NOT UNDERSTANDING ANDY: A METAPHORICAL ANALYSIS OF STUDENTS' RESISTANCE TO LEARNING

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"How the heck do you get that?"

"What the? There's no …"

The fourth-grade students complained and groaned as one student, Andy, presented his work. The students were discussing representations of different fractions. Rather than using the expected fractions of 1/4 or 1/3, Andy presented unfamiliar fractions such as 1/24, 1/96 and 1/768. As he explained these fractions, the other students expressed their skepticism and frustration. They were unsure about how he got his fractions and even questioned whether those fractions existed.

Students' resistance to learning is not a new or isolated event. Teachers frequently encounter moments in which students reject or fail to make sense of more advanced mathematical ideas, especially ideas articulated by peers. While there may be many reasons for students' resistance to learning, the major complaints from Andy's peers were that his mathematics did not make sense and that what he was doing was not possible or permissible. A discursive analysis of the metaphors used by the students makes it possible to see how these objections may arise as students' use of different metaphors results in different expectations for doing and talking about mathematics.

Conceptual metaphors

My use of metaphor in this article draws upon the notions of conceptual metaphor articulated by Lakoff and Johnson (1980) and Lakoff and Núñez (2000). These researchers suggest that much of our thinking is metaphorical and that metaphors structure our interpretations of and interactions with ideas, including mathematical ideas. According to their theory, we come to understand and structure an abstract idea (the target) by drawing upon the features of a familiar domain (the source). For example, when we talk about *getting into (or out of) trouble,* we use our familiarity with containers to structure the abstract domain of trouble (Johnson, 1987). We know that trouble is not an actual space that we enter or exit. However, we use our experiences with physical containers to help us conceptualize trouble as a *bounded entity* with consequences for those who are *in it.*

This theory of conceptual metaphors has been critiqued from multiple perspectives, with some arguing that metaphors cannot account for all understanding and others charging that the theory too rigidly declares plausible source domains (*e.g.,* Goldin, 2001; Howe, 2008; Madden, 2001; Schiralli & Sinclair, 2003). While keeping these important critiques in mind, I follow the lead of Williams and Wake (2007), who both acknowledge the limitations of conceptual metaphors and argue for their usefulness as a lens for making sense of mathematical understanding.

There are several features of conceptual metaphor that are important to the analysis in this article: the use of experience as a source domain, the recognition of metaphors in commonsense phrases, and the blending of two metaphors to create a new metaphor. Lakoff and Johnson (1980/2003) and Lakoff and Núñez (2000) argue that our physical experiences are an important source of structures for making sense of abstract ideas. One example of a physical experience that structures fractions is the notion of part-whole (Lakoff & Núñez, 2000). We have experiences taking objects and disassembling them into parts or taking parts and using them to construct an object. We use our experiences with parts and wholes to structure abstract ideas like days: we partition days into mornings, afternoons, evenings, and nights and we understand that together these parts constitute a day. We also draw upon part and whole to make sense of fractions: we partition wholes into equal size pieces which we call fractions. Sfard (1997) refers to this use of part–whole as the FRACTION-AS-PIECE metaphor.

Another aspect of conceptual metaphors is that they may sometimes become so commonplace that we forget their metaphorical origins. This makes it difficult to recognize that these "dead" metaphors (see Lakoff & Johnson, 1980, 1999) were once (and are still for some people) live and challenging to comprehend. For example, when we add 1/2 and 1/2, we sum fractions in the same way we sum whole numbers. As we do this, we draw upon the domain of number to help us make sense of the domain of fractions. In other words, we easily use the dead (to us) metaphor FRACTION AS NUMBER. However, for students just learning fractions, this metaphor is not at all dead or intuitive and may be hard to make sense of or use (Sfard, 1997).

One more point about conceptual metaphors is important to this article: conceptual metaphors can sometimes result from a blend of two source domains, resulting in what is called a conceptual blend (Lakoff & Núñez, 2000; Sfard, 1997). Rather than mapping a single source onto a target, two different domains are integrated and their combined and individual structures are used to make sense of the target. For example, when we think about fractions, we may use a conceptual blend of FRACTION AS NUMBER and FRACTION AS PIECE. The fractional amount, say 1/3, conjures images of pieces of a whole, perhaps a circle divided into three equal pieces. We can perform arithmetic operations on 1/3. For example, we could divide 1/3 by 2. We can easily translate the result into 1/6 and simultaneously imagine the consequences of the division on our visual image (perhaps a 1/3 piece has been partitioned into two pieces, each representing 1/6). As we coordinate the visual image with the numerical operation, we are blending the metaphors of FRACTION AS NUMBER and FRACTION AS PIECE.

The notion of conceptual metaphors provides a means for examining how a group of students thinking about the same idea (in this case, fractions) might, because they are drawing upon different source domains, have different metaphors and therefore different understandings of the target domain. What remains is to find a lens that will render different source domains visible and offer an explanation for challenges in communication.

Metaphor and discourse

Because metaphors are discursively constructed, it seems analytically productive to explore their presence and function by using a tool that focuses on discourse. As Sfard (1997) notes, the discursive structures of a metaphor's source domain guide the talk about the target domain. For example, when students use the metaphor FRACTION AS NUMBER, they talk about fractions in ways that are similar to how they talk about numbers. The utterance, "One-half plus one-fourth equals three-fourths," uses fraction words as things to be added together, just like other numbers. Careful attention to classroom discourse should suggest the presence of a metaphor along with its source and target domains.

The discursive analysis in this article borrows from the work of Ben-Yehuda, Lavy, Linchevski, and Sfard (2005). These authors describe how mathematical discourses can be differentiated by attending to word use, visual mediators, discursive routines, and endorsed narratives. I used these four discursive features to differentiate and describe metaphors in the classroom I studied. However, I found that one feature, endorsed narratives, was most useful in analyzing the disagreement between Andy and his peers. I will focus on that feature in this article.

Endorsed narratives are statements that participants in the discourse believe to be true (Ben-Yehuda *et al.*, 2005). Math facts and definitions are two examples: for individuals who are proficient at mathematical discourses, statements like "1/4 + 1/4 = 1/2" and "a square is a special case of a rectangle" are true and are thus endorsed narratives. However, for students who do not yet understand fractions or who see squares and rectangles as different objects, these two statements may not be seen as accurate and are thus unendorsed by the student.

By probing classroom discourse for patterns in discursive features, it is possible to differentiate discourses, to consider what conceptual metaphors might be underlying and structuring different discourse, and to conjecture about the relationship between those different metaphors and students' reactions to each others' ideas.

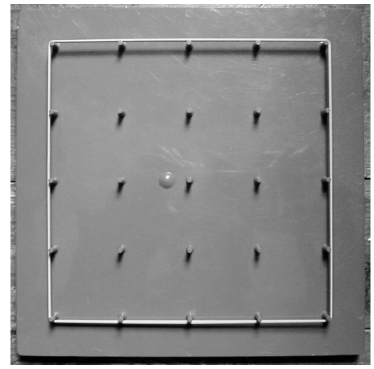

Figure 1. Plastic 16-square geoboard

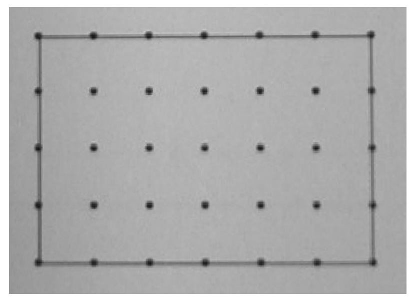

Figure 2. Paper 24-square geoboard

Background on the lessons and analysis

This article focuses on six lessons in this classroom (out of 23 total lessons in the fractions unit) that used geoboards. These devices were the predominant manipulative used in the unit (other manipulatives were only used for one or two days), so it seemed important to analyze the discourses connected to them. Also, my initial analysis of lessons in this unit revealed that members of the classroom talked about fractions in distinctive ways as they used the geoboards. These patterns seemed to contribute to the contentious interactions between Andy and his peers and thus seemed important to explore.

The students used two different size geoboards, which sometimes had consequences for the classroom talk around metaphors. One had four rows of four squares (16 squares in all), while the larger geoboard had four rows of six squares (24 squares). (See figs. 1 and 2.)

I was able to videotape 19 of the 23 fractions lessons including each of the six geoboards lessons. Each videotape was transcribed and analyzed using the four-part framework of Ben-Yehuda *et al.* (2005). I grouped moments where the students and the teacher consistently used similar words, mediators, routines, and endorsed narratives. I then determined what source domain was framing this talk and labeled those instances with the appropriate metaphor. By attending to patterns in discourse, I was able to limit my metaphorical analysis to talk that exhibited routines, rather than seeking a metaphor in every moment of talk.

Metaphors used across the classroom

Across the six geoboard lessons, there were 673 instances in which the discourse in the classroom used a particular metaphor. The metaphors FRACTION AS PIECE (146 uses, 21%) and FRACTION AS CONTAINER (428 uses, 64%) accounted for most (84%) of the metaphor use. (The next most frequently used metaphor was that of FRACTION AS NUMBER with 40 uses or 6%.) In the sections below, I will focus my analysis of classroom metaphors on these two metaphors not only because of their frequent use but also because these two metaphors seemed to be the basis for the students' disagreement with Andy's ideas.

Fraction as piece

The students and the teacher used the FRACTION-AS-PIECE metaphor on numerous (146) occasions. For example, in the excerpt below, the class discussed whether to use one-half or one-eighth to name the partitions created when a 16-square geoboard was divided into two partitions each containing eight squares.

Excerpt 1

Line	Speaker	Words and Gestures
1	Teacher:	Okay, Kurt, how could you explain [*i.e.,* why the partition isn't an eighth]?
2	Kurt:	Because it [the partition] has to be one of eight pieces [in order to be one eighth].
3	Teacher:	One-eighth means one of eight equal pieces. Do all pieces have to be the same?
4	Students:	Yes
5	Teacher:	Well, this has eight in it. [She points to the half of the geoboard, indicating the eight squares.]
6	Kurt:	It has to be one piece.
7	Student:	That's two pieces
8	Kurt:	It has to be eight pieces in all.
9	Student:	And that's only two pieces.
10	Teacher:	… So what fraction is it?
11	Students:	One-half.

On lines 2, 7, 8, 9 and 11, the students argued that partitions of the geoboard were named based on the number of pieces in the whole: because the board was divided into two pieces and not eight pieces, it was divided in halves and not eighths. These students' statements metaphorically connect the notion of the physical parts and wholes to fractional names.

This excerpt demonstrates the main *endorsed narrative* of the FRACTION-AS-PIECE discourse: the unit fraction is named based upon the number of equal pieces into which the whole is partitioned. For example, the fraction name one-eighth meant the whole had been divided into eight equal pieces. An important feature of this endorsed narrative is its general nature: rather than apply this narrative to a limited number of known fractions, students used it to evaluate any unit fraction. For example, when students were presented with the new fraction, 1/32, one student argued that this fraction was not possible because "There's not thirty-two pieces in the whole thing" (See Excerpt 4, line 41). This student rejected the new fraction based upon the endorsed narrative that the fraction name must match the number of pieces.

Fraction as container

The students in this classroom used a second metaphor that was more prevalent than the FRACTION-AS-PIECE metaphor. This second metaphor, FRACTION AS CONTAINER, used notions of containment and boundedness to structure talk about fractions. The notion of containers was implicit: the students and the teacher did not literally think of fractions as three-dimensional containers, nor they did use the word 'container'. Instead, they used notions of boundedness to differentiate the interior and the exterior of a space. The rubber bands (or, when the geoboard was represented on paper, the lines) created this boundary. Students then counted the squares in the interior. In this metaphor, fractions were containers that held a certain number of squares.

This metaphor was developed from the first day of work with the geoboards as the teacher directed the students to justify fraction names by referring to the number of squares in the fractional shape. For example, the students were dividing 16-square geoboards into eighths. The teacher asked for the class for their attention.

Excerpt 2

1	Teacher:	How many squares are inside an eighth? Dquan?
2	Dquan:	Two
3	Teacher:	So when you do different eighths, as many as you can think of, they all have to have two squares in them?…
6	Dquan:	Yes
7	Teacher:	Two go in an eighth.

The teacher's statements "squares inside an eighth" (line 1), "two squares in them" (line 3), and "two go in an eighth" (line 7) refer to an eighth in the same way one might refer to a box or container that holds or fits a certain number of squares.

These statements provided a discursive template for talking about fractions that the students adopted as they worked to identify and name fractions on geoboards. For example, the following excerpt occurred during the sixth lesson using geoboards. Students were working on the 24-square geoboard, dividing it into thirds, sixths, and twelfths. Allison and Tamika had finished when I asked Tamika to explain her work to me.

Excerpt 3

1	Tamika:	… You have to use the shapes like more than once, 'cause you gotta use the twos twice, the fours once and the eights twice …
2	Marcy:	Okay, so when you say you have to use the twos twice, what do you mean by that? …
3	Tamika:	[She looks at the back of her paper and then points to her work on the geoboard on the front of the paper.] You gotta use the twelfths, the one-twelfths twice.…
5	Tamika:	[Tamika continues to flip her paper over, looking at the back and then returning to the front each time she mentions a different fraction.] One-thirds, no the one-

twelfths twice, the one-thirds twice and the one-sixths twice, one-sixths once I mean.

6 Marcy: So what are you looking at on the back of your paper that's helping you figure that out?

7 [Tamika turns her paper to the back where the following is written:

1/3	8
1/6	4
1/12	2]

8 Allison: No, wait Tamika, we did it wrong. One, two spaces inside. It's supposed to be three, not two!

9 Tamika: [Tamika writes on the front of her paper as she talks.] One-third equals eight and one-twelfth equals two and one-sixth equals four.

10 Marcy: Okay… how does that help you figure out what these spaces are here?

11 Tamika: [Tamika points to the partitioned geoboard.] So see how much one-third, how big is it, so how many squares I need to use in it. So I was like first you can't use all of these once, so you use the one-thirds twice, the one-twelfths twice and the one-sixths once.

This excerpt is a particularly explicit example of the ways in which students connected fractions to containers. Tamika partitioned her geoboard by counting squares and creating shapes containing two, four, or eight squares. She used those numbers of squares because they were the equivalent of the fractional amounts she was supposed to use (1/3, 1/6, and 1/12). Her writing on the back of the paper (line 7) and then on the front of the paper (line 9) illustrates the connections she made between fractions and numbers of squares. In addition, both Tamika and Allison used language that indicated their thinking about FRACTIONS AS CONTAINERS. Allison talked about "spaces *inside*" (line 8, emphasis added) and Tamika explained how her writing (from lines 7 and 9) helped her figure out how big one-third was or "how many squares [she needs] to use *in* it" (line 11, emphasis added). By using "inside" and "in it", Allison and Tamika invoked a sense of containment related to fractions. Fractions were not just labeled areas, they were shapes that contained squares. In this instance (and others), Allison and Tamika were metaphorically thinking about FRACTIONS AS CONTAINERS that held specific numbers of squares.

The *endorsed narratives* for the FRACTION-AS-CONTAINER discourse linked the fraction name with the number of squares in the fraction shape. Tamika's list (lines 7 and 9) illustrated the endorsed narratives for the 24-square geoboard: 1/3 held 8 squares, 1/6 held 4 squares, and 1/12 held 2 squares. Unlike the FRACTION-AS-PIECE discourse, the students did not generalize this endorsed narrative to new fractions. Instead, the endorsed narratives for the FRACTION-AS-CONTAINER metaphor were limited to the fractions used by the students: one-half, one-fourth, one-eighth, one-third, one-sixth, and one-twelfth.

An important feature of the container metaphor was the focus on squares. Students only created fraction shapes that contained whole squares. Students would sometimes draw shapes that included triangles, but the shape had to have an even number of triangles so that the triangles could be counted in pairs as whole squares.

Both metaphors, FRACTION AS CONTAINER and FRACTION AS PIECE, were accepted and expected ways of talking about fractions across the classroom. Thus, when one student, Andy, began to talk about fractions in a different way, using a different metaphor, there was considerable disagreement about his ideas.

Andy's blended metaphor

During these lessons, Andy developed a conceptual blend metaphor that integrated FRACTION AS PIECE and FRACTION AS NUMBER. A conceptual blend integrates two domains and uses their individual and combined structures to make sense of the target domain (Lakoff & Núñez, 2000; Sfard, 1997). Andy demonstrated his conceptual blend as he explained to the class how he partitioned his 16-square geoboard into halves, fourths, eighths, sixteenths and thirty-seconds. This excerpt begins with Andy at the overhead. He first drew and labeled the one-half shape indicated in figure 3. He then drew the vertical line creating the vertical rectangle labeled 1/4. As he did this, he created the small triangle on the bottom row. A student commented on this.

Excerpt 4

1 Student: Oh, he ain't using the whole square

2 Teacher: [Talking to Andy] One, what's that?

3 Andy: One-fourth [Andy marks off the square in the upper left and writes 1/16 in it.]

4 Teacher: Okay. Tell me how you got one-slash-one-six right there. [She points to the 1/16.] I don't know how you got that. Tell me what you're thinking.

5 Student: Oh, yeah

6 Andy: Sixteen plus, er, um, eight plus eight is sixteen.

7 Teacher: Okay, so we know there's sixteen altogether.

…

9 Andy: And since there's sixteen squares, I can make one-sixteenth out of one square.

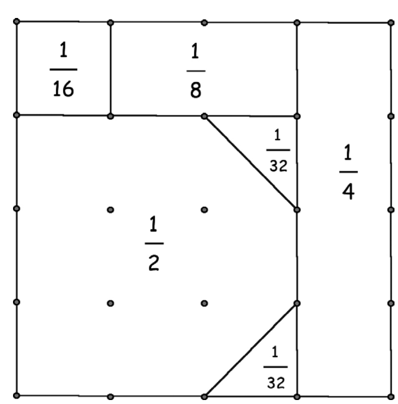

Figure 3. Andy's work for Excerpt 4

10	Teacher:	Okay
11	Andy:	And then I've got another
12	Teacher:	Did you guys understand that?
13	Students:	No!

Andy continued to draw as the teacher discussed one-sixteenth with the class. He marked off 2 squares and wrote 1/8 and then wrote 1/32 inside the two remaining triangles. A student asked about the two triangles labeled 1/32:

21	Student:	But what about them two little ones.
…		
23	Teacher:	So you're saying you can't do this. [She points at the 1/32.]
…		
25	Teacher:	[to Andy] Tell me how you got one-thirty-two.
26	Andy:	Because sixteen plus sixteen is thirty-two.
…		
28	Andy:	And I made it half of sixteen. [Andy points to the bottom 1/32 and to the 1/16.]
29	Teacher:	Okay. So since you made it half of sixteen. If you did the whole board in halves, that would make thirty-two?
30	Andy:	[Andy nods.] Thirty-two pieces.
…		
32	Students:	I don't think that's right... Awww. Ohhh.
…		
34	Andy:	It is right!
35	Teacher:	[to Andy] You're sure it's right?
36		[Andy nods. Other students are groaning and saying "uhhh." Lots of talking.]
37	Albert:	I don't think that's right.
38	Teacher:	Andy's sure.
39	Tamika:	It's not one thirty-two.
40	Teacher:	Tell me why you think that.
41	Student:	There's not thirty-two pieces in the whole thing.
42	Student:	There's not thirty-two squares.

This excerpt illustrates how Andy blended the metaphors of FRACTION AS PIECE and FRACTION AS NUMBER. Andy explained that the *number* 16 could be doubled (line 26) while, at the same time, the 1/16 *piece* could be halved (line 28). These two connected activities created a new fraction with a symbolic representation of 1/32 (resulting from FRACTION AS NUMBER) and a shape that was half of the previous shape (resulting from FRACTION AS PIECE).

Lines 6 and 9 offer additional evidence of Andy's blended metaphor. In line 6, "eight plus eight is sixteen" suggested that the sixteen in Andy's one-sixteenth came from doubling the number eight in the fraction one-eighth. In line 9, Andy connected his talk about doubling to the notion of pieces by using "and" followed by an explanation of how sixteen squares meant one-sixteenth. In this instance, Andy did not directly describe halving one-eighth to create a sixteenth, but his use of addition of eights followed by his explanation of pieces of sixteenths suggests that Andy easily connected the ideas of fractions as number and fractions as piece.

Andy's use of this blend was not isolated to these two examples in this one excerpt. He demonstrated the same connection of halving pieces and doubling denominators on two other days, describing how to use this process to generate fractions such as 1/48, 1/96, and eventually, 1/768.

The primary *endorsed narrative* for Andy's blended metaphor was a rule for generating fractions. Andy succinctly stated this rule, "[E]very time that you do times one of the number [times two], it's half as much as the bigger number." This narrative specifically connects doubling the denominator with halving the fractional piece in order to name and create a new fraction. While this narrative was limited in the possible *operations* relating fractions (*i.e.,* halving and doubling) it was not numerically restricted. In this way, Andy's blended metaphor was somewhat like the piece metaphor in that both could be generalized beyond the fractions familiar to the students.

While Andy was familiar with the FRACTION-AS-PIECE and FRACTION-AS-CONTAINER metaphors used across the classroom, he was also able to use his blended metaphor, which was more mathematically sophisticated than these other two metaphors. Unfortunately, Andy's blended metaphor was not well received by other students in the class.

Student reactions to Andy's metaphor

Andy presented his ideas to the class on two different instances, one of which is Excerpt 4. During these presentations, there were 36 instances in which the other students in the class articulated lack of understanding or disagreements with Andy's ideas. For example, in Excerpt 4, lines 32, 36, 37, 39, 41, and 42, students groaned or commented that they did not think Andy was right. The tone and quantity of negative feedback from other students was unprecedented in comparison with the 19 lessons I observed, leading me to label the students' activity as resistance to learning. In using the word *resistance*, I don't mean to imply that learning was not possible. Indeed, several days later, many of the students indicated an interest in learning "Andy's fractions". However, during these initial presentations of Andy's ideas, many students seemed unconvinced that Andy's ideas were worth learning.

While there may be multiple reasons why students objected to Andy's work, some of the students' comments suggested that their lack of understanding was related to differences in metaphor use. I identified three categories of metaphor conflict. First, students protested when Andy made fraction shapes that were smaller than one square. In Excerpt 4, students commented, "Oh, he ain't using the whole square" (line 1), "But what about them two little ones" (line 21), and "There's not 32 squares" (line 42). These comments seem to reflect a difference between the FRACTION-AS-CONTAINER metaphor and Andy's blended metaphor. In Andy's blended metaphor, fraction shapes could be any size as long as they were half the size of another fraction. In contrast, the CONTAINER metaphor had a minimum unit, the square. Pieces that were smaller than a square were only possible if they could be joined with other piece to make a square. Thus, a container smaller than a square contained nothing that could be counted and could therefore not be labeled with a fraction. Students may not have reasoned to this logical extreme, but their objections indicated that they felt Andy's shapes were too small to be used for fractions. It seems reasonable to tie this objection to the endorsed narratives for the CONTAINER metaphor.

Second, students objected that the geoboard did not have the number of pieces required by the denominator of Andy's fractions. For example, when Andy proposed 1/32, a student stated, "There's not 32 pieces in the whole thing" (Excerpt 4, line 41). Students voiced similar objections during Andy's second presentation of his fraction when he used 1/24 and 1/48. These objections seemed to reflect students' use of the endorsed narrative arising from the FRACTION-AS-PIECE metaphor, which required that the whole be divided into the number of pieces indicated by the denominator. Students could imagine this division when the fractions were familiar such as 1/4, 1/2, or even 1/12. However, students felt that it was not possible to divide the whole into the number of pieces required by Andy's large denominators.

Students may also have protested Andy's large denominators because of their use of the CONTAINER metaphor. As discussed above, the CONTAINER metaphor suggests a minimum size for a fraction shape. If students believed that fraction shapes had to contain whole numbers of squares, they might well struggle to see how a whole made of 16 squares could be partitioned into 32 pieces.

Finally, Andy suggested during his second presentation, "the bigger the bottom number is, the smaller [the shape] is." He made this statement while trying to explain how he had used 1/24 to make 1/48. Many students announced that they did not agree with this statement. Their disagreement may arise from the lack of opportunities for students to explore this idea. Neither the PIECE nor the CONTAINER metaphor encouraged students to explore questions about possible patterns involving fraction symbols and fraction size. In contrast, Andy's blended metaphor was based upon the relationship between the denominator and the size of the fraction. It seems quite reasonable for students who had not considered this relationship and whose metaphors provided rules about minimum sizes but not comparative sizes to reject the notion that there were relationships between fraction symbols and fraction size.

Intellectual flexibility

In reflecting upon this story of metaphors and metaphor use, two questions arise: how did Andy develop his more sophisticated metaphor and what would need to happen so that his peers might learn from him? I believe that these two questions are related through the notion of intellectual flexibility.

It is possible that Andy's new metaphor was the result of his flexibility in working with each metaphor and the features of each discourse. Rather than view the routines and "truths" as rigid, Andy was playful and willing to push on these "rules". I offer just one example of Andy's rule-breaking: the students had a routine for filling the board with fractions that required students to make containers until the board was filled with same-size containers. If any part of the board was not included in a container, the board needed to be rearranged. Early in the fractions unit, Andy violated this routine. He made shapes that were each one-eighth, but were positioned so that he could not fill the board with eighths. Andy was chastised by the teacher and students for this experiment. However, it may be that these smaller, isolated pieces led Andy to questions about how they might be named and perhaps led to his ideas about doubling and halving. While the evidence for this connection is scant and circumstantial, it is possible that 'bending' or 'breaking' discursive rules led Andy to his novel metaphor.

Indeed, breaking the rules was what Andy's classmates used to argue against his ideas: he was accused of not being right and of engaging in prohibited activity. However, his classmates' rigid adherence to routines and "truths" about fractions seemed to prevent them from exploring patterns and investigating other ways of thinking about the fractions. Perhaps if the students had focused more on variations and exploring the edges of routines and "true" statements, they would have been more open to Andy's ideas.

Conclusion

Metaphors seem to play an important role in the learning process. In addition to structuring student talk, they also provide rules and routines that students might either bend to develop new metaphors or use to evaluate (and potentially reject) others' ideas. While this case of Andy is (hopefully) an extreme example of students' unwillingness to learn new ideas, it calls our attention to the possibility that metaphors might play similar roles in learning situations that are less energized. For example, many students quietly struggle to make sense of fundamental mathematics. Perhaps these students are holding onto unproductive metaphors or too rigidly applying rules and not looking for patterns. This article suggests that discursive analysis of metaphors might provide an additional tool for making sense of and perhaps improving student mathematical learning.

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