MATHEMATICAL ARTIFACTS AS COMMUNICATIONAL FACILITATORS BETWEEN INDIGENOUS AND NON-INDIGENOUS CULTURES

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In this article, we discuss how the artifacts used by Maxakali Indigenous teachers to represent and operate with numbers contribute to overcoming linguistic constraints and to improving communication in a mathematical activity in non-Indigenous context. We are teacher/researchers in a teacher education course for Brazilian Indigenous People from different ethnicities, offered by our university. In our teaching experience, we have observed several powerful examples of how linguistic competencies and knowledge are required to participate in classroom mathematical activities (Tomaz, 2013; Tomaz & Campos, 2018). There is a complexity to mathematics teaching in Indigenous intercultural contexts because the languages are predominantly oral, so the symbolic expressions of mathematical relations do not match the syntactic structure of the corresponding oral expression (Parra, Mendes, Valero & Ubillús, 2016). Here, we consider the mathematical activity of members of a group of Maxakali-speaking teachers with little mastery of Portuguese and a singular worldview who strongly challenged us to develop new forms of communication.

In recent works devoted to analyze classroom activities, we used cultural-historical activity theory perspectives (Engeström, 2015) in order to clarify the role of visual representations for structuring mathematical activity in a monolingual classroom (David & Tomaz, 2012; David, Tomaz & Ferreira, 2014). In this article, we adopt a similar perspective in order to analyze mathematical activity of Maxakali-speaking teachers in a mathematics outreach course, since it proves to be a framework of analysis capable of capturing the complexity of the multilingual classroom activity and illuminating tensions between different worldviews in order to discuss how mathematical communication constraints can be attenuated.

The Maxakali Indigenous people

In 2010, there were almost 2000 Maxakali individuals in Brazil (IBGE, 2010). Their territory is located northeast of Minas Gerais, Brazil. They speak the Maxakali language and live in patriarchal nuclear families in houses arranged in a semi-cycle around the *kuxex* (religion house), with the *mimãnãm* (ritual mast) in front of it. This is where the main social and spiritual activities of the community take place. In their system of knowledge, there is usually no clear separation between the material and the spiritual/sacred world. In line with this, Maxakali knowledge is collectively produced from people's experiences in these two worlds. Thus, teacher education with the Maxakali demands that the researcher/instructor grasps that, as Wagner (1981) claims, "every understanding of another culture is an experiment with one's own" (p. 19).

Viveiros de Castro (2014) states that creating a relation with Indigenous thought and practice is no simple task, because the Amerindian world operates quite differently from the non-Indigenous. There is no notion of a vital principle or a spirit attached to the body, but rather a transformational principle. This transformative principle is the point of liaison between the human and animal species.

> Why is it that animals see themselves as humans? Precisely because we humans see them as animals, while seeing ourselves as humans. Peccaries cannot see themselves as peccaries (or, who knows, speculate on the fact that humans and other beings are peccaries underneath the garb specific to them) because this is the way they are viewed by humans. If humans regard themselves as humans and are seen as nonhumans, as animals or spirits, by nonhumans, then animals should necessarily see themselves as humans. (p. 69)

The human sees the jaguar as an animal rather than a person; the jaguar sees itself as a person and the human as an animal. The social life of the Maxakali can be seen as aligned with Amerindian thought, as it is structured around their ritual-ceremony life, which is guided by the *yãmĩyxop* (singular *yãmĩy*), which are compounds of spirits, ritual songs, and the events in which the spirits show themselves. That is, "They are at the time the groupings of ritual song repertoires, the images of *spirit-people* who visit the village, and the situations of encounter and exchange occurring between them and other people" (Tugny, 2014, p. 160, our translation). The *yãmĩyxop* regulate a wide array of activities through socio-political mechanisms, supported and reinforced by their cosmological-ritual system.

Among the *yãmĩyxop*, the *Po'op* (monkey spirit-people) sings in the *kuxex* mimicking when the *kemĩy* (bird), another

yãmĩy, arrives with the returning hunters in the village. In this ritual song, the *Po'op* counts the number of animals hunted to share with the women, guaranteeing that all receive the same share. This *Po'op* also sings to count the money brought to the village by the non-Indigenous people. According to Maxakali teachers we have spoken with, the ritual song of this *Po'op* can be seen as related to mathematics.

The *kuxex* and the *mimãnãm* mark that the *yãmĩyxop* are in the village. In the *mimãnãm*, there are inscriptions that represent songs as drawings, words, pictures, sounds and colors (Tugny, 2011, p. 8) to be not only seen but *read*. The Maxakali use other metaphors from sacred contexts to understand the reading and writing processes. For instance, *Kax'ambix* (writing) can be literally translated as 'drawing the song'. They also often compare writing with the *Mĩmkũĩn* (or *taquara*, in Portuguese), which is a bamboo stick used to sing and record their ritual songs, such as the *Po'op* song. It has marks, like a sound scale, which represent a song as well as the order in which to sing it, associated with the numerical order (1, 2, 3, …) (Vieira, 2006, p. 171). According to Matos (2001), Maxakali language has only three words to express quantities: one (*ũmxet*), two (*ũtix*) and three (*ũtikoyuk*). When bigger quantities are needed, such as to measure time to comply with post-childbirth restrictions [1], they formerly used at least two procedures: adding dashes on a bamboo stick or breaking sticks to register the days and months. In school, to express greater amounts, they adapted words from Portuguese such as *tinit'ta* for thirty (from *trinta*, in Portuguese) and represent them with Hindu-Arabic numerals (30).

The Maxakali people are not isolated; they are in permanent contact with non-Indigenous society. Therefore, two sources of knowledge, traditional and nontraditional, come into contact and connect to produce a frame of understanding and validation, that gives meaning to the world around them, but not without conflicts and prejudices. According to Tugny (2014) the Maxakali-speaking people live among the worst social and environmental conditions faced by Indigenous peoples in Brazil. They are faced with serious environmental and health problems (for instance, alcoholism, diarrhea, worms) because of forced displacement from their land by non-Indigenous people, pollution of their water, and inhospitable land. Therefore, they suffer extreme food shortages and insecurity.

Nevertheless, they are known for their strong cultural resistance to the different forms of violence inflicted by non-Indigenous society. These are some characteristics that ascribe to the Maxakali a distinct view of the world when compared to other Indigenous peoples of Minas Gerais, which is a challenge for public agents, teachers and researchers, who are seeking to understand their way of life and to respond to their needs.

The scenario

The mathematical activities discussed in this article occurred in an outreach course developed following an assessment made during a visit to the Maxakali's territory, in 2015, aimed at knowing better their schools and living conditions. In the four previous years, some Maxakali attempted the entrance exam to our university [2], but only one was approved. We decided to investigate the reasons for their lack of success.

We observed that in the public schools in the villages, Maxakali teachers, mostly without formal teacher education, teach in Maxakali to students up to the age of 14. After that, non-Indigenous teachers teach all subjects in Portuguese, treating the students as if they had no prior schooling. The Maxakali school can be seen as a bridge between the Maxakali world and the outside world, but although there are legal guarantees for bilingual and differentiated schooling, the government does not recognize the Maxakali public schools as equivalent to high school. Even if they have attended a Maxakali public school for more than 10 years, students do not receive a high school diploma. Consequently, these students do not have access to higher education in Brazilian universities. As a result, in 2015, there were only five Maxakali who had a high school diploma, and, thus, were allowed to study at university.

At school, the Maxakali children first learn how to read and write in Maxakali. After that, when they are introduced to Portuguese at about age 14, some notions of division and addition are taught to be able to share the goods received, captured, or produced through their experiences within and/or out of village. The teaching of mathematics in the Maxakali school is done using Portuguese translations of the Western school materials, aligned with what Burton (2009) calls "near-universal conventional mathematics" [3]. In this context they learn aspects of mathematics they will need to deal with the challenges of the external world, such as how to write dates, and the Western counting system, but most Maxakali children attend these Portuguese classes for a short period of time and then drop out.

In a meeting during the initial assessment, the Maxakali teachers said they wanted to improve their mastery of Portuguese and to have a Maxakali mathematics teacher in their community, arguing the need to learn mathematics earlier in school in order to address difficulties with commercial transactions with the non-Indigenous and with bureaucracy. They suffer economic exploitation by non-Indigenous people, as access to and use of the government grants they receive demands technological and market procedures they had not mastered well.

As a result of this meeting, an outreach course prioritizing activities of reading and writing both in Maxakali and in Portuguese was developed. The main objective was to enable the Maxakali teachers to apply for a high school equivalency diploma to overcome one of the barriers preventing them from studying at the university level.

In the course, some activities were proposed aimed at the mathematical language and procedures required by banking and commercial transactions using Brazilian currency (Reais). We avoided the use of conventional school algorithms in those activities as we supposed that the Maxakali Indigenous teachers might not be acquainted with them. The teachers received calculators and were taught how to use them in commercial transactions. Nevertheless, some opportunities were also created for the group to count, quantify, and record numbers.

There were three phases in this course amounting to a total of 200 hours both at the university and in the Indigenous villages. A group of 10 Maxakali teachers without prior teacher education (4 women and 6 men) was selected to

attend this course [4]. The course instructors were a mathematics educator (Vanessa Sena Tomaz) and a professor of Portuguese literature (Josiley Francisco de Souza) [5]. Both had been instructors in education courses for other Indigenous people, but it was the first time they were working together, and neither had mastered the Maxakali language. The skills of the group of teachers varied a great deal regarding their mastery of Portuguese and their arithmetic skills. Both Maxakali and Portuguese were used in classroom conversations and, when needed, translated by a Maxakali undergraduate student (Lucio). Records were made in both languages.

One source of empirical data used for this research were videos recorded by the Maxakali teachers themselves, at times they chose. These video records were used primarily as pedagogical material to monitor the performance of the teachers during the classes, when new subjects were introduced, or to reflect on those subjects. Selected excerpts of these videos were exhibited to the teachers, to discuss what was being used, why, and other ways to solve and explain a calculation. This methodology created favorable conditions for developing teaching activities with this group of Maxakali teachers. After one class, which is the focus of this article, Vanessa began to record the classes herself, with the authorization of the Maxakali teachers. Both sets of videos, those recorded by the teachers and by Vanessa, were used in the analysis. The empirical material also includes copies of tasks solved by the Maxakali teachers, and one videotaped interview with Lucio, who was a translator in the classroom, that took place six months after the first videos were recorded.

Tikmũ'ũn xohi' texĩy pip? (How many people are there?)

The activity "How many people are there?" was designed to help us understand the poor performance of the Maxakali Indigenous candidates in the Entrance exam in 2014, even though the test was translated from Portuguese to Maxakali.

Hõnhã' nũxi' 'ũxit kukpa' 'ũxohi' texĩy yũmũg xate' nõm yũmũg tikmũ'ũn 'ũxohi' texĩy.

Tikmũ'ũn yõg hãm xomã'ax yũmũgã'ax kax'ãmix	**'Ũxohi' texĩy?**
Tikmũ'ũn xohi' texĩy pip?	
Yãyta' xohi' texĩy pip?	

Agora **PREENCHA** o quadro abaixo com a quantidade aproximada que você acha que tem na sua comunidade.

Informações sobre sua comunidade	**Valor numérico**
Número de pessoas	
Número de famílias	

Now **FILL IN** the table below with the quantity that you think that there is in your community

Information about your community	Numerical Value
The number of people	
The number of families	

Figure 1. The first questions in the original task from the Entrance exam [6], which was presented in both Maxakali and Portuguese. The English is our translation of the Portuguese version.

In the first class, the Maxakali teachers were asked to fill in a table with the estimated quantities of people, families, children, women, men, and schools in their communities. This activity was based on a similar task in the Entrance exam (Figure 1). It offered possibilities for them to deal with large numbers, reading, recording and making calculations with them.

In the first class, Vanessa explained that the question they were supposed to answer in each case was 'How many?' and clarified the sort of response they were expected to give. She also explained that they should write the number in the space provided in the table. These explanations were necessary since she noticed that some of the Maxakali teachers did not understand the questions, although they were in Maxakali. She also noted that it was not clear to them what was meant by 'community' in this context. Edmonds-Wathen, Owens, Sakopa and Bino (2014) call attention to the difficulty in translating words into Indigenous languages because "words that initially present as good translations for a mathematical term may have different scope of use in Indigenous languages" (p. 209). A related difficulty of translation is represented in the task from the Entrance exam shown in Figure 1. The Portuguese noun phrase "valor numérico" (numerical value) becomes a question in Maxakali "'Ũxohi' texĩy?" (How many?).

To discuss the numbers estimated and recorded by the teachers, Vanessa drew a summary table on the blackboard, writing row by row the numbers of people in the community, for each teacher (Figure 2).

The first number, presented by Isael [7], was 412. After writing this number on the table, Vanessa reviewed its decomposition: 412 = 400 + 10 + 2, orally and on the board. When she asked if there was another way to write 412, Marilton went to the blackboard and wrote the number 412 in a place value chart.

Below an excerpt of this lesson.

Marilton I forgot to make, I also did this too, to know, neh? Because there is, mathematics, neh? like this *[drawing the place value chart on the board; see Figure 3a]*

Vanessa Ahhhh, the place value chart!

Number of People	
Isael	412
Marilton	840
Pauliana	45
Marquinhos	850
Major	604
Lucio	642
Damião	640
Jami	603
Elizângela	414
Paulinho	414
Maiza	414
Sueli	414

Figure 2. The table with the number of people estimated by each teacher.

Marilton So, when I, here, ehhhh, unit, tens, and, hundreds.

Vanessa All right,

Marilton So, I have to, count, neh? *[showing the decomposition of the number 412 in units, tens and hundreds]* after you place, here is the unit, two, neh? here is ten *[Isael repeats Marilton's speech]* one, here is hundred, neh?

Vanessa This is what we call the place value chart.

Marilton So there is, there is, to the left, neh? *[making gestures with his arm to the left, indicating higher powers of ten]*

[Marilton speaks in Maxakali, while adding the thousands column (see Figure 3b) and filling in the digit 1]

Vanessa So, what is the number now? *[pointing to the number 1412 suggested by Marilton]*

Marilton One thousand, four hundred and twelve *[Isael repeats his words]*

Vanessa Okay, so, pay attention, to write numbers in mathematics, each place has a different value, pay attention,

[Vanessa begins to use Marilton's chart and Marilton speaks, in Maxakali]

Marilton One real, *õheap mõg tu nãm tu kux ['one, two, up to nine', showing the units place]*

Vanessa One *real*, one unit,

Marilton One unit,

Vanessa Two, two units *[indecipherable]* when he counts the tens,

Marilton Ten, ten,

Vanessa If he counts tens it is one ten

Marilton Up to nineteen,

Vanessa But the 1 here is one ten, it isn't one unit, its value is 10, if I put the number 1 here *[showing the hundreds place]* it is in hundreds place.

Marilton One hundred *reais, nexyẽn mõg tu nõmet tu kux ['one ten up to ninety']*

H	T	U
4	1	2

TH	H	T	U
1	4	1	2

Figure 3. Marilton's place value chart: (a) initial representation, (b) extended representation.

Vanessa If I put 1 here *[showing the thousands place]*

Marilton One thousand *reais [indecipherable]*

Marilton *Pu ũkax ãmix õmxop õhõm huyũmũg tatu nõom a hõnhã jũmũga'a panopeaxot'te payã'xex kã'õgma ãte ũg'putox kopa. [Marilton speaks to his peers: 'you have to write it so that you can remember, because a long time ago a teacher wrote it for me, now I am remembering'].*

Marilton spontaneously expands the chart to higher powers of ten and adds the numeral 1 to represent one thousand units. Marilton reads in Portuguese *"fica mil, quatrocentos e doze"* (one thousand four hundred and twelve) and Isael repeats his words. After that, Vanessa resumes the explanation of the meaning of each position: units, tens, hundreds. Marilton actively participates in this dialogue associating each numerical place value to monetary values (1 unit is 1 *real*; 1 hundred is 100 *reais*; 1 thousand is 1000 *reais*).

Following Marilton's lead, Vanessa starts using the place value chart to check if the numbers (in Figure 2) were represented correctly, stressing the principle of positional value, and making the same association, as Marilton did, between the numerical place values (U, T, H, TH) and monetary values. As a final task, she asked them to compare the numbers in the table shown in Figure 2. When making the comparison between the number of people in Major's community (604) and Lucio's community (642) several teachers, at first, said that the number of people in Major's community was larger. After some discussion they concluded that the opposite was true and Vanessa asked how many more people there were in Lucio's community when compared to Major's community. Vanessa wrote both numbers in the place value chart, while the teachers discussed, in Maxakali, the meaning of the question. After that, one teacher answered 40 and Maiza said that it was 38. Vanessa drew marks on the blackboard and they counted one by one from 604 to 642, concluding that the difference was 38. Next, Vanessa questioned the teachers if there were other ways to calculate that difference, for example, using a calculator. At this point, Marilton suggested doing the conventional subtraction algorithm, because for him it was the easiest way to answer this.

After watching videos of the first phase some months later, in the second phase of the course, they discussed the procedures they used to represent, compare and record the numbers in the table shown in Figure 2. They commented about the choices they had made at that time, including the use of the place value chart to represent the numbers. Marilton explained that with the place value chart it is easier to learn, because he could put any number (meaning up to nine) in the units or tens positions. Two others, Lucio and João Bidé [8] echoed Marilton's opinion. Other ways of doing the calculations became evident when the two instructors were discussing the videos with the teachers and they were commenting on how each one had done or could have done the calculation.

For example, in the following transcript Maiza explains how she got the answer 38, starting from a colleague's incorrect answer (40), through an alternative procedure:

Maiza	I started like that, going down *[gestures downwards]* and then going up,
Vanessa	Like Lucio?
Maiza	And I went down *[gestures]*
Vanessa	How did you go down? Did you go one by one?
Maiza	No, *[speaks in Maxakali]* because the one from Lucio was 642 and the one from Major was 604, so I reduced Lucio's (number) to make it equal (to Major's number) because, they said that they should increase 40, neh? So I thought, because 40 was not going to work, then I thought, I'm going to take 42 and I'm going to diminish, I took 42, 41, 40, 39, and 38, and it was right.

As Engeström (2015, p. xxiv) argues, every human activity is by definition a multi-voiced formation. Because it is a collective formation, it is not possible to reduce an activity to a single perspective, to a single subject, as its multivocality is a source of innovation, richness, and resilience. From the dialogue between subjects with different linguistic repertories and different skills with numbers and arithmetic calculations, emerge several ways to make the calculation 642 – 604, enabling the expansion of the linguistic repertory and the arithmetical skills of the subjects.

Bridging the gap: multimodal mathematical writing?

In this analysis we characterize the activity 'How many people are there?' as an activity system (Engeström, 2015) whose object is the estimation, comparison and calculation of the number of people in the teachers' communities and its representation in Hindu-Arabic numerals. According to Engeström's (2015) perspective, an activity is always understood as a collective phenomenon in a community, and individuals can only perform actions inside a larger system of collective activities. Every activity system is characterized by its object that can be expressed as concern, motivation, effort, and meaning, that is, the 'problem space' towards which the activity is directed. The activity system described here involves the use of several artifacts including the tables (that restrict the space for the representation of the numbers), the place value chart and the addition and subtraction algorithms. In this theoretical perspective, artifacts are the results of a process of communication between the self and the other. Artifacts encompass signs of historical and cultural contexts in which they are created and they are continuously recreated through their own use, making the culture a system of accumulated artifacts of a community. Indeed, artifacts are considered 'unfinished projects', mediating the relationship with the outside world in a new and dynamic way (Engeström, 1990).

The Maxakali teachers were led to use Hindu-Arabic numerals, directing their actions towards a system of linguistic codes which they had not totally mastered, and in which they could express 'big numbers' for which they did not have available Maxakali words. This created some tensions, as well as opportunities for new learning. According to Barwell, tensions are an inevitable part of linguistic diversity in classroom (2012, p. 317). In this case, the artifacts the teachers mobilized allowed them to organize the numbers' representations and to consider amounts up to nine, column by column, without considering/perceiving the global quantity. In addition, these artifacts seem to have aligned their actions with school procedures, "to pacify the white man" (Vieira, 2006, p. 182).

According to Engeström (1990, p. 171), we may take two views of artifacts: the system view and the personal view. Under the former, typically taken by the researcher, the artifact is seen to enhance the performance of the entire activity system. According to the personal view, taken by the subject, here the Maxakali teachers, the mediating artifact changes the nature of the task the person is facing.

Initially, as Vanessa was focused on the activity system, and she was the authority, she defined the linguistic code that should be used to register the quantities estimated by the teachers. She decomposed the numbers in order to discuss the quantity represented, taking this decomposition as a mediating artifact.

But, when Vanessa asked if there was another way to represent the number 412, after making its decomposition, Marilton introduced a new artifact (the place value chart), revealing the relation he made with other artifacts and the rules that govern the writing of numbers in non-Indigenous school. Marilton's recall of such a conventional school artifact initially came as a surprise to Vanessa, given what she knew of the school experience of the teachers, but she ended up understanding why this number organization was a better choice for them than the decomposition, because they were not used to interpreting such big numbers as quantities.

We believe that Vanessa's decomposition influenced Marilton's use of the place value chart as a number writing structure, without having to deal with the number globally, as an amount. This shows how the use of an artifact always implies more possible uses other than the one originally intended (Engeström, 1990, p. 174). In this case, it is important to pay attention to the personal view of the artifact, because it reflects Marilton's choice and his use of the artifact.

For some time Marilton directs the activity, using the empowerment gained with the new artifact, even to the point of redirecting the instructor's actions. Vanessa starts using the place value chart to structure the discussion about the rules for writing 'big numbers' in the decimal system, and tries to help the teachers relate the written number to the quantity represented. Later on, Marilton introduces one more artifact (the subtraction algorithm) as an easier way to make a specific calculation (642 – 604).

The number of students

In the next situation, the teachers were required to fill in a table with the number of students in the classes of the 10 teachers and of Lucio (the undergraduate student and translator), and the total number of students they were teaching. All writing was in Portuguese (Figure 4). They then had to find the sum of these eleven numbers.

Maxakali teachers	Number of students
Jami	16
Maiza	23
Elizângela	24
Paulinho	26
Damião	11
Major	15
Marquinhos	30
Sueli	17
Pauliana	25
Lucio	20
Marilton	27

Figure 4. Translation of table showing the number of students of each teacher.

Vanessa discussed the number of students of each teacher and how they calculated the total. Damião, Elizângela and Maiza used the conventional school algorithm.

Vanessa To add, someone was able to add?

Vanessa Oh! Did Damião make the addition? *[Marilton says something in Maxakali]* what was the total?

Damião Two hundred and thirty and seven

Vanessa Two hundred and thirty and seven, someone else also added?

Maiza/ Elizângela We added, we added

Vanessa Okay. The total was two hundred and thirty and seven *[Vanessa writes the number on blackboard]* they *[Elizângela, Maiza and Damião]* added, good, Damião, could you explain to us how you did it?

[Damião remains silent for 13 seconds staring at his paper]

Vanessa Do you want to go to the blackboard to explain?

[Damião stands up and goes to the blackboard]

Damião I am going to stay here for a long time. *[He talks to his colleagues, in Maxakali]*

After that, Damião wrote the addition algorithm on the blackboard and explained his procedures, in Maxakali, showing the addition by place value (first units, then tens, *etc*., see Figure 5). His colleagues helped him with the calculation.

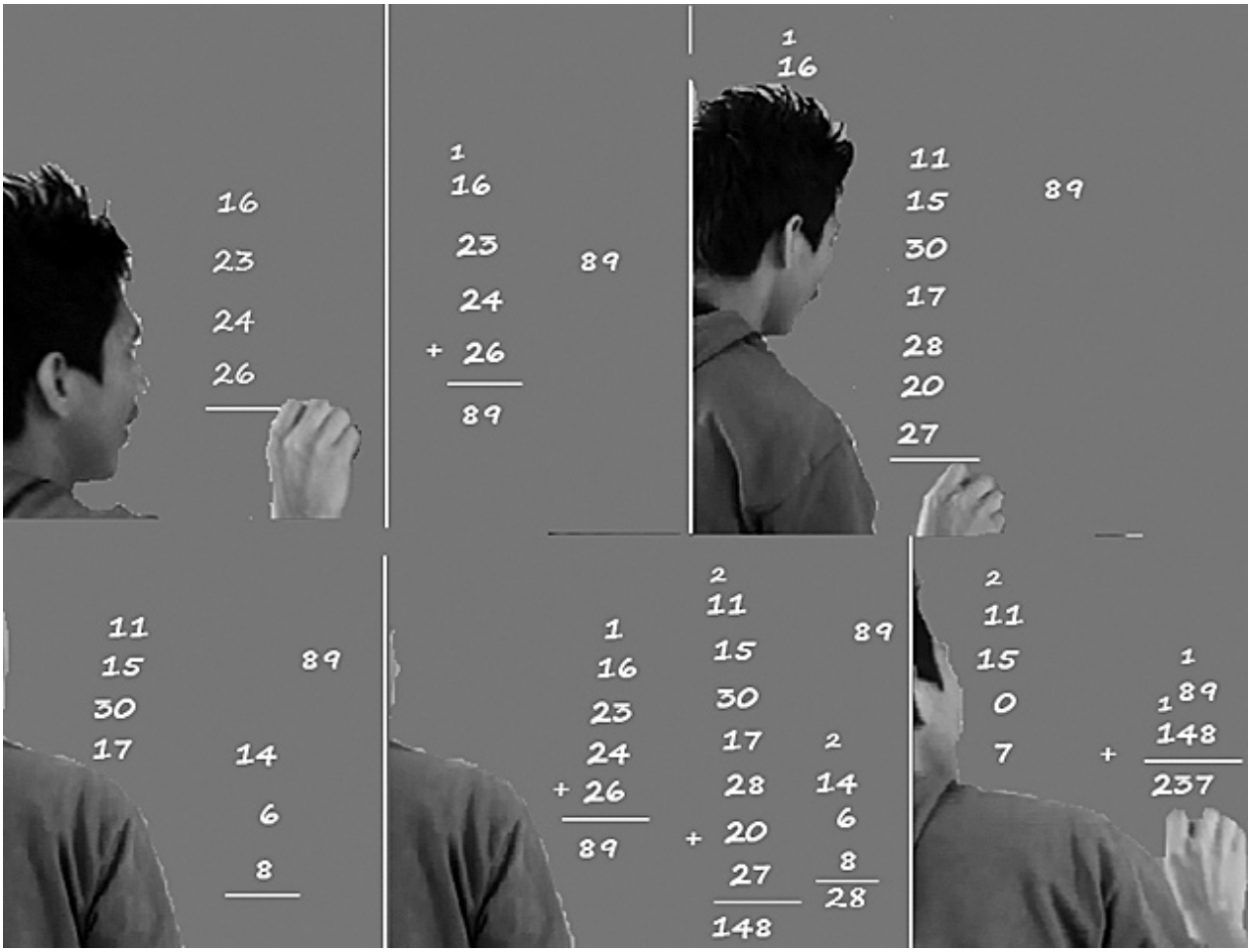

Figure 5. Damião's writing of the addition algorithm (writing enhanced for clarity).

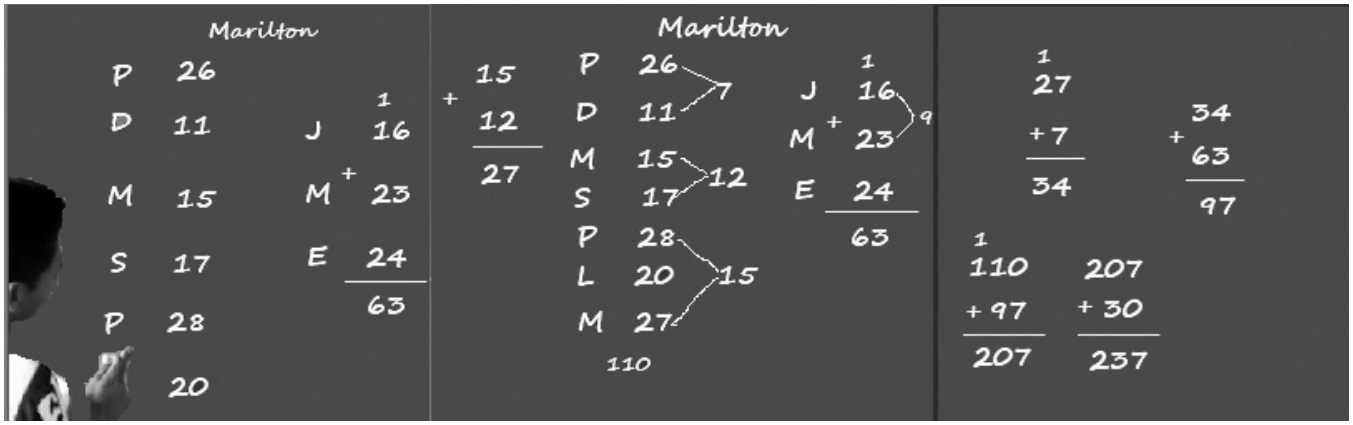

Figure 6. Marilton's writing the addition algorithm (writing enhanced for clarity).

Marilton also went to the blackboard to show his calculation. He too called upon the conventional school algorithm to add the number of students, but not in a very straightforward manner, and without using a calculator or mental calculations. His calculation can be divided into six steps (Figure 6). In the first step, he added the first three numbers in the table (16+23+24=63), adding first the units and then the tens. In the second step, he added the tens digits of the remaining numbers, and recorded the total of tens (110) below. In the third step, he added the units, in groups of two or three, and then he added the total of the units (15+12=27; 27+7=34). In the fourth step, he added the total of the units from the third step with the total of the first step (63+34=97). In the fifth step, he added the total of the tens of the second step and the final total from the fourth step (110+97=207). In these last three steps he added units first and then tens. He then noticed that he had made a mistake, as his total was 207 and not the expected 237. With the help of his colleagues and the instructors he saw that he had forgotten to add one number (30), so, in the final step, he added 207+30=237.

All the teachers were engaged in those activities although some of them, like Maiza, explained that they had never studied arithmetic algorithms and did not teach them in their schools. Despite that, Maiza gave several suggestions for the calculations while Damião and Marilton were using the addition algorithm.

In the third phase of the course, Marilton, who declared he had already learned these procedures at school, calculated the difference of the numbers of Indigenous schools, from 2007 to 2011, represented on a graph (3036 – 2460), explaining his algorithmic procedure in Maxakali. He wrote the letters U (units), T (tens), and H (hundreds) and organized the numbers according to place value. Using gestures with his fingers pointing to the place values, he subtracted from right to left. Thereafter, Maiza and other colleagues successfully used the same algorithm to carry out the calculation.

Six months later, we interviewed Lucio, showing him a first version of this article, and we had the following conversation while he was staring at a place value chart with the initials for units, tens and hundreds in English (U, T, H):

Vanessa And in this case, do you think that when Marilton remembered that (chart) it was easier for me to understand what he was trying to explain? Do you think that this chart, even if it was in Maxakali, is something that I understand and you understand, in spite of the fact that I speak Portuguese and you speak Maxakali?

Lucio Understand

Vanessa Why is it possible to understand?

Lucio Because similar to Portuguese, resembles but it is not, but it is the same, but it is different in the writing, for example, U, D and in the Maxakali *[While he speaks, he makes a place value chart, in Maxakali, see Figure 7]*

Vanessa It's neither Portuguese, nor Maxakali *[both were looking now at a chart in English]*

Lucio For example, look, it's not Portuguese nor Maxakali, but I know that

Vanessa What do you know here?

Lucio Here for example, one, two units, and there is T, seems that T is tens *[dezena, in Portuguese]* and H is hundreds *[centena, in Portuguese]*

Vanessa *[indecipherable]* And because each one is in a cell, then you think, this here is units,

Lucio Yes

Xẽy (xẽykõn)	N (nexyẽn)	Ũ (ũmxet)
4	1	2

Figure 7. Place value chart in Maxakali, made by Lucio.

Lucio agreed that the visual representation facilitates communication between languages and he reiterated this on several occasions.

Place value charts and algorithms as boundary objects

Damião also called upon a conventional algorithm (artifact) to solve another calculation (16 + 23 + … + 27), as did Marilton, who introduced a different way to use it that minimizes difficulties in speaking Portuguese as well as dealing with big numbers. These situations were not anticipated by Vanessa, since she did not expect such artifacts, typical of non-Indigenous schools, to arise in this context. The choices of artifacts made by these teachers modified the participation of other teachers, redirecting their actions to school experiences subsequent to the introduction of Portuguese, when this type of record was presented in some Maxakali schools. The new directions of the activity also modified the division of labor, making it more horizontal, since the Maxakali teachers acquired more authority, as they shared with the instructor a linguistic code originated in non-Indigenous culture.

Marilton's choices, when analyzed from within the activity system, show how the artifacts (place value chart and algorithms) worked as "boundary objects" (Engeström, 1990, citing Star, 1989, p. 46), that is, "objects that are both plastic enough to adapt to local needs and constraints of the several parties employing them, yet robust enough to maintain a common identity across sites. They are weakly structured in common use, and become strongly structured in individual site use" (p. 190). They facilitate interconnections between multiple practices. In the present case, we stress mainly their contribution to facilitate the communication in the activity.

Considering the specificity of Maxakali writing, the place value chart and the algorithm could also be working as 'semiotic artifacts'. These are signs (particularly linguistic) created and used in the different spheres of activity, which organize the way of life of a culture and enable people to 'think together' (Impedovo, Andreucci & Ginestié, 2017).

According Gadanidis, Borba and Scucuglia (2016), the use of new media and arts offer an interesting cognitive and affective scenario for students' mathematical communication, mainly students who do not speak the official language well. In this study, as Marilton, Damião, and the others could not speak Portuguese well, they combined different semiotic artifacts, such as talking (in Maxakali and Portuguese), using gestures, the place value chart and the algorithms, to communicate their ideas. They created a multimodal writing that bridged the gap between the different textual worlds, in which they were living in those classes. That is, the Maxakali teachers were seemingly using the algorithms and the place value chart to compose a multimodal text, namely a multimodal mathematical writing, allowing them to communicate mathematical ideas.

They have used mediating artifacts that revealed new connections between different ways to record and operate with big numbers. The Maxakali multimodal writing, as part of their intense ceremonial/ritual life, was a basis for expressing big numbers in a school mathematical activity, which is introduced in the Maxakali schools as a translation of near-universal conventional mathematics. The expansion of communicational means by the Maxakali-speaking people shows that we must learn from each other about the possibilities of interaction between different worldviews to express specific ways to deal with common issues of people's lives, such as recording and operating with large numbers.

Final remarks

In the present study, the artifacts mobilized by the teachers offered them a visual organization that facilitated expressing quantities and calculation procedures. As they could use those artifacts as 'drawings' or as pictorial writing, we believe that this amplified the possibilities of interconnections between the different forms of rationality—Maxakali and non-Indigenous. According to Mendes (2007), for the Indigenous people drawings do not serve only as a visual support, for them, "the narrative incorporates the visual and the arithmetical text in the action of identifying the situation" (p. 237). About Maxakali writing, Tugny (2011) defends that it is not a representation that considers only phonetic writing, as it cannot be seen as something that simply follows the speech. On the contrary, according to Tugny, there are different ways of Maxakali writing, which include pictographic writings, hieroglyphs, ideograms, and others, as can be seen in the *mimãnãn*. Thus, the choice of these sorts of artifacts enables consistency and communication between different cultures and, in the present case, it contributed to align different linguistic codes, modifying the participation of the teachers, redirecting the instructor actions and, expanding the communicational repertory of both.

To conclude, it is worth to emphasize an unexpected result of this study: the fact that Maxakali teachers with limited schooling in Western culture spontaneously resorted to conventional school artifacts, which one would expect to be alien for them, as a way to connect the different forms of knowing and acting in the world created in the teacher education course activity system and to gain a more powerful position in this multilingual classroom mathematical activity. According to Radford (2012, p. 340), "language operates as a marker of differences in the social, political, and economic arenas of culture. Even more, language is constitutive of its subjects whose subjective existence can only be realized through it and the worldviews it conveys". In this case, we have privileged some communicational artifacts related to quantification and numerical records, which conveyed a much more political message—reaffirming the teachers' ability to obtain certification, to be recognized as knowledgeable from the perspective of the non-Indigenous mathematics, and to stand out from the rest of the group as someone who already knows what mathematics is about—going beyond simply learning about arithmetic. In fact, the teachers succeeded in the certification test and became empowered to the point that they could influence both the process and the test so that they were more attuned with multimodal mathematical writing, but still acceptable by the educational system to which they are subject. The teachers influence on the process of certification, resulting from the outreach course, deserves further investigation.

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Notes

[1] According Vieira (2006), post childbirth there are restrictions on the new mother and father; for instance, she must rest for 30 days and he must not eat meat with blood in it.

[2] Since 2008, our university has offered an Intercultural Indigenous teacher education course for different Brazilian groups of Indigenous people.

[3] Burton (2009) argues that mathematics or mathematical should "[concern] a system for dealing with quantitative, relational, or spatial aspects of human experiences" (p.10), which should "be used much more widely than just to refer things in mathematics texts or journals" (p. 6). On the other hand, when he wants to talk about the smaller, formal, conventional world of academic mathematics presented in schools all over the world, he uses the words "near-universal conventional mathematics" or NUC mathematics.

[4] These 10 teachers were selected according to the following criteria: they had already participated in another university project, they were teaching for a longer time and they would represent the three different regions of Maxakali territory.

[5] Manuela is a researcher in a research project from which the outreach course is a development. She did not play an active role in teaching, but participated as a senior researcher.

[6] Entrance test. https://www.ufmg.br/copeve/Arquivos/2014/PROVAS/fiei/. Accessed on 16/8/2018.

[7] Isael, a Maxakali teacher and a former student at the university, attended some classes and was also a translator.

[8] João Bidé is a Maxakali teacher who attended just the second phase, since he was not applying for certification.

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