SYMBOL SENSE BEHAVIOR IN DIGITAL ACTIVITIES

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Over the last twenty years the relationship between procedural skills and conceptual understanding has been widely debated. This relationship plays a central role in the 'Math Wars' discussions (Schoenfeld, 2004). An important issue in this debate is how students best acquire algebraic expertise: by practicing algorithms, or by focusing on reasoning and strategic problem solving activities. The former approach sees computational skills as a prerequisite for understanding mathematical concepts (US Department of Education, 2007). In the latter approach, the focus is on conceptual understanding (*ibid.*). Even if the idea is shared that both procedural skills and conceptual understanding are important, there are disagreements on their relationship and the priorities between the two.

The last decades can also be characterized by the advent of the use of technology in mathematics education. In its position statement the National Council of Teachers of Mathematics (2008) acknowledges the potential of ICT for learning. The advance of technology may strengthen the relevance of 'real understanding' in mathematics (Zorn, 2002). Still, there is a firm tradition of educational use of ICT for rote skill training, often referred to as 'drill and practice'; such a tradition is lacking for symbol sense skills. The issue at stake, therefore, is twofold: how can the development of procedural skills and symbol sense skills be reconciled, and how can the potential of ICT be exploited for this ambitious goal?

Procedural skills 'versus' conceptual understanding

The distinction between procedural skills and conceptual understanding is a highly researched field of interest. Kilpatrick, Swafford and Findell (2001) synthesize the research on this issue in the concept of *mathematical proficiency,* which comprises five strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. They define conceptual understanding as "the comprehension of mathematical concepts, operations, and relations" and procedural fluency as the "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately" (p. 116). Furthermore, "the five strands are interwoven and interdependent in the development of proficiency in mathematics" (ibid.).

Arcavi (1994, 2005) provided a breakthrough in the thinking on procedural skill and conceptual understanding in algebra. In 1994 he introduced the notion of *symbol sense,* which includes "an intuitive feel for when to call on symbols in the process of solving a problem, and conversely, when to

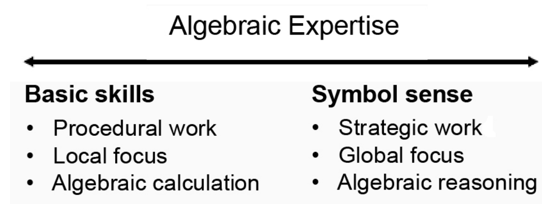


Figure 1. Algebraic expertise as a dimension (Drijvers & Kop, 2008)

abandon a symbolic treatment for better tools" (p. 25). Illustrating with examples, Arcavi described eight behaviors in which symbol sense manifests itself. The examples showed the intertwinement between procedural skills and conceptual understanding as complementary aspects of algebraic expertise. Both procedural skills and symbol sense need to be addressed in algebra education, as they are intimately related: understanding of concepts makes basic skills understandable, and basic skills can reinforce conceptual understanding (Arcavi, 2005).

In line with the work of Arcavi, Drijvers (2006) sees algebraic expertise as a dimension ranging from basic skills to symbol sense (see fig. 1). Basic skills involve procedural work with a local focus and emphasis on algebraic calculation, while symbol sense involves strategic work with a global focus and emphasis on algebraic reasoning.

One of the behaviors identified by Arcavi (1994) concerns flexible manipulation skills. It includes the versatile ability to manipulate expressions, not only technically but also with insight, so that the student is in control of the work and oversees the strategy. Two important, and interlinked, characteristics of flexible manipulations skill behavior are the gestalt view on algebraic expressions (Arcavi, 1994) and appropriate ways to deal with their visual salience (Kirshner & Awtry, 2004; Wenger, 1987).

A *gestalt view on algebraic expressions* involves the ability to consider an algebraic expression as a whole, to recognize its global characteristics, to 'read through' algebraic expressions and equations, and to foresee the effects of a manipulation strategy. Arcavi (1994) claimed that having a gestalt view on specific expressions makes symbol handling more efficient, and emphasizes that 'reading through' expressions can make the results more reasonable. A gestalt view on algebraic expressions is a prerequisite for carrying out basic procedural skills and for deciding which type of manipulation to perform.

Flexible manipulation skills also involve dealing with visual cues of algebraic expressions and equations, their socalled *visual salience.* Kirshner and Awtry (2004) provided a definition of visual salience and tabulated several expressions with greater and lesser visual salience, respectively. They claimed, "visually salient rules have a visual coherence that makes the left- and right-hand sides of the equation appear naturally related to one another" (p. 11). This coherence is strengthened by two properties of the equation under consideration: (i) repetition of elements across the equal sign and (ii) a visual reparsing of elements across the equal sign (Awtry & Kirshner, 1994). Visual reparsing "manifests itself as a dynamic visual displacement of elements" (p. 11). Take for example:

A $\left(\frac{x}{y}\right)\left(\frac{w}{z}\right) = \frac{xw}{yz}$, and

B $(x - y) + (w + z) = (x + w) - (y + z)$

In identity A, the right hand side seems to follow immediately from the left hand side. In identity B this is not so much the case. However, the two identities are structurally similar: replacing multiplication and division signs in A by addition and subtraction, respectively, yields identity B. In spite of this shared structure, identity A is more visually salient than B. Awtry and Kirshner concluded that many errors in algebra are not the result of conceptual misunderstanding, but of an over-reliance on visual salience. The way Awtry and Kirshner perceived visual salience seems to be closely related to our perception of gestalt.

In line with Wenger (1987), who describes salient patterns and salient symbols, in this study we distinguish two different types of visual salience: pattern salience and local salience. *Pattern salience* (PS) concerns the recognition of patterns in expressions and equations, and as such is close to the ideas of Awtry and Kirschner described above. If a pattern is recognized by the student by means of a gestalt view, it may recall a standard procedure and invite its application. *Local salience* (LS) concerns the salience of visual attractors such as exponents, square root signs and fractions. Whether it is good or bad to resist the local visual salience depends on the situation. Using our extended definition of visual salience, developing a feeling for when to resist or succumb to both pattern and local visual salience is part of the acquisition of a gestalt view and thus of algebraic expertise. In short, a gestalt view includes both pattern salience, involving the recognition of visual patterns, and local salience, involving the attraction by local algebraic symbols. In both cases, a gestalt view is needed to decide whether to resist or succumb to the salience. A gestalt view, therefore, includes the learner's strategic *decision* of what to do next. This is graphically depicted in figure 2. It should be noted that visual salience is not a matter of "yes" or "no": algebraic expressions may have different degrees of visual salience that also depend on the context and on the knowledge of the student.

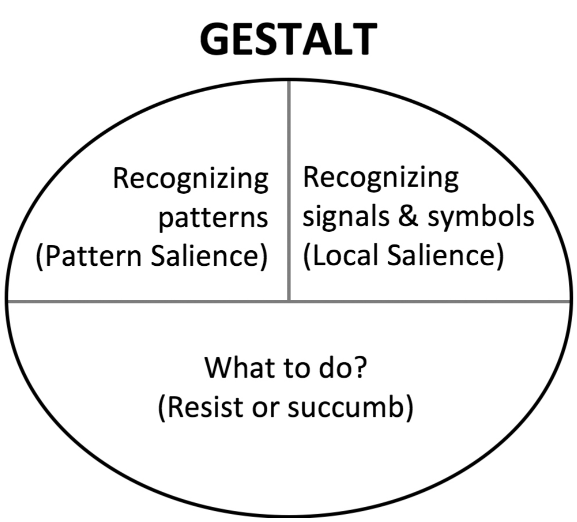


Figure 2. Gestalt view: pattern salience, local salience and strategic decision

The *resistance to visual salience* refers to the ability to resist visually salient properties of expressions, and their implicit invitation to carry out specific operations. For example, students who perceive brackets may be tempted to expand the expression, whereas this does not necessarily bring them closer to the desired result. Another example is the sensitivity to square root signs in an equation, that in the students' eyes 'beg to be squared', even if this may complicate the equation. The opposite can be said for exponents on both hand sides of an equation: here taking roots can or cannot be an efficient operation.

How might technology fit in?

Now how about the role of technology in the acquisition of algebraic expertise in the sense of both procedural skills and symbol sense, and with a focus on a gestalt view on, and the visual salience of, algebraic expressions? Educational use of ICT often consists of 'drill-and-practice' activities, and as such seems to focus on procedural skills rather than on conceptual understanding. However, research in the frame of instrumental and anthropological approaches shows that there is an interaction between the use of ICT tools and conceptual understanding (Artigue, 2002). This interaction is at the heart of instrumental genesis: the process of an artifact becoming an instrument. In this process both conceptual and technical knowledge play a role. To exploit ICT's potential for the development of algebraic expertise, it is crucial that students can reconcile conventional pen-and-paper techniques and ICT techniques (Kieran & Drijvers, 2006). Important characteristics of ICT tools that can be used for addressing both procedural skills and conceptual understanding are options for the registration of the student's solution process, and the possibility for the student to use different strategies through a stepwise approach. This enables the student to apply his or her own paper-and-pencil reasoning steps and strategies (Bokhove & Drijvers, 2010).

The opportunities that technology offers for the development of such algebraic expertise so far remain unexploited. Our goal, therefore, is to design and pilot digital activities that cater for the development of both procedural fluency and conceptual understanding. More specifically, we try to observe symbol sense behavior in digital activities. Do the concepts of symbol sense, gestalt view and pattern and local visual salience, described in a pre-digital era, help us in understanding what students do in a digital environment?

This is the main topic of this article. In answering this question we do not focus on the characteristics of the digital tool (Bokhove, in press; Bokhove & Drijvers, 2010); rather, we focus on the mathematical aspects.

Categories of items with symbol sense opportunities

To address the above issue, we first have to decide what we want to observe. We want to be able to see which strategic decisions students make while solving algebraic tasks in a digital environment. We want to know what salient characteristics – be they pattern salience or local salience – students resist or succumb to. This can only be done if the tasks offer symbol sense opportunities. For the task design, we used sources related to the transition from secondary to tertiary education, such as exit and entry examinations, remedial courses, textbooks and journals. Several suitable 'symbol sense type items' were identified and selected according to their focus on gestalt view and visual salience and supported by theoretical reflections from literature. The main criterion was that items would invite both procedural skills and symbol sense. This yielded a collection of thirty items, grouped into four categories, addressing both procedural skills and symbol sense, with an emphasis on the latter. We defined four categories of items, (1) on solving equations with common factors, (2) on covering up sub-expressions, (3) on resisting visual salience in powers of sub-expressions, and (4) items that involve recognizing 'hidden' factors. Even if these categories may seem quite specific, they share the overall characteristic of an intertwinement between local and global, procedural and strategic focus.

Category 1: Solving equations with common factors

Items in this category are equations with a common factor on the left and right-hand side, such as:

> Solve the equation:
>
> $(x^2 - 7x + 12) \cdot (8x - 11) = (x^2 - 7x + 12) \cdot (3x + 14)$

A symbol sense approach involves recognizing the common pattern – in this case the common quadratic factor. This is considered as a sign of pattern salience, involving the pattern $AB = AC$. After recognizing the pattern, students have to decide whether or not to expand the brackets. The decision not to expand the brackets is seen as a sign of gestalt view and of resistance to the pattern salience of the pairs of brackets on both sides of the equation. After deciding not to expand, students could be tempted to just cancel out the quadratic terms on both hand sides of the equation, relying on the rule $AB = AC \Rightarrow B = C$ and thereby forgetting that $A = 0$ also yields solutions. This could be the result of a wrong rewrite rule applied to a recognized pattern. A non-symbol sense approach would involve expanding both sides of the equation, in this case yielding a third order equation that cannot be solved by the average student.

Category 2: Covering up sub-expressions

In this category, sub-expressions need to be considered as algebraic entities that can be covered up without caring for their content. A well-known example is:

> Solve for v:
>
> $v \cdot \sqrt{u} = 1 + 2v \cdot \sqrt{1 + u}$

A symbol sense approach consists of noticing that the expressions under the square root signs are not important for the solution procedure (gestalt) and can be covered up. This requires a resistance to the local salient square root signs. In addition to this, a resistance is needed to the tendency to just isolate the v on the left hand side of the expression by dividing by the square root of u, which would leave a v on the right hand side. Thus, resistance to pattern salience is required as well, and not doing so shows a limited gestalt view.

A non-symbol sense approach might focus on the visually attractive square roots and try to get rid of them by squaring both sides. This would be a strategic error, and does not bring the solution any closer.

This equation is presented by Wenger (1987), who explained the issue as follows:

> If you can see your way past the morass of symbols and observe the equation #1 ($v \cdot \sqrt{u} = 2 + 2v \cdot \sqrt{1 + u}$, which is to be solved for v) is linear in v, the problem is essentially solved: an equation of the form $av = b + cv$, has a solution of the form $v = b/(a - c)$, if $a \neq c$, no matter how complicated the expressions a, b and c may be. Yet students consistently have great difficulty with such problems. (p. 219)

Recognizing the salient pattern of a linear function $AV = B + CV$ and what to do with it is deemed a gestalt view, as defined in our conceptual framework. Gravemeijer (1990) elaborates on the same example and emphasizes the importance of recognizing global characteristics of functions and equations.

Category 3: Resisting visual salience in powers of sub-expressions

This category is about recognizing when to expand expressions and when not. It contains equations with sub-expressions that just beg to be expanded because they are raised to a power:

> Solve the equation:
>
> $(x - 3)^2 + 4 = 40$

A symbol sense approach would include the recognition that after subtracting 4, both sides are squares, of $x - 3$ and $\sqrt{36}$, respectively. One should resist the temptation of expanding the left-hand side of the equation (resistance to pattern salience). Expanding the square to get rid of the brackets would be quite inefficient, and therefore is considered a non-symbol sense approach. Once the two squares of the pattern $A^2 = B^2$ are recognized, it is a sign of good gestalt view to succumb to the pattern salience by taking the square roots of both sides of the equation.

This item has several variants. For example, what if $(x - 3)$ is raised to the seventh power in the above example? The amount of work involved expanding this expression may stimulate students to look for alternative solutions.

Category 4: Recognizing 'hidden' factors

This category concerns the recognition of factors that are not immediately apparent (gestalt). An example is the following item adapted from Tempelaar (2007):

Rewrite $\dfrac{x^2 - x}{x^2 - 2x + 1}$

A symbol sense approach would involve recognizing a common factor in both numerator and denominator and noticing that both numerator and denominator can be factored by $(x - 1)$. A pattern $\frac{A \cdot B}{A \cdot C}$ is then recognized. A further manifestation of what to do next, a gestalt view, facilitates further simplification and may lead to an equation resembling those of the first category. Not recognizing these factors results in complex rewriting. A non-symbol sense approach would involve the manipulation of algebraic fractions without much result.

The design of a prototypical digital environment

The next step was to design a prototypical digital environment containing the items we defined. For this we carried out an inventory of digital tools for algebra and chose to use the Digital Mathematical Environment DME (Bokhove & Drijvers, 2010). Key features of DME that led to its choice are that it enables students to use stepwise strategies and that it stores these stepwise solution processes. It also offers different levels of feedback, allows for item randomization and has proved to be stable.

For the design we used the DME's authoring tool. Figure 3 shows some of its main features: the question text, the initial expression, the answer model, navigation, scoring and the possible use of randomized parameters. Figure 4 shows the implementation of an item from the first category. It is important to note that the algebraic steps are provided by the student, while the tool formats the steps, checks them algebraically and provides feedback. [1]

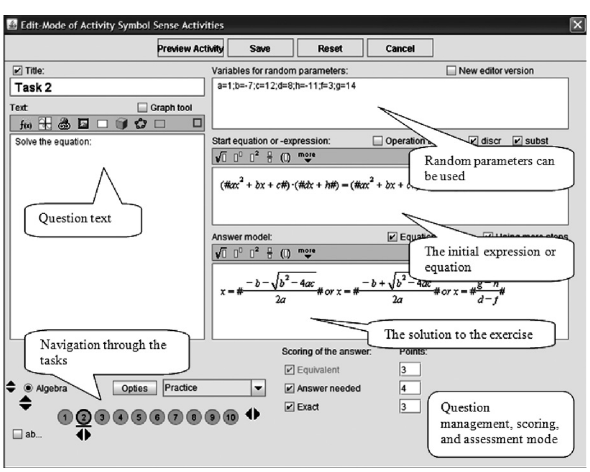


Figure 3. Authoring an item on the equation $(x^2 - 7x + 12) \cdot (8x - 11) = (x^2 - 7x + 12) \cdot (3x + 14)$

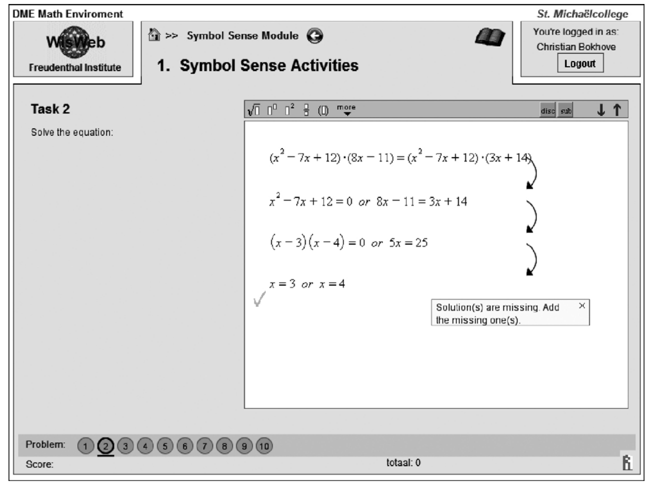


Figure 4. Example of student steps and feedback provided by the tool

Piloting through one-to-one sessions

To find out whether the concepts of symbol sense, gestalt view and pattern and local visual salience help us in understanding what students do in a digital environment, five one-to-one sessions with pre-university grade 12 students (17 year olds) were held. The students all had C+ grades for mathematics.

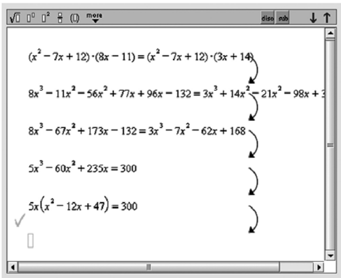
During the two-hour sessions, students worked through the digital activities. They were asked to think aloud while working. If a student was not able to complete (part of) a task, the observer asked what information would help in proceeding. On occasions where student used wrong strategies or made specific procedural choices, the observer asked the student what he or she was thinking. This informed possible feedback for a future revision of the prototype. After completing the session, the observer and the student went through the student's work and reflected on the solutions, discussing the student's arguments and alternative solution paths.

Data consisted of audio and video registrations and computer screen recordings. Data analysis focused on the types of behaviors shown by students while working with the digital activities, and in particular on signs of (a lack of) symbol sense, and was carried out with software for qualitative data analysis. One first round of analysis concerned students' technical behavior when performing algebraic activities: factoring expressions, rewriting expressions, aggregating terms, expanding expressions and canceling terms. A second round of analysis concerned the identification of gestalt and/or visual salience features in the students' behaviors. [2]

We now summarize the findings of the one-to-one sessions for each of the item categories. Per category one typical example of student behavior concerning gestalt and/or visual salience is given, as well as an overall description of the observed behaviors. We provide a rough time indication Δt in minutes per step, the technique used and comments on behavior related to gestalt view and visual salience.

Student behavior on category 1 items

Figure 5 shows the work of one of the students, Martin. Martin did not recognize the common factors on the left- and right-hand side. In the first step he expanded the expressions



Δt	Technique	Interpretation
3	Expand	The student does not recognize the common factors and expands to remove the brackets
2	Aggregate common Terms	
3	Rewrite	
3	Factor	The student recognizes the common factor $5x$
		Student realizes he cannot solve a third degree equation, and gives up.

Figure 5. Student solving an equation containing common factors

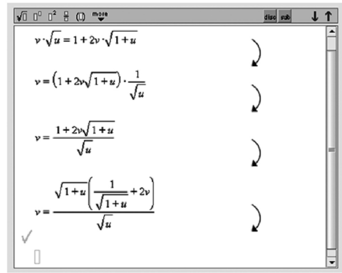
on both sides, a strategy that he incorrectly described as "always works". His inability to notice the common factor, and the pattern salience of the pairs of brackets, lead to his expanding strategy, a strategy which he used successfully – though not efficiently – in the previous task. In the second and third step Martin looked for terms that could be aggregated and rewrote the expression in the form 'expression = number'. Next he tried to factor the left-hand side. Although he showed good rewriting skills, and was even able to factor the expression later on, he gave up eventually. Martin realized that he could not solve a third degree equation. His approach does not reflect a gestalt view on the initial equation.

In general, student behavior on this task and similar ones in this category showed that both too much routine and a lack of self-confidence play an important role in obstructing gestalt. For example, student Laura solved several equations correctly, but always worked towards the Quadratic Formula. She also solved one equation correctly with a symbol sense strategy, but when confronted with a similar equation with fractional terms, she was reluctant to solve it as she immediately stated she "was not skilled enough". Only later did she recognize that, although the equation looked different, a similar technique could be used. Ideally a student would recognize the zero product theorem here. Another solution involved 'just' canceling out the common factors. As described in the category descriptions this indicates that on the one hand there is gestalt and resistance to pattern salience ("I'm not expanding both hand sides of the equation"). On the other hand, however, students also succumb to an incorrect pattern salience, a buggy rule, of $AB = AC \Rightarrow B = C$ by just canceling out the common factors.

From this category we conclude that a gestalt view, and the observation of the salience of the common factor pattern in particular, is not evident for many students. Even skilled students show a lack of gestalt view on encountering this type of task in a digital environment.

Student behavior on category 2 items

Figure 6 shows Barbara's digital work on equation by Wenger (1987). Barbara was instantly alerted by the square root signs, knowing that squaring these would not bring a solution any closer. Thus, she resisted the local salience of the square root signs. She was triggered by the task to write the expression in the form $v = \ldots$ and first divided by the symbol in front of the v. This corresponds with Wenger's observed strategy: "Divide the equation by the coefficient of



Δt	Technique	Interpretation
0 3	Rewrite	The student isolates v by dividing by $\sqrt{u}$
4	Rewrite	
5	Factor	The student realizes $2v$ poses a problem and tries to lose this term by factoring
9		The student tries a next step, backtracks, tries again and finally gives up

Figure 6. Student behavior on the Wenger equation (Wenger, 1987)

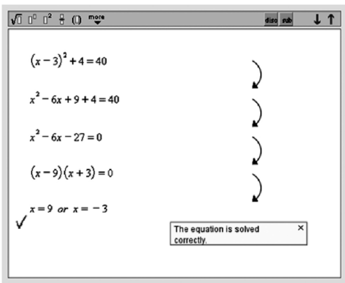
one of the occurrences of v in the given equation" (p. 230). This can be seen as succumbing to pattern salience. Asking for an expression in the form v = is directly transferred to the expression, and the quickest path to a solution in that form is dividing by $\sqrt{u}$. Barbara used one more step to rewrite the right-hand side as one fraction. She then took some time and circled numerous times round the term $2v$ exclaiming, "I want to get rid of this term". She then started to rewrite the numerator, stating: "I want to simplify the numerator. I think this helps" and "I often do this to create a sort of hunch. That I look at the exercise in a different way, as to see what can be better or must be done". She used her procedural skill to rewrite terms hoping that this might provide insight into the correct solution path. After backtracking, she tried another approach, but again ended up with a term $2v$ on the right-hand side. She then gave up.

In general, the students started with similar steps as Barbara did, focusing only on the v on the left-hand side of the equation. Some gave up because of circularity: "… the process of symbolic manipulation which results in an obvious or tautological identity, which is uninformative and unproductive" (Arcavi, 1994, p. 29). Two of the students backtracked after unsuccessful attempts and seemed to have a better idea what to do, finally ending up completing the task correctly. This was facilitated by the fact that the tool provided feedback on the correct or incorrect nature of an answer. This can be seen in figure 4, where the system responds with the comment that solutions are missing. Other versions of this type of task, presented right after this one, but with different variables, were recognized by most students. Remarkably, the students with the higher marks for calculus saw some of these tasks as completely new ones. These students solved them correctly, but in a very inefficient way. Apparently, showing a high procedural skill mastery does not necessarily imply that a student sees the general in the particular.

From this category we conclude that in the digital environment students show the same specific behavior when covering up irrelevant sub-expressions as Wenger reported earlier: students show resistance to local salience but fall victim of pattern salience. The chosen actions by the students reveal a lack of gestalt view.

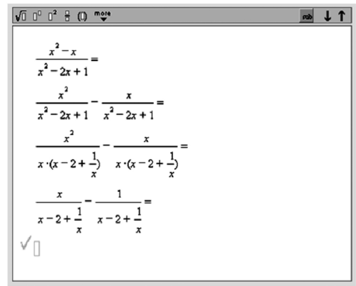
Student behavior on category 3 items

Figure 7 shows the work by Laura on a category 3 item. Laura did not recognize 36 as a square, and expanded the left-hand side. The diagonal juxtaposition, as described by Kirshner (1989), was too strong to withstand: the square



Δt	Technique	Interpretation
1	Expand	The student cannot resist the temptation of expanding the expression on the left hand side
1	Rewrite	
1	Factor	The student recognizes the expression can be factored
		The equation is solved correctly.

Figure 7. Student not resisting visual salience



Δt	Technique	Interpretation
3	Rewrite as a sum of fractions.	The student does not recognize the common factor and applies knowledge of the sum of fractions
2	Factor the denominator	The student expects to be able to cancel out the factor x
1	Cancel	
1		The student reflects on the task and backtracks to the initial assignment

Figure 8. Student not recognizing a hidden common factor

must be eliminated and this was done by expanding the term (non-resistance to pattern salience). Laura preferred to use her standard procedure for quadratic equations: to first rewrite as a quadratic equation equaling zero. Then she factored the expression on the left-hand side. Laura stated that this was possible because of the "nice numbers", which can be seen as a gestalt view. Otherwise she would have used the Quadratic Formula that "always works". This process finally yielded the correct solutions. She could have reached this solution more efficiently if she had recognized 36 as a square, and then had noticed that both sides of the equations could be considered as squares – that is, she would have observed the pattern $A^2 = B^2$. In this case her standard procedure obstructed any thoughts on alternate strategies.

In general, students *did* recognize both sides as squares. In contrast, the previous task involved the equation $x^2 - 6x + 9 = 36$, in fact the same equation with expanded left-hand side. It was remarkable that no student noticed these tasks were similar. From this category we conclude that lack of gestalt view on the initial equation, and lack of resistance to pattern salience, obstructs students thinking about alternate strategies, as is the case in a pen-and-paper setting as well.

Student behavior on category 4 items

Figure 8 shows Barbara's work on a category 4 task. Barbara instantly started rewriting, applying her knowledge of fractions. Instead of recognizing a common factor in both numerator and denominator – the pattern $\frac{A \cdot B}{A \cdot C}$ – she started with what she did best: rewrite the expression as a sum of fractions – the pattern rule $\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$. We see this as succumbing to a weak form of pattern salience and a lack of gestalt view. Next she factored the denominator and canceled out x in both terms. After several steps she noticed that the expression was becoming increasingly complex. The tick symbol denoted that the algebraic operations so far were correct. This, however, did not bring her to a more simplified expression. While carrying out these operations, Barbara became aware of the fact that $x - 1$ played an important role in both numerator and denominator. She then backtracked, rewrote the initial expression with $x - 1$ as factors and canceled them out.

In general, students showed trial-and-error behavior on this item. In some cases, this method seemed to provide the student with global insight into the expression. From this category we conclude that these students have difficulties in recognizing common factors in nominator and denominator (lack of gestalt); however, the tool offers opportunities for a trial-and-error approach, which can provide insight into these factors.

Conclusion and discussion

The issue we wanted to address in this article is whether the notions of symbol sense, gestalt view and visual salience, described in a pre-digital era, help us in understanding what students do in a digital environment. The design process and the one-to-one pilot sessions suggest that these concepts remain extremely relevant when deploying digital activities. The observations show that students using a digital environment exhibit both symbol sense behaviors and behavior lacking it. The notions of gestalt view and visual salience are helpful in analyzing student work. Although students work in a digital environment instead of with paper and pencil, these results are in line with past findings in traditional pen-and-paper settings (Arcavi, 1994; Wenger, 1987).

While solving algebraic tasks in the digital environment, the students can use any strategy, and thus can show sensitivity towards gestalt and visual salience aspects, and further develop such sensibility. The tool seems to facilitate this development through its mathematical interface and feedback opportunities, which would be more difficult to offer in a paper-and-pencil environment.

The exemplary tasks also point out that observing symbol sense is not a straightforward affair. It often is quite hard to recognize whether students are relying on standard algebraic procedures or are actually showing insight into the equation of expression, in line with the gestalt view or visual salience notion. Using standard procedures at least implies that a student recognizes the form of an expression. Recognizing patterns, and subsequently deciding what action to take, reflects a gestalt view. However, (over-)reliance on standard procedures can also be seen as a matter of 'succumbing' to routine patterns: when a student encounters an expression with brackets he wants them to be eliminated. Extending the concept of visual salience to patterns provided by standard routines students already know could perhaps relieve the tension between the application of standard routines and succumbing to salient patterns.

Are we suggesting that digital tools are the panacea for algebra education? Things are not as simple as that. Crucial to the issue of how to design such activities is of course appropriate content – that is items inviting symbol sense, as proposed by the designed categorization. If the tasks are not appropriate, the intended learning will not happen. The potential is in the combination of task design and digital implementation. If the tasks do invite for adequate procedural techniques and appropriate theoretical thinking, a powerful environment is designed. The Task-Technique-Theory model

(Chevallard, 1999; Kieran & Drijvers, 2006; Lagrange, 2000) may help designers to keep this aspect in mind.

The digital environment itself is a next crucial factor. High demands are put upon the digital tool in use. Students can get stuck by limitations of the technology. This being said, the potential added value of technology is promising: compared to carrying out the tasks with paper and pencil, we now have opportunities for different levels feedback and correction, for construction and exploration room for students, private and 'endless' practice and room for multiple stepwise strategies. With these conditions, the student is not restricted to strategies proposed by the digital tool itself, but can make his or her own correct or incorrect reasoning steps.

These conclusions suggest some guidelines for further research and development. Three issues for future development emerge: the sequencing of tasks, the extension of feedback, and scaffolding. First, future development should involve the design of *outlined sequences of tasks,* which appeal to symbol sense, and range from 'solvable with procedural skills' through 'inviting symbol sense' to 'only solvable with symbol sense insight'. Second, cues for developing gestalt view and the ability to deal with visual salience could be provided by relevant *feedback*. This issue asks for further elaboration. Feedback needs to be designed in more detail, concerning both the amount and the type of feedback (Hattie & Timperley, 2007). This also includes timing issues. As we saw students just starting a task without taking the time to actually think about it, it might be a good idea to include a cue for first scrutinizing the item carefully. When addressing feedback we can build on research by Nicaud (2004) and Sangwin (2008). Third, it might be worthwhile to build *scaffolding* into the sequence of activities, through initial activities that are structured and provide much feedback, that are then followed by items that gradually offer less structure and feedback. Support for this idea of formative scenarios (Bokhove, 2008) can be found in the notion of fading (Renkl, Atkinson, Maier, & Staley, 2002). It is in the line of these three issues that we plan to continue our research.

We do not pretend that the final word in the debate on procedural skills and symbol sense skills has been said. We do believe, however, that an optimal educational strategy is to focus on both simultaneously, and that technology may provide appropriate environments for this.

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Notes

[1] An English version of the prototype can be found at http://www.fi.uu.nl/dwo/en/. For storage of the results, registration is required, but one can also enter as a guest user.

[2] Data is available through http://www.fi.un.nl/~christianb.

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