, 1994).

The 'graphic calculator group' was allocated their own tutorial slot during which I guided and encouraged them in the use of the graphic calculator. (The tutorial period was generally about 45 minutes per week and provided an opportunity during which students could consult with a lecturer about problems in the tutorial exercises.) Since the use of the graphic calculator was not allowed in examinations and tests, I felt morally obliged to limit the use of the graphic calculator to the *regular* tutorial problems from Larson which all first-year major students were expected to do. Thus, the students were encouraged to use the graphic calculator to *verify and support* analytic results rather than as a tool in its own right. The other students attended their own similar tutorial sessions (of course, without graphic calculators).

This limited use of the graphic calculator turned out to be fundamental to an interpretation of the relationship between graphic calculator and learner.

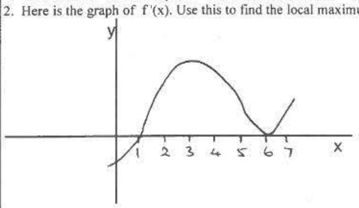
The data

Two types of data, in the form of answers to a set of mathematical problems (the task) and in the form of seven distinct mathematical interviews were collected in order to distinguish between the possible roles of the graphic calculator as a mediating sign in the ZPD. The purpose of the statistical analyses of the task data was to corroborate or inform the analysis and interpretation of the qualitative data from the interviews, rather than serve as a stand-alone measure.

The task

The task was a written mathematical test consisting of five mathematical problems, and was completed by sixteen graphic calculator students and forty-eight other students (who happened to be attending one of my classes) in September 1995. The questions are reproduced in Figure 1.

1. Find the area bounded by xe^{x^2} and the x-axis from x = -1 to x = 1.
2. Here is the graph of f '(x). Use this to find the local maximum/a and local minimum/a of f(x).



3. Find $\lim_{x \to 0^+} (\tan x)^x$
4. If $F = \pi\int_0^2 x^2 dx$, roughly sketch a region whose area is given by F.
5. Find the area enclosed between $y_1 = x^5 + x^4 - x - 1$ and $y_2 = x^4 - 4x^3 + 4x^2 - 1$.

Figure 1 Questions of the task

Broadly speaking, three of the questions from the task (1, 3, 5) can be solved more efficiently and easily if appropriate graphic calculator skills are employed; their purpose was to ascertain whether students with access to the graphic calculator show improved performance when using the tool when compared with non-graphic calculator students. Thus, these questions were designed to test whether the graphic calculator functioned as an *amplifier* of the ZPD.

The other two questions (Questions 2 and 4) could not be solved directly on the graphic calculator. Their solution required conceptual knowledge and a non-algorithmic solution. Their purpose was to determine whether the prolonged use of a different symbol system (as embodied in the graphic calculator) had resulted in a different quality of mental activity, evident even when not using the tool. In other words, had the graphic calculator functioned as a *cognitive re-organiser*?

The interviews

At the end of the academic year, I interviewed individually seven different students (four who had access to the graphic calculator; three who had not). In the interviews, the students were audio-taped while solving a mathematical problem out loud.

The students were selected so that for each student from the graphic calculator group whom I interviewed (students VT, RS, RP) a similar student without a graphic calculator was interviewed (JB, MK, RM). This 'matching' was in terms of lecture group, matric rating, socio-economic background (in terms of previous school) and gender and facilitated a qualitative comparison of the interviews. In addition, I interviewed one student, EP, who was not part of my graphic calculator group, but who had his own graphic calculator since school days.

The interview was designed so that it was possible to observe the way in which the student used different semiotic systems (representations) in order to solve a mathematical problem. The mathematical problem (see Figure 2) was a non-trivial question which required conceptual knowledge and which could be solved using algebraic and/or graphical representations.

EXERCISE

(You may use a graphic calculator for this exercise, if you wish).

Given $F(x) = \int_0^x (4t-t^2)dt.$

Let g(x) = F '(x).

a) Find g(x).

b) Represent or show F(x) and g(x) in any way you want on sets of axes.

c) Explain why F(x) = 0 for x = 0.

d) Explain why F(x) = 0 for x = 6.

e) Explain why F(x) < 0 for x = 7.

f) For what value of x (x ≥ 0) does the maximum value of F occur? Why?

g) Explain why F(x) > 0 for x = 2.

Figure 2 The interview problem

Ultimately, the student was required to link these different representations. The graphs that were possibly of interest were simple enough to be drawn by hand or with the use of the graphic calculator. Details of the interview protocol (which derived from the Russian psychologist, Luria) can be found in Berger (1997).

The actual mathematical problem is of a type frequently encountered in the literature of calculus reform, such as in the Ostebee and Zorn (1995) textbook.

Results

Qualitative data

The graphic calculator functioned primarily as an amplifier of the zone of proximal development for the individual learner in the interviews: on the whole, the graphic calculator students used the calculator to generate the graphs of F and g relatively effortlessly and quickly, thereby:

circumventing low-level limitations of human computational ability. (Perkins, cited in Salomon, 1990, p. 191)

Although I had also hoped to witness some cognitive re-organisation effects (perhaps the graphic calculator students would approach the problem in qualitatively different ways from the non-graphic-calculator students), this was mostly not the case. What primarily distinguished the graphic calculator students from the non-graphic-calculator students was the speed with which the student could access the appropriate graph when required to do so.

For example, RP used his graphic calculator as a straightforward amplifier of the ZPD. RP explicitly expressed his preference for algebra:

> you could have checked on your graph [...] or by using the table [...] [but] I just prefer to work it out [...] algebraically.

Nevertheless, he used his graphic calculator to set the scene for a discussion of the mathematics by first and foremost generating the (polynomial) graphs of F and g. He then went on to solve and discuss the different sections of the problem algebraically using the calculator-generated graphs for verification when required to present an alternate explanation. What was interesting in this interview was not so much that RP preferred an algebraic to a graphical mediation, but rather that despite his preferences he had the graphs easily available when he required support for his algebra. RP did not waste time or energy in hand-drawing the graphs; he could thus conserve his intellectual energies for the algebraic thinking which he seemed to prefer and he could then use the graphs for verification as needed.

VT was the only graphic calculator student who seemed to use her calculator in a manner which was qualitatively different from that possible with pencil and paper (and then only once). VT was struggling to find and explain where F attains a maximum by relating the different representations of F (as an area under g, or as a polynomial). So VT drew F as a polynomial on the graphic calculator. Looking at the polynomials F and g on the graphic calculator screen, VT suddenly 'saw' the link between the different graphical representations of F and g and their algebraic expressions:

> MB: So you are trying to find what now?
> VT: [looking at graphic calculator; graphs of $F(x)$ and $g(x)$] I'm trying to find. Oh, you know where it is? [excitedly] It is where, um, $g(x)$ cuts the graph, cuts the x-axis, so it will be at $x = 4$.
> MB: OK.
> VT: OK. I don't know how. *I just saw it with the calculator here* ... let's zoom out ... there ... it would be a maximum ... it would be a maximum value of $F(x)$ would be where $g(x)$ cuts the x-axis.

Such an episode may well have resulted in some sort of cognitive re-organisation for VT, thus illustrating how amplification effects may precede cognitive re-organisation effects.

The interviews with the non-graphic-calculator students stand in sharp contrast to the interviews with the graphic calculator students. Both JB and MK spent a lot of time, effort and intellectual energy hand-drawing and correcting the graphs of F and g. For both these students, these actions cluttered the zone of proximal development thereby leaving less space in which mathematical conceptions could develop. JB eventually managed to solve the problem using both algebra and hand-drawn graphs, but I suggest that her task would have been that much simpler had she not had to generate the graphs of F and g by hand. MK never really explained any of his reasoning in a satisfactory way and it remains unclear how much of the mathematics he understood. Although RM sketched the parabola fairly effortlessly, and was able to discuss the different aspects of the problem fairly easily, he never managed to draw the graph of F as a polynomial and so never adequately formed the link between F as a polynomial and the area representation. In this way, his mathematical experience was diminished and possibly an opportunity for cognitive re-organisation was lost.

In summary, there was only one concrete instant in all the interviews (VT's interview) in which it could be argued that the student was significantly using the graphic calculator as a tool for cognitive re-organisation rather than as a tool of amplification. Once the students (both graphic calculator and non-graphic calculator) had the necessary graphs available, they all approached the problem in a similar way. Other than the VT episode quoted above, no student appeared to use the graphs on the graphic calculator in any substantial way which might have been different from that of paper and pencil (other than to speed up the process of generating the graphs).

Quantitative data

Although there was little evidence of cognitive re-organisation effects in the interviews, I did hope that this effect would reveal itself in the statistical analyses of data from the task. After all, Questions 2 and 4 of the Task (see previous discussion) were designed to test whether the use of the graphic calculator over the year had resulted in long-term restructuring or re-organising of awareness. However, this was not to be.

Although the graphic calculator students did do substantially better than the non-graphic calculator students on questions 2 and 4 (an average score of 55% compared with 39% respectively), this difference in performance was not statistically significant. Given this statistical data together with the absence of qualitative data pointing to cognitive re-organisation effects, the interviews and the task provide insubstantial evidence of a cognitive re-organisation role for the graphic calculator in this course. In fact, I contend that the cognitive re-organisation effects of the graphic calculator were limited by the socio-cultural context of the study (discussed below).

Furthermore, if the use of the graphic calculator over the year had resulted in some sort of cognitive re-organisation for the graphic calculator students, one could also have expected students from the graphic calculator group to perform differently from those students in the non-graphic-calculator group in the their year mark over tests and exams (in which the use of the graphic calculator was not allowed).

For example, in terms of claims made by various practitioners and researchers, the graphic calculator students should have had a better and deeper understanding of basic calculus concepts and thus it would be reasonable to expect their year mark to differ in some measure from the non-graphic-calculator group. However, a statistical comparison over the year mark for tests shows no significant difference between the scores of the graphic calculator group (mean percentage: 61.6%) and the scores of the non-graphic-calculator group (mean percentage: 58.8%).

With reference to the statistical analysis of the data concerning amplification in the task (questions 1, 3 and 5), the graphic calculator group again did substantially better than the non-graphic-calculator group (a mean of 34.5% compared with 23.9% respectively) - but again these differences were not statistically significant. I attribuite this failure to achieve quantitatively significant results despite the existence of qualitatively significant differences in the interviews to the blunting effects of the institutional setting of this study (discussed below).

Interpretation and conclusion

As mentioned previously, the distinction between cognitive re-organisation and amplification was suggested to me by a reading of the literature. However, although my study provided ample qualitative evidence of an amplification role for the graphic calculator, it provided little qualitative evidence of a cognitive re-organisation role for this tool; also, the quantitative evidence was not statistically significant. Why was this?

I would like to suggest that it is necessary to interpret the evidence further in terms of the socio-cultural context in which the graphic calculator was used. Furthermore, I maintain that a shortcoming of much of the research about graphic calculators is precisely this failure to focus on the context of the study.

Both Gomez (1996) and Penglase and Arnold (1996) argue for the situating of graphic calculator research within its relevant context. Gomez focuses on the differences between the use of graphic calculator in developed and developing countries:

> The contribution of graphing calculators [...] depends on the stage of development in the society in which it is happening and on the conditions that are found in mathematics education in that society. It is for that reason that there will be important differences in the social, institutional and educational effects that the introduction of graphing calculators will have, and they will depend upon the level of development in the country that is carrying out such a curricular innovation.

Penglase and Arnold argue that:

> the [graphic calculator] itself [...] is meaningless in isolation from its use [...] Of more benefit then may be those studies which directly attempt to address the issues of graphic calculator use within particular learning environments. (p. 79)

In particular, I maintain that it is important to distinguish between research projects which use the graphic calculator as an add-on tool (as in my study) rather than as an integral part of the course and curriculum. If the graphic calculator is used for support and verification rather than as a tool in its own right (which is frequently the case in developing countries where technological and financial resources are scarce and curricula traditionally focus on symbolic manipulations), and if its use is not permitted in tests and examinations, this may substantially contribute to the type of relationship that learners can form with the graphic calculator.

Previous studies have shown that when the purpose of learning is socially unimportant, little learning and transfer of skills takes place. For example, Scribner and Cole (cited in Salomon and Globerson, 1987) failed to find transfer from the Vai literacy training to cognitive tasks. Apparently, Vai literates rarely needed or used their literacy skills other than for an occasional letter to a relative. Similarly, many studies on children's programming have failed to show measurable cognitive effects beyond some mastery of programming itself (Pea and Kurland, cited in Salomon, 1990). For internalisation to take place, it is not sufficient that a student is merely exposed to a new technology; rather, he/she needs to engage thoughtfully with the technology (Salomon, 1990). In order to interact in such a mindful way, he/she has to use the technology actively and consciously in a socially or educationally significant way. Vygotsky, in fact, regarded conscious reflection or intellectualisation as so fundamental to human thought processes that he deemed it a necessary condition for internalisation and mastery of an activity.

As stated, the data in my study provide little evidence of a cognitive re-organisation role for the graphic calculator. I contend that the reason for this was the use of this technology as an add-on tool in the course, and its corresponding lack of social or educational significance.

At this point, it is also useful to consider Wertsch's (1990b) concept of *privileging*. This notion contextualises the choice and use of semiotic mediator within the particular socio-cultural setting. It debunks the idea that, given a choice, a student consciously selects the semiotic mediators which are:

> somehow more basic, more 'cognitive,' or more important than others. (p. 111)

Rather, it is the social setting and values which may elevate one form of mental functioning over another and in this way privilege a particular form of mental operation such as algebraic or graphical reasoning. In fact, for the last two centuries algebra has been privileged as a semiotic system and it is still privileged as the most appropriate mode of mathematical discourse, particularly in developing countries. Of course, as more and more countries integrate graphic calculators and computer graphics into their curricula so this pattern of privileging will change

Thus, the perceived and in fact the actual status of the graphic calculator in any mathematics course profoundly effects the influence of this technology on the students. Certainly, a proper integration of this technology into the course and curriculum would significantly alter the traditional patterns of privileging and hence the potential relationship between the user and the technology.

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