Communications

A three-tier teaching model for teaching mathematics in context

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> Home mathematics we don't write but school mathematics we write with either pencil or pen.
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> School mathematics is studied in English but home mathematics is different.

These quotes from Ghanaian primary school pupils reflect how they perceive the relationship between in-school and out-of-school mathematics. Most of them, especially from very poor socioeconomic background and rural areas, see everyday and school mathematical practices as very different and mutually exclusive. This perception has serious consequences for their opportunities to learn mathematics meaningfully, in Ghana as well as in other sub-Saharan African countries.

Making a link between in-school and out-of-school mathematics is related to the dilemma of teaching minority/indigenous children an international subject like mathematics while retaining the integrity of a minority/indigenous world view (Barton, 2008), as well as the assumption of cognitive modes in the primary school mathematics curriculum and the language of instruction that are different from those available to the child in their everyday experiences and home languages (Berry, 1985).

Drawing on Bishop's (1988) distinction between *mathematical enculturation* and *mathematical acculturation*, Vygotsky's (1987) distinction between *everyday* and *scientific* concepts, and Lancy's (1983) stages of cognitive development, I have developed a teaching model that draws on everyday mathematical conceptions to scaffold pupils' higher understanding of school mathematics. The following example illustrates the three tiers of the model: *enculturation* of pupils into their own mathematical culture, *transition* from everyday mathematics to school mathematics, and *acculturation* into school mathematics.

An example of the use of the three-tier model in teaching fractions

In this introductory lesson the aim was to systemically enlarge the pupils' notion of halves (which was everyday) to include the school concept of halves, using the three-tier teaching model. The children were in primary two in Ghana. Their ages ranged from 7 to 8 years old. There were 49 pupils in the class and the lesson was conducted in their local language.

Figure 1. Identification of fraction of a stick.

Session 1: Enculturation

The pupils were first asked to name the parts of each of three sticks broken into two, as shown in Figure 1. All the pupils named each of the parts of the three sticks as *fã* (a half) because "you have broken it [each stick] into two". Pupils were then asked to name the fraction of water in a bottle (which was three-fourths full). About two-thirds of them identified it as *fã* because the level is about the mid-point, while the rest maintained that it is *sin* (less than a whole). They could use their everyday knowledge to answer these questions, but their answers to the second question reveal the limits of their everyday language. In contrast, one pupil, who was repeating the grade, answered "one and half" because "the bottle is one and the water is half." Her answer reflects a school mathematics that is disconnected from her everyday notion of fractions.

The thirty minute session ended with the pupils being asked to consider how tourists from Europe or America might identify halves in their home countries. The pupils imagined how they could identify halves in a way that would enable them communicate halves to tourists such that the tourists would understand. Asking the pupils to look at how other cultures identify halves at the end of the session prepared them to accommodate an expansion in their schema of halves. This ushered them into the next stage of the three-tier teaching model (the transition stage).

Session 2: Transition and acculturation

This was a sixty minute session, which began with connections to the previous lesson. The teacher asked the pupils: "how do we obtain halves?" and the pupils answered in chorus "when we break things into two". Pupils were asked how tourists from Europe identify halves in their home country and one pupil answered *kyemu per* (break into two equal parts). This part of the lesson was intended to begin the acculturation of pupils into the school notion of half by drawing on their everyday knowledge of halves (the enculturating stage of the three-tier model) and their knowledge of how halves are experienced elsewhere (the transition stage of the three-tier model).

Pupils were then organised in groups of five or six and asked to shade half of drawn objects based on *kyemu per,* which they could do accurately. The formulation *kyemu per* allowed them to shift their understanding of half towards the school notion of half. The groups looked again at the three sticks broken into two in Figure 1 and were asked which represent half from the international perspective. All the groups identified the stick that was broken into two equal parts, and provided answers such as:

the others do not show halves because for each of them the two parts are not equal, one part is bigger than the other

If you want halves you divide it into two equally (*kyemu ebien pepeper*) but this was not done for the other two

The pupils were asked if they could write 'half' symbolically, but they were not familiar with this notation, so the teacher introduced the pupils to the usual symbolic representation. There was the potential in this exchange to again move from everyday experiences (as the pupils might have seen the ½ symbol used on a shop sign) but in this case they had not been enculturated to the use of the symbol, and so instead the acculturation built on their school based knowledge of half as *kyemu per.*

To end the session, the teacher drew a rectangle on the chalkboard and divided it into four equal parts. He shaded one part and asked the pupils whether the shaded portion was a half. Pupils answered "no", in chorus. The groups were asked to name the fraction and all identified the shaded portion as one part of a whole divided into four equal parts. Again, they were able to move conceptually to the notion of a fourth, but only one group (which included a pupil repeating the grade) could name the fraction. As with 'half', symbolic representation of a fourth was difficult for pupils because symbolic representation of this term does not exist in their everyday mathematical practices.

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The three-tier teaching model draws its theoretical support from Lancy's (1983) cognitive theory, which says that, regarding cognitive development, it is societies rather than individuals which make transitions from one level of cognitive functioning to the other (p. 169). Although propounded over three decades ago, Lancy's theory is important for this model because it helps position mathematical thinking as a kind of meta-cognitive knowledge which is affected by both culture and values. Specifically, the three-tier teaching model draws its strength from the last two stages of Lancy's three-stage theory: the *enculturation* stage and the *metacognition* stage. The first stage of the three-tier teaching model relates to the second stage of Lancy's cognitive theory, in which "what happens to cognition [...] has much to do with culture and environment and less to do with genetics" (p. 205).

The second and the third stages of the three-tier teaching model have connections with the third stage of Lancy's cognitive development theory, the metacognitive level. In Lancy's third stage pupils "learn what kinds of knowledge are important for what purposes; they learn the relationship between knowledge and status; they learn the appropriate occasions for knowledge acquisition and display; and so forth" (p. 208). In other words it focusses on relationships between the different cultures of mathematics which is explicit in the meaningful presentation of mathematics to pupils in tiers two and three.

Although this study involved only two sessions in one school, the results illustrate how the three-tier model can be used to introduce fractions meaningfully to children, in contrast to starting with the school notion of halves, thirds and so on, as stated in the Ghanaian primary school mathematics curriculum (Ministry of Education, 2012). The three-tier teaching model can bridge gaps between pupils' practical knowledge and theoretical knowledge.

Note that the intent is that the three-tier model be used throughout grades one to four. Subsequent to primary school level it becomes more difficult because the policies of Ghana, as in some other African countries, do not permit the use of local languages beyond the primary school level. However the model should not be interpreted as focussing on everyday knowledge in the early grades and shifting to school mathematics later. As in the example given above, each topic moves through the three tiers. Otherwise, this makes the transition between schools in different contexts difficult. In fact, all three tiers come together in each lesson to help pupils to understand the mathematics they learn in school, and make links between the mathematics they learn in school and mathematics in their society.

Clearly, there are implications for teacher education in this proposal. Existing teacher training does not generally equip teachers to draw on pupils' social and cultural background to inform their teaching (Akyeampong, Lussier, Pryor & Westbrook, 2013). Further work is needed to identify the best ways to introduce such teaching approaches to new and practicing teachers.

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From the Archives

The following is an excerpt from *Learning mathematics in a second language: some cross-cultural issues* by John W. Berry, in issue **5**(2), pp. 21-22.

The issues and questions raised above clearly have relevance far outside the African context. As Austin and Howson have noted, they arise in one form or another whenever there is a mismatch between the language, the culture, or the logic and reasoning system of the student, the teacher or the textbook author. Most of the research to date has been conducted by psychologists and anthropologists and has had surprisingly little impact on educators. Indeed, little concrete action at the instructional level has occurred. This is in part understandable, since the problem is most severe in those countries like

Botswana which have few trained mathematics educators of their own and are still largely reliant on expatriate assistance. It underscores the urgent need to train nationals of the country to assume the task of designing curriculum and instructional strategies which respect the natural modes of thought which arise from the language and culture of their own people.

One of the goals of such a curriculum would be to encourage the students to think mathematically in Setswana. If 'thinking mathematically' requires assimilation of 'western' cognitive modes and strategies then it is clear that this can only be a long term goal of the school system in a country such as Botswana. The present curriculum from primary through secondary school and university is based completely on cognitive modes derived from English (or other Indo-European languages), even though the language of instruction (at the primary level) is Setswana and the child's natural patterns of thought are derived from Setswana. What we propose is the development of an alternate mathematics curriculum which builds on the child's natural thought modes and gradually and continuously encourages the child to assimilate the cognitive skills required by the 'western' curriculum to his own Setswana schemata. There should be no abrupt changes at any point, except perhaps for a switch in the language of instruction at the elementary-secondary interface. Even here one can conceive ways of making that shift gradually (as is often the case in practice, particularly when the teacher is a Motswana) [...]

What this points to is a model of curriculum building different from that normally assumed in mathematics. Such a model would begin from a starting point of assumptions about the learner's cognitive structures rather than from assumptions about elementary mathematics as a 'given'. Since the goal would be to end up with recognisable, 'standard' mathematics, such a model would have to be dialectical, pulling the child's thinking toward the processes required by standard mathematics, while at the same time pulling the mathematics toward the current cognitive style of the learner. In the early years of school the gap might be quite large, and one of the guiding principles of such a model would be to narrow the gap as the child progresses. But at any rate, one would have to abandon the ideal of 'doing everything the correct way' (in terms of standard mathematics) right from the start.

In the primary grades, such a design would require major changes in the curriculum objectives away from many which have been stressed in modern mathematics programmes developed since the 1960's. But we stress once more that the very nature of this type of curriculum change dictates that it must be carried out [by] the Batswana themselves. No expatriate advisor would bring to the task the necessary sensitivity to the cultural aspects of cognition which we have stressed above. Their input may be useful, indeed essential, but it cannot be the final determinant of the content or methodology used. It follows that the training of nationals of the country to an advanced level in mathematics education methods and curriculum design is a matter of the highest priority. Only as this is being accomplished can the curriculum design task begin.

The development of such a mathematics programme should be carried out right up to the university level. A longitudinal study comparing the effectiveness of such a curriculum to more traditional (i.e. European translated into Setswana) forms would be of great interest. We have discussed this curriculum as a continuous process leading up to the university level study of mathematics. However, we believe that the type of curriculum change advocated here has major significance for the society as a whole, not just the minority of students destined for university. Indeed, it may be even more important for the majority who leave school at an earlier stage. As the influence of 'western' technology impacts more and more on the daily lives of the Batswana, the type of cognitive adaptation we have discussed will grow enormously in importance. Indeed, one can argue that unless it takes place the progress of the country's economic and technological development will be severely impeded. It is absolutely crucial that the schools encourage and assist this change rather than impede it, and that they do so in a manner in consonance with those factors which determine the cognitive landscape: the traditions, culture and language of the Batswana themselves.

On the challenges of multilinguisme in mathematics education research

VINCE GEIGER, CLAIRE MARGOLINAS, RUDOLF STRÄßER

This comment is a response to the collection of short communications in **37**(1) headed *Challenges and opportunities related to linguistic and cultural diversity in research and publishing,* two of which cited Vince and Rudolf's article in **35**(3), *The challenge of publication for English non-dominant-language authors in mathematics education.* Vince and Rudolf have been joined here by Claire Margolinas in order to provide a complementary (French) perspective to their Australian and German outlooks on this topic. The comments of the three of us can be grouped into two broad themes, namely, those related to language issues including translation and those related to research in mathematics education or didactics of mathematics. We argue that that these two issues are not separate, but can be seen as closely linked to each other.

Together with Osnat Fellus and Florence Glanfield (*Reflections on the FLM pre-conference*), we see the notion of building relationships and connectivity as central to our productive professional collaboration. Vince and Rudolf have had a longstanding relationship, as colleagues with common interests in mathematics education, which led to a personal friendship. It was the *connectivity* of this relationship that allowed Rudolf to feel comfortable asking Vince for a language check of a planned presentation—which led to our FLM article. As noted by Fellus and Glanfield, the concepts of *proximity, activation,* and *space* (Axelrod & Cohen, 2000) played a role in starting this collaboration: *proximity* as there was a context in which our relationship was forged; *activation* because of Rudolf's need for a language check of his paper; and the *space* that was related to our shared interest in promoting mathematics education.

We argue, however, that building such relationships depends on personal, real connections with others (*e.g.,* in non-virtual conferences). For example, when Rudolf and Vince decided to share ideas about diversity in language within mathematics education research via an annotated translation of their original FLM article for *Recherches en didactique des mathématiques,* they asked a French colleague, Claire, to join them. Rudolf and Vince knew Claire from different work relationships and meetings at conferences, and so it was easy to ask for her collaboration and profit from it—even if she was not an obvious member of their 'usual' or 'standard' community of research practice.

By putting our ideas on translations to the test with French, we (now Claire, Rudolf, and Vince) have learnt about new challenges, such as *false friends* and *black holes* (*voids* as Osnat Fellus calls them) in different languages, in addition to the question of differences in granularity of languages. We have also been forced to think about the challenge of positioning research in different scholarly communities, including the positioning of our own work within less familiar research landscapes; that is, shaping our ideas to articulate with new (for us) paradigms and formats (the three of us are now preparing an article *On translating research in mathematics education*). In reworking the FLM article for RDM, we also experienced various difficulties associated with the translation of technical terms into another language (like the one described by Cole, mentioned by Fellus and Glanfield). Thus, we have seen first-hand that publishing a scientific paper in another language requires more than professional translators, as specific nuances of language associated with a discipline or scientific field are important when communicating research.

Why are professional translators not able to breach language and cultural barriers in a scientific field? One might understand why it is so difficult to translate poetry, but not why it is so difficult to translate a scientific text that seems, at first, a much more straightforward task. In the past, Claire had the experience of sending a paper translated by a British professional translator: the reviews came back twice with a comment to "check the language with a native". The translator was furious! Claire (and her co-author Floriane Wozniak) had tried to produce, in French, a text which was acceptable for an anglophone environment, doing exactly what France Caron explains in her short comunication, that is, writing a very different paper than what she might have published in French by drawing on what was thought acceptable for the target journal. Sentences were even cut into shorter ones to facilitate the translator's work and to try to imitate, in French, the final English text. What was produced was a sort of *alien*: not a French and not an English text. Because of all these efforts the paper was finally published, but it was a difficult experience—perhaps due to the challenge of establishing our legitimacy through the use of delicate wording that is part of managing implicit meaning through *indexicality* (Dressen-Hammouda, 2014). Even in a scientific publication, the reviewer is influenced by unconscious linguistic phenomena which are not related to the scientific content of the paper and this is certainly why Claire and Floriane were asked repeatedly to ask a native English speaker for help.

So what maintained Rudolf's and Vince's interests in working together on the issue of language diversity in mathematics education research after Rudolf had presented the reaction paper on which he had originally asked for Vince's help? We think that the notion of *Wicihitowin,* helping each other to develop in shared responsibility and stewardship, as discussed in Fellus and Glanfield's communication might go some way to explaining our continuing efforts, as we saw others facing the same challenge. As mature and maturing academics we felt responsible for helping others facing the same plight. Thus, there was both a sense of responsibility to others and of custodianship towards the field of mathematics education. We would add, however, that this was not just a matter of altruism, particularly in Vince's case, as many good ideas and ways of thinking are currently not accessible to the English-only speaking community. Thus, finding ways to mitigate the barriers that inhibit the publication of non-dominant English language authors in international journals means that all parties will benefit. Claire's introduction to our team strengthened our resolve to share our reflections about the translation of research and consider this work as a legitimate scientific production and not just a necessity that all the researchers have to undertake alone. This is difficult work but essential as a translation of a concept impacts on all the efforts of a research community to communicate within the research community.

The issue of shared responsibility and stewardship also connects with the difficulties reported by Caron when deciding in which language to publish, when the option of a real choice is available. Caron added to this by showing that the decision is not just a matter of ability and preference, but one which may be dependent on the topic in focus. Different publication languages may feel differently appropriate to describing different issues. The choice of theoretical framework, for example, may be constrained by language and culture and not just the nature of the research itself.

Moreover, when we choose to publish in a language which is different not only from our own but also from the dominant language of our scientific community, we are confronted with the translation not only of words but of concepts expressed in words which have been chosen or coined within a specific language. For instance, the French word *didactique* does not have the same negative connotation as the English *didactic* (nor does the word *Didaktik* in German). On the contrary, as it is the name of a research field which is now known by teachers, educators and education researchers, this word is not only acceptable but the only one we use in our (French and German) linguistic environments. The fact that another possibility has been used in English (*Education/ Mathematics Education*) limits the possibility to use *didactics* as a legitimate word and people may say, "in English it has a negative connotation". But this has some important implications, for instance, *didactics* indicates a possibility to link *mathematics didactics* with all the fields which have an interest in teaching discipline specific knowledge, whereas *education* is a totally open field which refers to all the community which has an interest in education in general, not necessarily related to specific knowledge.

At the same time, as Caron expressed, there is a tension between sharing scientific ideas internationally—influencing the field—and the need to publish in the dominant language of the community you wish to influence in terms of practice, for example, changing everyday school practice for teaching and learning mathematics.

From our reading of the communications in **37**(1), one feature stands out. Although all are concerned with the teaching and learning of mathematics, there are only very faint allusions to mathematics itself. A search for statements on mathematics as such produces two assertions: Caron quotes Karine Godot, who was surprised by the relative absence of mathematics in an international conference on mathematics education ("j'ai été surprise par le peu de place accordée aux mathématiques dans la plupart des exposés."). On the other hand, the "Math Guy" in the play (or is it a protocol of the pre-conference discussion?) by Maheux gives the impression that mathematical language is the only place without ambiguity—an assertion, which was not accepted among the audience (see especially page 23). What is the relationship between mathematics and its (re)presentation by language? Is it a tenable epistemological position to take the presentation of mathematics as the whole of mathematics (as is basically done by some didacticians like Willibald Doerfler from Austria)? What about consequences for teaching and learning mathematics? Regardless, it seems reasonable to start from the assumption that there is a special relationship between scientific, disciplinary mathematics and language, be it written or spoken and that this has been underplayed in the discussion to date.

A final question by Caron reminded us of the relationship between the concepts of centrifugal and centripetal forces—"The question here is where does the center lie?" Originally, the two constructs are used to describe the tendency to foster (or not!) a use of words and grammar consistent with standards set by official guardians of a given language (*e.g.,* Académie Française for French, the Duden for German, the Oxford Dictionary of Current English). Hence, centrifugal or centripetal forces (in the narrow sense) effect only language issues. However, the very same constructs, centripetal and centrifugal, can be used, in a metaphorical way, for the tendency to promote habits and approaches, if not paradigms, from certain scientific schools (*e.g.,* ATD or TDS in French "speaking" research or a traditional approach to empirical research by means of mathematical statistics in Anglo-Saxon research). At least in some countries and within certain research communities, certain scientific journals are said to be more or less open to specific approaches to research or are more or less tolerant towards divergent types of publications. A major reason for choosing FLM for our first step into the world of *language diversity in mathematics education* was this journal's reputation for being open in terms of publication languages and also having a willingness to promote new or *avant-garde* directions in mathematics education research. As this example demonstrates, a single publication can be centrifugal language-wise, while being centripetal in terms of its scientific approach and vice versa.

We end our short remarks by endorsing the indigenous perspective of taking responsibility for bringing people together. This is a very interesting cultural perspective, which stands in sharp contrast to a world that invites competition. Should this responsibility be part of an academic's training above that of writing a thesis, which in itself is a self-absorbing enterprise? In a globalised and competitive world, what scope is there for elders and/or scientists to take responsibility for bringing people together? Would a world more oriented to co-operation be a better, more productive, and a more intellectually stimulating place to live in?

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