Mathematics Teaching: What is It?

BARBARA JAWORSKI

Mathematics teaching in the classroom involves the creation of opportunity for children to learn mathematics. Elements of this include:

providing a supportive learning environment

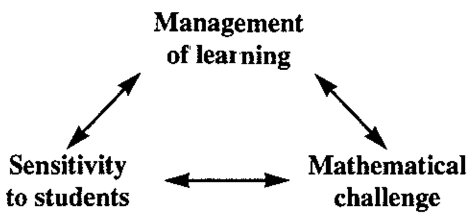
offering appropriate mathematical challenge;

nurturing processes and strategies which foster learning.

As a result of extensive observation of mathematics classrooms, I should like to offer *the teaching triad*, a synthesis of the three elements above, as a means of describing mathematics teaching which I find pervasive and powerful. Elements of it appear throughout most of the teaching situations which I have analysed, and it seems to illuminate and clarify the teaching process. The classroom observations to which I shall refer, were made during a study of investigative approaches to the teaching of mathematics, and this in its turn was related to a constructivist view of knowledge and learning. Thus, after first introducing the teaching triad and relating it to classroom situations in which I see aspects of it to be manifested, I shall show how it might also be seen as a means of linking a constructivist philosophy with the classroom teaching of mathematics.

The teaching triad

The essence of mathematics teaching may be seen to lie in three domains: the management of learning (ML); sensitivity to students (SS) and mathematical challenge (MC). These are distinct in theory, but rarely manifested singly in practice. Thus in any teaching situation, some elements of each domain are likely to be founds, and they are likely to be related in some way.



The teaching triad

First, briefly, what are they?

Management of learning (ML) concerns the creation of a learning environment. This includes classroom organisation and curriculum decisions. However, crucially it involves the establishing of ways of working and classroom values and expectations. Sensitivity to students (SS) involves the developing both of a knowledge of students, their individual characteristics and needs, and of an *approach to working* with students, consistent with these needs. It influences relationships and classroom ethos. Mathematical challenge (MC) involves stimulating mathematical thought and enquiry, and motivating students to become engaged in mathematical thinking. It influences the designing of activities and the style in which they are presented.

However, this is a theoretical description of what was initially a synthesis resulting from analysis of the practice, and what I want to claim is that this triad forms a powerful tool for making sense of the practice of teaching mathematics. I shall therefore move into instances of practice, to provide manifestations of the teaching triad in situations from lessons which I have observed.

The strange billiard table

The billiard table in the illustration below is a little odd. It only has four pockets and the base is divided into squares. The rules of the game are a little odd too. Only one ball is used and it is always struck from the same corner at 45° to the sides (The ball always rebounds at 45° to the sides.)

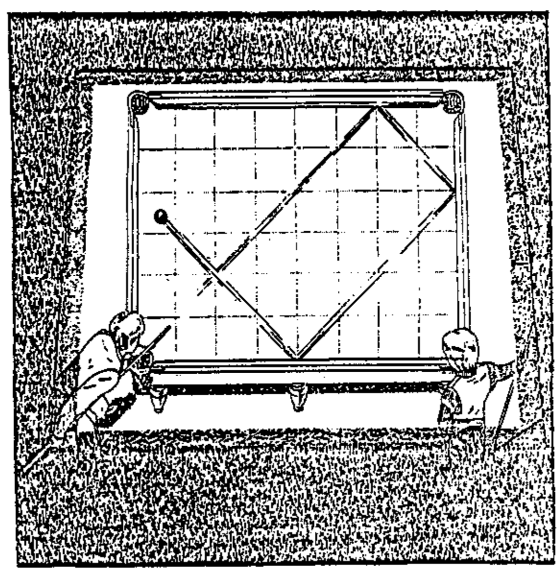


Figure 1

Management of learning

A number of manifestations arose in Situation 1, which concerns a teacher, Mike, and his Year 9, age 14, mixed-ability class.

For the Learning of Mathematics **12**, 1 (February 1992)
FLM Publishing Association, White Rock, British Columbia, Canada

Situation 1: "Introduction to Billiards"

At the beginning of a lesson, the class were given a sheet of paper on which 'The strange billiard table' was introduced. [See Figure 1] Mike asked them to read the sheet, and then said:

1 "Run through, in your mind, what happens — silently — *don't* put hands up yet"
2 "Everyone got something?"
and when there were nods, he asked for their contributions. After a number of contributions had been made, he asked,
3 "Anyone going to say anything different?"
After this initial discussion of what the strange billiard table was about and what sorts of aspects students might explore, he set a task:
4 "In groups, decide on a different thing to try, and ask: 'What happens?'."
5 "While you're doing it. *what am I going to ask you to do...?*"

There was a pause between the beginning and the end of these words. He started off giving an instruction, seemed to think better of it, and instead asked the class what instruction he had been about to give. One response from the class was, "Keep quiet", which he acknowledged with a nod, but other hands were up and he took another response which was, "Ask questions". His reply was "YES!" Other hands went down. It seemed to me that others had been about to offer this response too

In (1) Mike asked students to consider the sheet before going further—to enter mentally into what was contained in it. The instruction seemed to carry more than just *what* they should do. It seemed to emphasise both the mental process, cued by the phrase "in your mind", and doing it yourself, "silently", "Don't put hands up yet". In this it seemed to* convey a philosophy for working, to emphasise the thinking process. With the question, "Everyone got something?" (2), he not only ascertained that the class were ready, he also indicated that each person was supposed to have achieved something during the thinking time. Now the hands went up and he invited students to contribute their ideas, after which he checked again, (3), "Anyone going to say anything different?" This offered opportunity for further contribution, but acknowledged that there may be people who had comments similar to what had been offered and who thus had done their thinking, but had nothing to add. I saw it as *respecting* the students' involvement—perhaps implicitly saying, "I know you *all* had something to contribute, but you may feel that someone else has said adequately what you would have said, and so are not attempting to repeat that." It may also have included implicitly, "If you did *not* have any thoughts to contribute, I won't embarrass you by asking you directly, but it's worth realising that I hoped you *would* have something." I saw all of this as the establishing of ways of working and of classroom values and expectations, and thus offering manifestations of ML.

In many of Mike's lessons, students worked in groups, mostly of their own choosing. It was a regular feature of these lessons that, after introducing an activity to the whole class, Mike then asked them to work in their groups on some aspect of the activity. In this case, the sheet of paper had described a scenario. As a result of student contributions, and Mike's comments on them, possible questions to explore had been suggested. He then set them a task (4)—told them what he wanted them to *do*—in groups, to try different examples of what they had found on the paper. He then started to say (5): "While you are doing it *(I want you to....)*". The words in brackets were never uttered. Instead, he asked them: "What am I going to ask you to do?" This seemed to be blatantly, "Guess what's in my mind", but it appeared that most of the class knew the answer, "*(You're going to ask us to)* Ask questions!"

As *I* hadn't known what it was that he wanted them to do, I was very struck by this. It seemed that a part of his classroom rubric was that the students should ask their own questions. He acknowledged later that he was always asking them to ask questions, hence they knew that *asking their own questions* was what he expected of them, and knew what he wanted without his having to spell it out. There seemed, here, to be an advanced form of ML in action. It involved recognition by the teacher of a valued aspect of working mathematically, *asking their own questions,* communication of this to the students, recognition of it by the students, its becoming a part of their way of working, and recognition by the students of it being something which the teacher expected of them and which they knew they should do without it being spelled out each time. Perhaps the ultimate stage would be when the teacher saw no need even to refer to it, because he could be sure that it would happen as a natural part of the class's working.

Another teacher, Clare, organised a series of lessons with her Year 10, mixed-ability class on fractions. Situation 2 includes a number of statements which the teacher made during two of these lessons.

Situation 2: "Fractions"

Statements from the teacher, Clare, in the course of two lessons on fractions and their decimal representations.
1 Can we have a "handsdown" think. I did 1/2. I want *you* to *think* what *you* might do next
2 Anyone who's ahead of this, try to think how to explain repetition in 1/7
3 While you're doing this, with another bit of your brain, do what Virginia did last week—look for other recurring patterns
4 All groups, pool what you've found—think of what questions to ask next.
5 In about three minutes, I want some feedback from you—just think what you're going to say.

Statement (1) arose as part of an episode in which, trying to link 1/2 an 0.5, Clare had asked a student, Katy, "What do you get if you divide one by two? Katy had replied *"Two."*

* I am making interpretations and judgements continually as I comment on particular situations. To keep saying "it seemed to me" interrupts and is unnecessarily repetitive; especially since everything I write is of something *as it seems to me*. It is up to readers to reflect on what I have written and to consider this against what is quoted and what interpretation their own experience leads them to make.

When Clare then asked her to key into her calculator 1÷2, Katy said "nought point five". Clare next said to her, "If you have one thing shared between two people, how much does each get?" Katy looked blank. Clare wrote 0.5 alongside 1/2 on the board, said "Can we have a "handsdown" think. I did 1/2. I want *you* to *think* what *you* might do next." (1) to the class, and then went over to talk with Katy.

I saw this as being a manifestation of a complex set of reasoning on the part of the teacher. Specific to ML was, first of all, a ""handsdown" think". This was a form of words used frequently by Clare to emphasise that she wanted them to think about something, but without the instantaneous hand-waving that might occur with quick superficial thinking. Then she indicated *what* she wanted them to think about. I interpret this as follows: "You have seen me do something to 1/2. This was just an example. What *else* might we do this to? How? What might we get?" I feel that the statement points towards a management of the learning situation, indicating to the students what she would expect of them at this instant—valuing their considered thinking, while giving a pointer to what to think about.

However, it seems that the motivation for this instruction was to give Clare space to go and talk with Katy who seemed to be having difficulty with the notion of fraction as an operation of division and as parts of a whole. Thus this overtly embodies management of the classroom—keeping the class productively occupied while allowing the teacher to give individual attention where and when it was required.

Finally the statement, and the teacher's associated action, embodies aspects of my other two major categories—being sensitive to individual students' needs, and offering mathematical challenge. In the first case, Katy seemed to need individual support, and Clare wanted the class to think themselves about the link between the fraction and the division operation, and saw the opportunity to offer this challenge. Thus, although I offer statement (1) above as a (multiple) manifestation of ML, it also carries with it elements of the other two categories, which I shall subsequently consider.

Statement (2) arose when Clare was working with the whole class on dividing 1.0000 by 7. She realised that, whereas some students needed more time with this, others were ready to move on—and could themselves work on an explanation for the repeating pattern in the decimal representation of 1/7—i.e. 0.142857142857142 ... Thus, again, she managed the classroom, catering to the needs of two distinct sets of students, and encouraging them to decide which group they wanted to join. They could continue to take part with the "whole" class in working on the division of 1 by 7, or they could opt out of this to consider the repeating patterns in 1/7 as a decimal. She did not instruct anyone as to which activity they should participate in. This seemed to indicate that she respected their willingness and ability to choose wisely, but at the same time it ran the risk that some might not make the best choice. As I gained further familiarity with the way in which Clare worked, I realised that in such a situation she would be monitoring students' activity, and if she felt anyone was making poor choices, she would not hesitate to recommend, or insist on, another course of action.

In the second lesson on fractions, the following week, the start of the lesson was spent in recalling examples of recurring patterns in decimals of fractions, for example, in 1/7 and in 1/11. She then set students the task of recording the decimal representations of all fractions up to 1/7. i.e. 1/2; 1/2, 2/3; 1/4, 2/4, 3/4; 1/5 ... As part of this task, she gave the instruction in statement (3), which could be seen as involving MC—"look for recurring patterns", or as sensitivity to students, valuing "what Virginia did last week" but it is the "with another bit of your brain" which I feel is a manifestation of ML. It seems to say, "you have brains—use them", and also, "you can often do more than one task at the same time", thus encouraging them to develop effective ways of working.

Statements (4) and (5) seem overtly managerial—telling groups to share ideas and to come up with their own questions; and to prepare themselves to provide feedback. However, these are more than just instructions for *what* to do, they also carry elements of a philosophy of *how* to work—perhaps that sharing ideas, asking questions and preparing to give feedback are valuable ways of working. And throughout, there was emphasis on *thinking*. Students in Clare's classes seemed to be left in no doubt that they were required to *think*

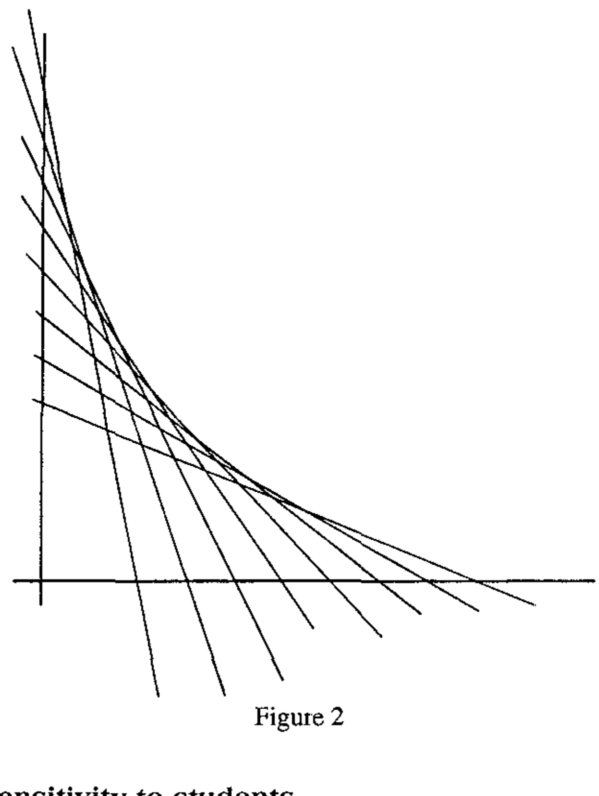


Figure 2

Sensitivity to students

Alongside the various manifestations of ML in the situations above, I see sensitivity to students (SS) as also present. Clare's concern for Katy, seemed to be one instance. Other manifestations lay in both Mike and Clare's respecting of the students' own preparedness to think and contribute. However, SS is less easy to characterise in terms of brief statements than ML is. Often my perceptions of sensitivity lay with the *way* in which the teachers talked with students—in their voice tones and gestures rather than in the actual words. Clare, particularly, knew her students individually and well, and often talked at length to me about their individual characteristics and how these affected her work with the student Once manifestations of SS arose in an interaction between Clare and a student Jaime

Clare had talked to me extensively of Jaime in the past. He came from a family whose language at home was not English, although he seemed to understand English and communicate with his peers in English. However, he did not do very well in mathematics lessons, seeming rather lethargic, uncaring and not eager to get involved. Clare had said of him in the past:

> "It would be too easy to say that it was just basic laziness, though I think there's quite a lot of that in it. There's a lot of other things as well. I mean, he's a very insecure sort of person and very complex. He's not happy with the English language any way: he doesn't use it at home."

I asked if he was fluent in English

> "Well, he is, but he doesn't realise he is, so he's mumbling. It's a bit like colour-blind people who don't understand any colours, because they've just given up on colours, whereas all they're confused about is red and green. But it's not just the language, it's the fact that he doesn't take any part in what's going on, plus the laziness, plus the language."

Situation 3: "Jaime"

In a lesson where students had been asked to consider the number of intersections generated when a number of lines crossed, Jaime had got very involved with the problem and had done some interesting and valuable work, which, particularly bearing in mind his usual attitude, was very exciting for Clare Unfortunately for me, the interaction had not been recorded, but Clare described it afterwards

"Very early on, and you haven't got this on tape, I had a look at what he was doing He called me over, after working for about five minutes, and I thought that perhaps he hadn't understood what he was supposed to be doing And when I went over to see what he was doing, he knew *exactly* what he was supposed to be doing, and he had practically gone straight to the heart of the problem. He was saying, I think, "Look I've got these (lines) and I know what is going to happen. This one is going to cross this many, and that one is going to cross that many." And *it was wonderful*

"Then later on, *not much later on,* he actually *got up out of his seat* and came across to me, which is again rather unheard of for him, and showed me these beautiful drawings which he had done, in which he had drawn the lines in a way such that they made a curve. So that his first two lines had been at right angles to each other, and his third line had been — if you imagine those as axes — his third line had had a very steep, negative gradient, so that it cut the vertical axis very high up, and the horizontal axis a little way along. So his second line had just been a little less steep, and his third line a bit less steep, and so they had made a curve as well. And, as well as getting the numbers of the crossing *quite easily,* he also discovered that these made a curve." [See Figure 2]

"After that lesson, I mentioned to him how very exciting I found that and *he was obviously very pleased as well,* because he was *smiling,* and he was *working,* and *he really felt he was getting somewhere.*"

This situation has a number of features which were typical of Clare's approach to working with students. The first is her intense interest in and caring for the student The above quotations comprise only a few excerpts from what she said about Jaime. She talked extensively of what she knew of him as a person, as a student in her lessons, and of his mathematical achievement in this particular case. Jaime was just one of many students of whom she spoke in this way. Secondly, I feel that her words above are revealing of her own approach to mathematics and to the mathematical thinking of her students. She believed that mathematics was exciting, and her enthusiasm came across in the way she spoke. Perhaps for Jaime, seeing her excitement in what he had done raised his self esteem and motivated him to tackle more challenging work.

Mathematical challenge

All of the situations I have detailed above incorporate mathematical challenges for the students. For example, in Situation 2, it seemed extremely ambitious that students should be asked to "explain repetition in 1/7". This seemed to incorporate a high degree of MC Yet the students did not seem to find the instruction strange, and many of them tackled it in interesting ways. There were rarely situations in the classes which I observed where no challenge was present; however, the degree of challenge varied very considerably from one situation to another, from one student to another. MC and SS seemed inextricably linked in many respects In order to know what degree of challenge was appropriate, it was essential to have a clear idea of what particular students needed or could cope with

In the fractions lessons mentioned above, Clare talked with a boy, Martin, who was thinking about "rounding off" and associated computer representations of certain decimals He noticed that recurring decimals often changed at the end, for example the decimal equivalent of 2/3 is 0 6-recurring. This might be written as 0.667, or 0.66667, or 0.666666667, none of which were exact representations. She said later of this that she had wanted him to talk about "approximation", although he had not used this word, and she was not quite sure just how much he understood. Should she leave him where he was, believing that he understood, or should she push him further, perhaps getting him to compare 2/3 with 0.67, or more provocatively 0.9-recurring with 1? This might bring him up against the notion of approximation, but also it might be too much for him to cope with at this stage. Thus, there were often difficult decisions for the teacher in finding an appropriate mixing of MC and SS.

The following situation from one of Mike's lessons on the topic of Pythagoras' theorem incorporates further manifestations of the SS/MC relationship. Mike had set groups of four in the class two tasks to work on in pairs within the group, hoping that cross-fertilisation would take place between the pairs The tasks were:

"Square sums"

$1^2 + 2^2 = 5$

What other numbers can be made by adding square numbers together? Investigate.

Triangle lengths

Draw a triangle with a right angle.

Measure accurately all three sides.

Can you find any relationship between the three lengths?

In Situation 4, a student, Phil, had been working on Square sums.

Situation 4: "Phil"

Phil talked animatedly to the teacher, Mike, about his work on "Square sums":

1 P. I've got 26, and I'm working on—if I want to get 27, I've . ., I have to try and get the closest number—to do the sum—I have to use something like 1.5, cause if I try to get the 2, then that'll make 4, if I try to use 2 squared plus 5 squared, er, that'll make 29, so I have to—cut 'em in half, obviously, cut 'em in half. (M. "Right") I'm going to try and keep the 5 and use 1.5 squared

2 M. That's a nice idea. So you're going to try to home in to 27. Is it 27 you're working on?

3 P. Yes.

4 M. Right

5 P. If I can't do that, I'll take 4.5, I won't take 5 and a 1/2, I'll take 4 and a 1/2, and use 2 here

6 M. OK. . . So you're going to have *something* squared—

7 P. Hm, what's the word for—1.5, er *decimal?* Yeah, I'm gonna use decimal—

8 M. Right. So, something squared, plus 5 squared equals 27

9 P. If it does. If it doesn't erm, that's what I think, if it doesn't I'll try, er, 4.5 squared, by, erm, 1.5. (Inaudible) Then I'll go back to the 2, then I'll go...

10 M. Well, let's try working to the 5 squared, for the minute, and let's say that equals 27, now, your problem is to find something squared, plus 5 squared equals 27. What can you tell me about this number, this *something* squared? Can you tell me anything about it so far?

11 P. Erm, well, I know this 5's important. That gets you into 20s.

12 M. Right, How far into the 20s?

13 P. Half way.

14 M. So what does 5 squared equal?

15 P. 25

16 M. Right. Now, something squared, plus 25 equals 27.

17 P. I need to get 2 out of here

18 M. Right. So something squared,—you've got to find *a* number, which . .—now that seems to be a short cut. Can I leave that with you—to look at?

My interpretation of Phil's thinking at statements (1) and (9) is as follows: At statement (1), Phil says he has got 26. I suggest this has arisen from $1^2 + 5^2 = 26$. Also $2^2 + 5^2 = 29$. So, to get 27, he would need something like $1.5^2 + 5^2$ or $1^2 + 4.5^2$. At statement (9), he says that if these don't work, i.e. if they don't give 27, then he will try $1.5^2 + 4.5^2$.

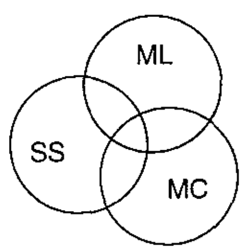
The teacher, after listening to Phil's initial statements, seems to think that Phil's approach is too random, so, at statement (10), he suggests staying with 5^2 and considering aspects of this first. He tries to get Phil to analyse what it *is* that he needs, rather than varying two things haphazardly. Ideally, Phil could realise that what he needs to add to 5^2 in order to get 27 is 2, and so the number squared which gives 2 is . The teacher could explain this to Phil, but is Phil ready to appreciate its sophistication? The situation is *not* ideal. Phil seems still to be in a position of believing that with persistence he will find the number which he wants, which he expects to be a simple terminating decimal. Hence he believes, implicitly, that there is such a number. It is likely that the realm of irrational numbers is beyond his current thinking. In the seconds which the teacher has in which to respond to Phil's statements, he has a very complex teaching situation to assess. Should he leave Phil to stab randomly, until perhaps Phil himself perceives the need for some other approach. Or should he try to move Phil towards what, according to his experience as a mathematics teacher, might prove to be more profitable?

It had been a direct challenge from the teacher which had resulted in the quite impressive thinking which Phil exhibits in Situation 4, where, despite the haphazard nature of his stabbing to get 27, he is nevertheless on the track of 27. It is likely that the teacher drew on this experience in pushing Phil further at statement 10, and this resulted in Phil finally observing, "I need to get 2 out of there" (Statement 17).

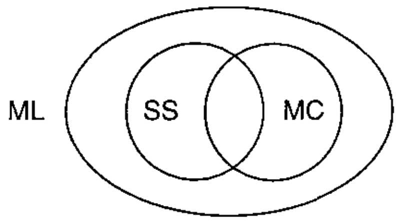
The teacher had to make judgements about the degree of challenge which it was appropriate to offer Phil, in order to enable him to move on. Too much challenge and the precarious position might be lost and Phil might have to recreate the thinking which he had already achieved. However, too little challenge may have resulted in Phil not making progress at the rate of which he was capable with the teacher's help. There has to be recognition of where a student stands and where she might reasonably reach. A great deal of knowledge of the student is bound up in this decision, and so MC cannot be divorced from SS.

Images of the teaching triad

As I have indicated, the domains of the triad interrelate in any teaching situation. My early images of the relationships were in terms of the following diagram. I had felt that the domains had equal status and "overlapped" with each other.



However, when I offered notions of this triad to another teacher Ben, his view was different. Ben said that he viewed *all* of his teaching as *management of learning,* and that the other domains were subsumed within this. Throughout our discussions of his teaching, Ben's allusions to the domains of the teaching triad supported the following image which for me, gradually took supremacy over the former one illustrated above



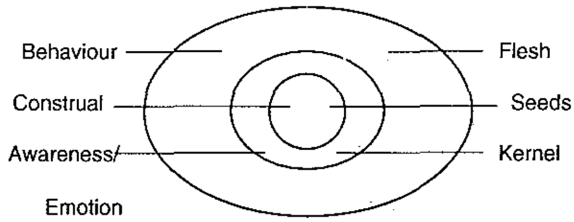
However, in offering this to colleagues subsequently, and in discussions with them, a three dimensional image of the triad* emerged. In this, each of three rectangular axes represents one of the domains of the triad, and any teaching situation can be represented in 3D space as incorporating differing degrees of the three domains. Further exploration may show that images vary according to teacher and context. The image is only important in so far as it is helpful to a person in envisaging the triad, the relationships between its elements, and its usefulness in describing teaching situations. I personally find the triad a helpful analytical device. I have evidence that at least one teacher, Ben, found it helpful in talking about his own teaching experiences and explaining teaching situations. In offering it to colleagues, I have had positive indication of its potential usefulness.

A constructivist perspective

My thinking about teaching is based on the belief that knowledge is constructed by the individual, not passively received from the environment, and that learning, or *coming to know,* is an adaptive process which tries to make sense of experience. It is not a process of discovering an independent pre-existing world outside the mind of the knower [von Glasersfeld, 1988; Kilpatrick, 1987]. Two immediate consequences of this are (1) that it denies the transmission metaphor of teaching and learning—i.e. that knowledge can be given by one person to another, as exemplified by phrases such as "I didn't get it over properly" [Davis and Mason, 1989]; and (2) that *if* there exists some absolute body of knowledge, based on some external ontological reality, then this can never be known. Knowledge, for any learner, can be no more than their own construction. This does not deny the possibility of "common knowledge" [Edwards and Mercer, 1987]. Through sharing and negotiation between people, individual meanings can be modified, and some measure of agreement about ideas and concepts can be reached [Bishop, 1984; Cobb, 1988]. What it recognises is that learning is a constant process of meaning making, and common knowledge is a negotiated synthesis of such things.

In the classrooms which I have studied, I have regarded students as meaning makers, and teachers as supporters of the process of meaning making by their students. This does not mean that I see teaching as some wishy-washy process of "letting it happen", the teacher being no more than a facilitator. This is as simplistic a view as is the image of teacher as the expert who hands over knowledge and skills. In any classroom situation students are making *some construal* of what they experience. The teacher has a responsibility (by *law* in may countries) to deliver the curriculum. Thus the teacher would like students' construal to include meaning-making relevant to that curriculum. Therefore the teacher needs to gain access to, and influence, students' construal.

I have found the following image useful in considering the interface between student construal and the teacher. It is of a soft fruit which has a succulent outer flesh, and a hard kernel which contains the seeds. The eater of the fruit gains ready access to the flesh, but comes up against the kernel which prevents access to the seeds. Analogous to the seeker of the seeds of the fruit, is the teacher, trying to gain access to student construal. The student's levels of transparency to the teacher may be seen as these layers of a fruit. Construal is represented by the seeds and behaviour by the outer flesh. The kernel, the hard shell between flesh and seeds, represents states of awareness and emotion of the student. Thus the teacher can readily gain access to behaviour, will come up against the student's awareness and emotion, but will have great difficulty in gaining access to what construal is taking place.



How then *does* the teacher gain access to and influence this construal? *How* does the teacher *teach?*

Observing teaching

It has been observed [for example, Brown & Cooney 1985] that there is a *gap* between theory and practice. For example teachers have been observed to espouse certain theoretical principles, but to give no evidence of implementing these principles in their classroom practice. My own experience is that theory and practice often do not fit well together. Part of the difficulty is the necessary immediacy of teaching, which depends crucially on social and epistimological expectations both of teacher and students, and on the prevailing ethos of the classroom. The demands and pressure on teachers is well known and as Desforges and Cockburn [1987] point out, despite firm beliefs in the development of higher-order levels of mathematical thinking, teachers find the implementation of these beliefs difficult. For example, the Cockcroft report [DES, 1982] pro-

* I am grateful to Candia Morgan, of South Bank Polytechnic, London, for this image

moted the importance of *classroom discussion*. The inclusion of productive discussion among students, and between teacher and students, has come to be regarded as "good practice" in the mathematics education community. Yet Desforges [1989] states, "Despite the exhortation of decades, fruitful discussions are rarely seen in primary classrooms. This is so even in the classrooms of teachers who recognise and endorse their value." One problem is that, although it is easy to talk of the value of such discussion, it is by no means obvious what practical manifestations of it look like. For the teacher who is being urged to incorporate discussions into mathematics lessons, the theory is unhelpful regarding what might actually be done, and what issues arise pertaining to its implementation.

My study originated from a decision to start from the practice of teaching, to try to identify aspects of practice which seemed to me to be significant in terms of my own theoretical standpoint, and from this to elicit issues which seem to be of importance to the teacher. I chose to work with teachers who gave some indication of espousing beliefs consistent with a constructivist philosophy of knowledge and learning. I studied just a few teachers in depth, collecting data through participant observation of their lessons, and talking with them at great length about their planning, their objectives, their reflections on lessons which I observed, and trying to gain access to their underlying belief structures. At the same time my own awareness of the practice of teaching and its relation to theory grew at a rate far beyond that during my many years as a practicing teacher. It was from my analysis of the data which I collected that the teaching triad emerged.

The teaching triad and constructivism

Only students themselves can construct their mathematical knowledge, relative to their own individual experience. In every moment of classroom action, some sort of construal occurs. A teacher needs to influence and interact with this construal. In all of the situations which I have offered above, I see attempts by the teacher to influence and interact. ML involves creating opportunity for influence. SS involves building the knowledge and opportunity for interaction. MC involves offering the content of influence and interaction in an interesting, attractive and motivating way. The three elements are inextricably linked. Good management depends on sensitivity, if it is to succeed. Challenge can not be taken up if it is inappropriate, or if strategies for handling it have not been created. Sensitivity alone might create happy situations, but challenge is required to enable mathematics to be done. Through good relationships which arise with students the teacher can gain access to their thinking. Through established ways of working the teacher can expect challenges to be taken up. By examining the response to challenge, the teacher can gain insight to levels of construal. This insight enhances knowledge of students and provides a basis for continued challenge. I suggest that my teaching triad is not only an illuminating means of regarding teaching situations, it also offers *an approach to teaching* consistent with constructivist views of knowledge and learning.

Notes:

I should like to thank the teachers concerned for their extreme generosity in allowing me to make observations, giving me their time, and talking to me about their beliefs. Without this the study I have reported would not have been possible.

I should like to thank David Pimm and John Jaworski for helpful comments on drafts of this paper.

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