# STATISTICAL KNOWLEDGE FOR TEACHING: EXPLORING IT IN THE CLASSROOM

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Although statistics is part of school mathematics curricula, it is generally a more recent addition. In New Zealand, statistics has been included in the primary school curriculum since 1969 and, as such, Watson (quoted in Begg *et al.,* 2004) suggests that New Zealand can be considered a world leader with regard to statistics in the school curriculum. The latest national curriculum (Ministry of Education, 2007) explicitly requires that students at all levels be engaged in learning statistics through investigations.

There is an extensive research literature on the teacher knowledge needed for teaching mathematics. Statistics education research has a much shorter history and its literature pertaining to teacher knowledge is relatively scarce. The study reported here goes some way to filling this gap.

## Literature review

Recent research on mathematics knowledge for teaching has focused on questions such as how mathematics knowledge is used in teaching. It has been argued that teacher knowledge is organised in a content-specific way, rather than around the generic tasks of teaching such as lesson planning, and so on (Hill, Schilling, & Ball, 2004). Furthermore, it is claimed (Ball, Lubienski, & Mewborn, 2001) that significant mathematical reasoning and thinking occurs as teachers go about their everyday work. Such work includes "figuring out what students know; choosing and managing representations of mathematical ideas; appraising, selecting and modifying textbooks; deciding among alternative courses of action; and steering a productive discussion" (p. 453). Such aspects of a teacher's role are indicators of different categories of teacher knowledge, and therefore help with framing teacher knowledge in terms of its use (Ball, Thames, & Phelps, 2008).

A model that categorises teacher knowledge and describes the knowledge required to deliver high quality instruction to students is outlined by Hill, Schilling, and Ball (2004). In this model, two categories of content knowledge, namely common knowledge of content and specialised knowledge of content, are differentiated. These two categories are further clarified by Ball, Thames, and Phelps (2005, 2008): *common knowledge of content* includes the ability to recognise wrong answers, spot inaccurate definitions in textbooks, use mathematical notation correctly, and do the work assigned to students; in comparison, *specialised knowledge of content* needed by teachers (and likely to be beyond that of other well-educated adults) includes the ability to analyse students' errors and evaluate their alternative ideas, give mathematical explanations, and use mathematical representations.

Similarly, pedagogical content knowledge is subdivided into two categories (Ball *et al.,* 2005): *knowledge of content and students,* and *knowledge of content and teaching.* These two types of teacher knowledge bring together aspects of content knowledge that are specifically linked to the work of the teacher, but are different from specialised knowledge of content. *Knowledge of content and students* includes the ability to anticipate student errors and common misconceptions, interpret students' incomplete thinking, and predict what students are likely to do with specific tasks and what they will find interesting or challenging. *Knowledge of content and teaching* deals with the teacher's ability to sequence the content for instruction, recognise the instructional advantages and disadvantages of different representations, and weigh up the mathematical issues in responding to students' novel approaches.

The categorisation of teacher knowledge into four components was developed from work in number and algebra. However, statistics is in some ways, different from mathematics, particularly because of the uncertainty surrounding the conclusions that one can draw from data (Pereira-Mendoza, 2002). So it is questionable as to whether such a model for teacher knowledge is applicable to statistics. There is no research literature that could be found that reports on classroom-based investigations into primary (elementary) teacher knowledge in relation to statistics. A statistical thinking model (Wild & Pfannkuch, 1999) that accounts for the uncertainty and non-deterministic processes of statistics provides a useful way forward for examining teacher knowledge in statistics.

Wild and Pfannkuch's (1999) model for statistical thinking has five main components: recognition of a need for data on which to base decisions, rather than reliance on anecdotal evidence; ability to 'transnumerate' the data, which includes capturing appropriate data, and being able to use different representations of the data in order to gain further meaning from the data; recognition of variation; reasoning from statistical models, such as tables, graphs, or more complex models; and integrating statistical knowledge with contextual knowledge.

Along with these fundamental types of thinking are more general types that could be considered part of problem solving (but are not exclusive to statistical problem solving). Wild and Pfannkuch's dimension of 'types of thinking' is one of four dimensions that explain statistical thinking in empirical enquiry. The other three dimensions are: the investigative cycle (problem, plan, data, analysis, and conclusions – these are the "procedures that a statistician works through and what the statistician thinks about in order to learn more



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| | Content knowledge | | Pedagogical content knowledge | |
| | Common knowledge of content (CKC) | Specialised knowledge of content (SKC) | Knowledge of content & students (KCS) | Knowledge of content & teaching (KCT) |
| Types of thinking: Need for data | | | | |
| Types of thinking: Transnumeration | | | | |
| Types of thinking: Variation | | | | |
| Types of thinking: Reasoning with models | | | | |
| Types of thinking: Integration of statistical and contextual | | | | |
| Investigative cycle | | | | |
| Interrogative cycle | | | | |
| Dispositions | | | | |

*Figure 1. Framework for teacher statistical knowledge*

from the context sphere"; Pfannkuch & Wild, 2004, p. 41), the interrogative cycle (generate, seek, interpret, criticise, and judge – this "is a generic thinking process that is in constant use by statisticians as they carry out a constant dialogue with the problem, the data, and themselves"; *ibid.*), and dispositions (including scepticism, imagination, curiosity and awareness, openness, a propensity to seek deeper meaning, being logical, engagement, and perseverance) which affect or propel the statistician into the other dimensions. All these dimensions constitute a non-hierarchical, non-linear model that encompasses the dynamic nature of thinking during statistical problem solving.

A framework that incorporates both statistical thinking and the four categories of mathematics teacher knowledge (as described earlier) was developed (Burgess, 2006). [2] This framework (see fig. 1) is proposed as suitable for identifying and describing categories of teacher knowledge in relation to each of the components of statistical thinking. Just as Ball, Lubienski, and Mewborn (2001) claim that many of the tasks of the mathematics teacher are essentially mathematical, this framework suggests that much of what a teacher engages in during the teaching of statistical investigations essentially involves statistical thinking and reasoning. Consequently the teacher knowledge categories can be examined in relation to each component of statistical thinking. The ways in which each of the aspects of statistical thinking interact with and exist as evidence for the various categories of teacher knowledge can also be examined.

The major question that arises, and which this paper addresses, is how effective is this framework for identifying teacher knowledge that is needed for teaching statistics through the use of investigations at the primary school level?

## Methodology

Because of widespread advocacy for conducting research on teacher knowledge in the context in which it is needed, namely the classroom, methods are needed that might provide such insight. Although students are also participants in the classroom, videotaping of the teacher during classroom lessons is considered sufficient for examining teacher knowledge (Schoenfeld, 1998) because a focus on the teacher explains a significant proportion of what takes place in the classroom, particularly as it relates to teacher knowledge.

Four primary school teachers, each teaching students from one or two year levels from Years 5–8 (approximately 9–13 years old), participated in the study. Videotapes of a sequence of four or five lessons from each teacher provided the first data source. Since it is recognised that a researcher's interpretations of what is going on can be problematic (Phillips & Burbules, 2000), one solution is to use stimulated recall interviews with the teacher, as a way of reducing the limitations of one researcher's interpretations, and as a type of data triangulation. An edited video of classroom 'episodes of interest', selected by the researcher, provided the stimuli for discussion between the researcher and the teacher. Such episodes of interest are considered 'critical incidents' on which to base stimulated recall interviews (Lyle, 2003). Gass (2001) points to the importance of minimising any delay between the original classroom event and the stimulated recall interview. Each audiotaped stimulated recall discussion between the teacher and the researcher was generally scheduled the same day as the lesson itself. These audiotapes provided a second source of data.

The two sources of data were coded using appropriate software. Coding categories were developed from the framework, with each cell of the framework having a unique code based on a category of teacher knowledge and a component of statistical thinking. As well as codes and timecodes being 'attached' to each segment (video or audio), notes could also be attached. These notes took the form of transcriptions or field-type notes. The software enabled searches by code or terms/phrases within notes, and the search returned all video or audio instances that contained the search criteria.

## Results and discussion

Descriptions of teacher knowledge in relation to both statistical thinking components and the four categories of teacher knowledge were developed. These descriptions are not however within the scope of this paper, but have been described elsewhere (Burgess, 2009). As part of the investigation into the suitability of the framework for identifying teacher knowledge as used in the classroom, or as needed but not used, a profile for each teacher was developed. Each teacher's profile showed which components from the framework were used by that teacher, which components were needed but not used, and which components were identified as not needed. One example of such a profile is shown in figure 2.

The key indicates that four types of situations occurred with regard to particular teacher knowledge. First, the grey-shaded cells of the framework indicate that direct evidence was obtained from the video or interview for that particular aspect of teacher knowledge. For example, Linda used *specialised knowledge of content: transnumeration* when she had to evaluate whether a student's sorting of data cards into two piles would be suitable to allow the student to compare these two piles and make a sensible statement from them. Second, for some cells on the framework, indirect evidence was found for that aspect of knowledge. These cells are shown with the 'hatched' shading. For example, *common knowledge of content: transnumeration* was not directly observed for Linda, as she did not sort data cards or otherwise transnumerate the data for herself. However, when Linda had to evaluate a student's sorting of the data, it was clear that if she had had to sort it

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| Types of thinking | Need for data | | | | |
| | Transnumeration | | | | |
| | Variation | | | | M |
| | Reasoning with models | | | M | |
| | Integration of statistical and contextual | | | | M |
| Investigative cycle | | | | | M |
| Interrogative cycle | | | | | |
| Dispositions | | | | | |

Key:
= direct evidence of that knowledge
= indirect evidence of that knowledge
M = missed opportunity related to that knowledge

*Figure 2. Summary of Linda's teacher knowledge*

appropriately for herself, she would have been able to do so. Consequently, it is inferred that she had *common knowledge of content: transnumeration*. Third, for some cells, the relevant knowledge was not identified in any episode, nor was it seen to be needed. Such cells remain unshaded. Fourth, some evidence was found when teacher knowledge could have been used in a particular interaction with a student or students, but for some reason that knowledge was not used. Such instances were referred to as 'missed opportunities', and are shown in the profile with an M in that cell. Missed opportunities are further discussed below.

The type of investigation that was undertaken by students involved multivariate data provided by the teacher (from a resource that all four teachers used), or that had been 'replicated' and gathered from their own class. As such, the students' statistical thinking did not require an understanding of a need to gather data to answer a question; they already had the data. So, the framework row that corresponds to 'need for data', as one aspect of statistical thinking, was not in evidence as part of teacher knowledge in this particular teaching unit. Dispositions, as another component of statistical thinking, although apparent during the teaching, was not able to be subdivided in relation to the four categories of teacher knowledge as were all the other components of statistical thinking. Consequently, in the profiles, that row of the framework is blank.

All remaining 24 cells of the framework were identified in at least one teacher's practices, with 21 of those cells represented in either three or four teachers' profiles (see fig. 3 for profiles of all four teachers). From the hatched shading in the profiles, it can be seen that common knowledge of content was the only type of knowledge that could be inferred as being present due to other types of knowledge, most commonly because of specialised knowledge of content. The other three types of teacher knowledge (specialised knowledge of content, knowledge of content and students, and knowledge of content and teaching) were considered to be more focused and with specific 'roles', and therefore it is unlikely that evidence of these can be inferred from other types.

In addition to the presence in the profiles of aspects of teacher knowledge, there were numerous instances of what

*Summary of Linda's teaching knowledge*

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| Investigative cycle | | | | | M |
| Interrogative cycle | | | | | |
| Dispositions | | | | | |

*Summary of John's teaching knowledge*

| Dimensions of statistical thinking | | Statistical knowledge for teaching | | | |
|---|---|---|---|---|---|
| | | Content knowledge | | Pedagogical content knowledge | |
| | | Common knowledge of content (CKC) | Specialised knowledge of content (SKC) | Knowledge of content & students (KCS) | Knowledge of content & teaching (KCT) |
| Types of thinking | Need for data | | | | |
| | Transnumeration | M | M | M | M |
| | Variation | | | | |
| | Reasoning with models | M | M | M | M |
| | Integration of statistical and contextual | | | | |
| Investigative cycle | | | M | | M |
| Interrogative cycle | | | M | | M |
| Dispositions | | | | | |

*Summary of Rob's teaching knowledge*

| Dimensions of statistical thinking | | Statistical knowledge for teaching | | | |
|---|---|---|---|---|---|
| | | Content knowledge | | Pedagogical content knowledge | |
| | | Common knowledge of content (CKC) | Specialised knowledge of content (SKC) | Knowledge of content & students (KCS) | Knowledge of content & teaching (KCT) |
| Types of thinking | Need for data | | | | |
| | Transnumeration | M | M | M | M |
| | Variation | | | | M |
| | Reasoning with models | | M | M | M |
| | Integration of statistical and contextual | | | M | M |
| Investigative cycle | | | | M | M |
| Interrogative cycle | | M | M | M | |
| Dispositions | | | | | |

*Summary of Louise's teaching knowledge*

| Dimensions of statistical thinking | | Statistical knowledge for teaching | | | |
|---|---|---|---|---|---|
| | | Content knowledge | | Pedagogical content knowledge | |
| | | Common knowledge of content (CKC) | Specialised knowledge of content (SKC) | Knowledge of content & students (KCS) | Knowledge of content & teaching (KCT) |
| Types of thinking | Need for data | | | | |
| | Transnumeration | M | M | | M |
| | Variation | M | M | | |
| | Reasoning with models | | M | M | M |
| | Integration of statistical and contextual | | M | | |
| Investigative cycle | | | M | | |
| Interrogative cycle | | | M | | |
| Dispositions | | | | | |

*Figure 3. Profiles of the four teachers*

were termed missed opportunities. Some of these corresponded to incorrect knowledge being used, while others related to non-use of teacher knowledge. For example, one group of students talked about questions that might be investigated in their data set. One student suggested: *Put them into year and age and gender.* The teacher asked what they were trying to find out. Student: *We will figure that out once we have sorted the cards. Like, if there are more girls who are year 6 than boys who are year 6.* Another student then said: *We can add them together and do averages.* At this point the teacher made no response to what the student meant or whether an average would have helped make sense of and 'get inside' the data. This incident indicates a missed opportunity in relation to *specialised knowledge of content: transnumeration*. Missed opportunities such as this show non-use of knowledge, which could be attributed to one of three reasons: a lack of statistics knowledge on the teacher's part, a lack of recognition that his/her knowledge could have been used in that situation to question the students, or a conscious decision (for a range of possible pedagogical reasons) not to use that knowledge. Some missed opportunities were further explored in the stimulated recall discussions, yielding a reason for that missed opportunity; in other cases, the reasons for the missed opportunities are unknown.

## Conclusions

The diagram of the four teachers' profiles (fig. 3) indicates the types of knowledge that were needed and/or used in teaching statistics through investigations. It is based on both the presence of evidence for all framework cells (excluding the two statistical thinking dimensions, as discussed) across the teaching episodes, and the missed opportunities in a high proportion of the cells that indicate that knowledge was needed in teaching episodes. This presents a strong case for the suitability of the framework as a means of identifying the types of teacher knowledge needed and/or used by primary teachers when teaching statistics through investigations. This knowledge can be classified in relation to a component of statistical thinking as well as one of four types of teacher knowledge.

The framework profile also provides a useful way of identifying for each teacher, through patterns of missed opportunities, aspects of teacher knowledge that are in need of development. In some cases, the missed opportunities extend along a row in the profile, which corresponds to a component of statistical thinking across the four knowledge categories. In other cases, the missed opportunities extend down a column in the profile, indicating that a particular category of teacher knowledge was deficient across a variety of statistical thinking components. Analysis of the missed opportunities suggests that such situations can impact negatively on the learning opportunities for students. [2] The profile could therefore be used to show areas of need for future professional development for a teacher.

It is acknowledged that in any classification of teacher knowledge, the boundaries between the various components can be difficult to determine. However, as Ball *et al.* (2008) describe, when working from the 'messiness' of classroom practice, decisions about which category of knowledge is appropriate depends on that particular context, and that rather than being a limitation, it can be argued that there is a strength from using classroom practice to develop theory about teacher knowledge. The distinctions between components of knowledge are open to further refining.

Together, the aspects of knowledge identified across the four teachers and the missed opportunities for each teacher indicate that the methodology adopted and the framework are successful and useful for identifying teacher knowledge needed for teaching statistics through investigations at the primary school level.

## Notes

[1] This paper is based on one presented to Topic Study Group 27, ICME 11, Monterrey, Mexico, July 2008.
[2] See Burgess, T. A. (2007) *Investigating the nature of teacher knowledge needed and used in teaching statistics* (unpublished doctoral thesis), Palmerston North, NZ, Massey University. Available at http://www.stat.auckland.ac.nz/~iase/publications/dissertations/dissertations.php].

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