Communications

Thinking about logic

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I read Reid and Inglis's article (FLM **25**(2), pp. 24-25) with interest. One of the questions (directed at readers) raised deals with the place of logical rules in our daily life. This is a question I have dealt with in some recently conducted research.

This research included interviews with mathematicians, logicians and leading people in mathematical education in Israel. The interviewees were asked about the uses of logical rules in our life. A few interviewees argued that life does not present situations that require the use of logic. On the other hand, most of those interviewed claimed that logic has a meaningful role in our life. According to them, we use logic in the processes of decision-making and argumentation. When asked to specify situations in daily life demonstrating such uses, their answers were divided into two:

- not being able to point to an example from daily life
- giving the following examples: use of transportation rules, law, and the game of chess. When I asked for additional examples, these interviewees had difficulties providing them.

When analyzing the examples provided by the interviewees who consider logic as having a role in our life, there seems to be a common ground for all of them. Chess, transportation and law are all closed systems based on predetermined rules, which the 'participator' must know and 'play' according to them. But it seems that in real life there are many and various situations that differ from the systems suggested by the interviewees.

Why is it that interviewees, who speak about logic's uses in life, provide examples which relate only to a certain part of life, or do not provide examples at all? There seems to be no congruence between the interviewees' claim that logic has uses in our life and their inability to reveal such uses with realistic examples (except for closed systems). In what I shall be saying below there will be no complete answer to the question of why interviewees find it difficult to offer examples of the uses of logic in life. Furthermore, I do not intend to rule out the possibility of finding such examples. I would just like to pay attention to this point.

In his story *The double*, Dostoyevsky tells a story about a party taking place at the home of the state consultant. The hero, Mr. Goliadkin, an employee who was not invited, counts, one by one, the reasons justifying his standing in the corridor for three hours before entering the party. Then he thinks about reasons for returning home. After deciding to return home, he suddenly pushes the door and bursts in. In this section, Dostoyevsky illustrates our inner system of conflicts and their irrational elements – just when Goliadkin uses logical arguments, he acts with another motive. Except for Dostoyevsky, who invented the hero, whoever was at the party and saw the hero bursting in, could not point out the logical chain that had taken place prior to the hero's entering the room. Other than the human soul, whose layers are difficult to explain, it is hard to point at a logical mechanism of occurrences in the world of phenomena; it seems that even if there exists a consistency in the way the world of phenomena "acts", it is doubtful that it is conditional on purely logical rules.

It seems that human beings (and life around them) are far more complex and are subject to constantly changing processes. Therefore, they cannot be generalized into the category of closed systems. Humans also activate analogical thinking (associations, comparisons) mixed inseparably with logical-deductive reasoning. Suppose we try to separate these two ways of thinking (logical and analogical) and conduct an exercise in 'thinking on thinking'. We observe from the outside and try to search for the logic in our thinking, as if there was a detective inside each of us, following us like a shadow. In such a situation, it is most probable, at least for those of us who are not used to such an exercise, that we will find it impossible.

There is a tale about a centipede. A grasshopper expressed to him his admiration of the fact that he can walk with all his legs in coordination. The centipede looked at his legs and tried walking, but could not move them. Thinking, like walking, is an activity, and it seems that merely trying to participate in such an exercise might cause problems with it. It may be possible to relate to the logic in our actions after the act, and conclude, only after the events have taken place, whether or not we have acted logically. Still, there is the question of whether all components of thinking, especially retrospection, have the same clarity with which we formerly experienced them. As to observing other people, it is perhaps possible to follow the logic in their behaviour, but obviously it is impossible to track their thinking and assumptions – these are invisible to us.

I shall re-emphasize that in the previous paragraphs I raised only preliminary thoughts, which wonder at the fact that people, dealing with mathematics and logic and who consider logic as a field that has importance in life outside mathematics, have difficulties giving examples for it. I would not want it to be understood from what I have said that logical thinking is nonexistent in our life. Even if it is difficult to point out, it does not mean that logical thinking is not there and that it does not support us in our life. It is more than likely that it will be easier for people with a higher logical capability to make more efficient and speedier chains of deductions. Thus they will tend to rely on logical thinking in situations where another person might tend to rely on conditioning and superstitions. My intention is to emphasize the fact that the interviewees could not raise examples for open situations in daily life worthy of further attention.

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Re-thinking real-world mathematics

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*A response to 'Word problems as simulations of real-world situations: a proposed framework', Palm, **26**(1)*: For all the talk about real-world mathematics, it seems like we still don't get it. In looking at Palm's framework for evaluating sample questions, two criteria seem marginalized or altogether absent:

- relevancy to students' lives
- the transformative nature of the problem for the purpose of making the world a better place.

The pizza party problem

Middle school (for students aged 13 and 14 years), where I work, is a wasteland of what I call 'pizza party' mathematics. You've probably seen the questions about finding fractions while cutting up a pizza, the youth in the textbook picture standing around looking gleeful. But I'm also speaking more broadly about problems that simply aren't relevant in students' lives. There are questions about the diameter of hamster wheels, the height of mountain peaks, the rate of CD sales in the month of June and the Swiss roll question in Palm's article. If the criterion is *relevancy to students' lives*, the lift question and the Little League bus question are also 'pizza party' mathematics.

Pizza-party mathematics is not transformative in nature. There is little, if anything, students will do with the knowledge that they need twenty lift loads or four busses. Heart rates have not crept above their normal resting rates because there is little, if any, of what Palm talks about as the affective domain. I can state with considerable confidence that if it is "crucial that the students significantly engage in the figurative context" (p. 46) affectively, most textbook mathematics questions that I have seen are completely inadequate. One of my students writes:

> The real difference is that pizza party math does not stick with you for life. What happens is that you just go through the different sections, learning all the material and then completing the test, but then all of the information goes out of your head because it doesn't really matter. It's kind of like being on water on a windy day. One wave comes and picks you up, and then keeps going and disappears.

Relevancy and transformative capacity as prerequisites

Relevancy and transformative capacity must be prerequisite to the other aspects. It is immaterial if the event "has a fair chance of taking place" (p. 44) if the event is not pertinent to the students. Whether or not the question "might be posed in the real-life event" (p. 44) is of little value when students are quick to realize that the chance that they themselves will pose it is miniscule. I can develop a question about finding the number of buses required to go to a Little League game that will have a great deal of realism and specificity, and the question I will no doubt (and rightly) be asked is "Who cares?"

As a teacher of students aged 13 and 14 (grades seven and eight), I know firsthand the consequences of mathematics tasks 'dressed up' with an out-of-school context. Eyes glaze over. Attention moves to graffiti on the desk or the latest *iPod* download. Keiran is quick to ask "Why are we doing this?" and Malcolm follows up with "When would we ever need to know this in our lives?" While it is true that one of the hundreds of students who have passed through my classroom doors *will* need to build a fence to enclose a herd of cattle and so will need to be able to problem solve this scenario, most students recognize that they are not destined to be ranchers. If Keiran and Malcolm, as skilled mathematics students, have doubts I worry deeply about those who join me at the beginning of grade seven with a clear distaste or downright fear of the subject.

Figurative contexts that put the real back in real-world problems

What, then, are real-world problems that are central to student lives that we as educators can use as the figurative context for mathematics learning? Actually, there are many, and in my experience the students engage with them intensely (and mathematically!). The real-world problems that provoke this response are those concerned with social justice. Usually when I tell people this, they respond with bemused patronizing smiles and polite small talk, but there are compelling reasons that justice issues and mathematics are a perfect pairing.

Why do my students return from lunch talking about how they've been followed around the convenience store by the owner and treated as petty thieves? If we looked at crime rates and compared youths to adults what would we find? What about if we began using statistics to look at racism in the criminal justice system? How about capital punishment? Or the privatization of the prison system?

What is the probability that those jobs that my students are looking for this summer will be minimum wage with no benefits and little training? If they work in the fast food industry or at one of the big mega-stores will they find unionized jobs? Why or why not? How do unionized jobs compare with non-unionized jobs? How many years does a minimum wage earner in Canada have to work to earn the same amount as a Chief Executive Officer does in one single year? What does the distribution of wealth in Canada look like? Do we think that the distribution is fair? These are all questions of justice, and ones where understanding the topic requires mathematics.

It's important to understand that mathematics is not an add-on to justice issues. How many of us have the number sense to fully appreciate the magnitude of the United State's military budget? I respectfully submit that it's very few of us. How many people know that we spend $319 billion dollars globally per year on advertising while the United Nations calls for an additional 19 billion a year to eliminate world hunger (this is a sure fire way to spark outrage in my classroom)? But mathematics can be used to "make the invisible visible" (to borrow from the title of Devlin's book) and in so doing set the stage for students and teachers alike to do something about the problem, to make the world more fair or more kind.

After studying the issue of domestic violence and interpreting graphs and charts, patterns and trends, my students develop a pamphlet on the issue and hit the streets of Toronto for a morning, talking to people and collecting donations for local women's shelters (we don't graph the number of pennies, nickels, dimes and quarters that we collect …). After studying global warming and carbon dioxide equivalents, we all look at how a diet of food that comes from within 100 kilometres of Toronto has so much less environmental impact than if we eat our grapes from California and our mangos from Central America. The point is that mathematics empowers students to make informed choices about issues central to their lives in a way that may transform the world for the better.

Keeping relevancy, in the most honest sense of the word, at the heart of real-world problems must be our goal. And if we're not going to betray the idea of education, the notion that problems will encourage students to transform our society for the better must also take priority. Students love talking about fairness, and given the chance and a good reason to do so, will move mountains to be kind to others. Let's give them a *real* reason to do so.

Communication: simulation, reality, and mathematical word problems

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*A comment after reading 'Word problems as simulations of real-world situations: a proposed framework', Palm, **26**(1)*:

In contemporary theory, terms like 'real world', 'simulations' and 'language' are anything but transparent and unambiguous. Our networked electronic media have restructured the balance of our senses and our sense of the relationships amongst self, others and reality. We no longer live in a world in which frameworks, grids and checklists can capture the complex relationship between human-made simulations and an assumed external reality.

Baudrillard (1981/2001) is a key theorist addressing issues of simulation and reality in our electronically-mediated world. Baudrillard presents the idea that simulations now *precede*, and in fact *supplant* reality, existing entirely without any corresponding or matching referent, and interacting primarily with other simulations. Baudrillard writes that

> [i]t is no longer a question of imitation, nor of reduplication […] It is rather a question of substituting signs of the real for the real itself […] A hyperreal […] sheltered […] from any distinction between the real and the imaginary, leaving room only for the orbital recurrence of models and the simulated generation of difference. (Baudrillard, 1981/2001, p. 170)

Simulations ('reality' TV shows, computer games, faked political crises, theme parks) precede and create events which may be indistinguishable from simulated events, and which interact with other simulations.

Baudrillard's characterization of simulations bears a resemblance to characterizations of genres in language, literature and film. Examples of any particular genre are made in imitation, not of life but of other exemplars of the genre – so exemplars of a genre interact primarily with one another, rather than with any external 'reality'. (Think of the train of imitative genre references generated by a series of films like *Frankenstein*, *Bride of Frankenstein*, *Son of Frankenstein*, *Curse of Frankenstein*.) Examples of a genre refer in only the most cursory way to 'real' objects and processes, and the intentions embedded in the history of the genre are carried forward with its use, regardless of the conscious intentions of the person using the genre as a communicative medium. Similarly, word problems refer only glancingly to the realities of the workaday world, referring primarily to other word problems (Gerofsky, 2004).

Baudrillard's simulations and simulacra go beyond genre to create cultural worlds where there is no boundary between real and imaginary. I will attempt to address the place of word problems in relation to Baudrillard's insights on simulations, and to relate this to Palm's framework for judging the degree to which word problems simulate "real-world situations".

Simulations and reality

Baudrillard writes about the historical relationship of representation (or image) with the real, and offers the following "successive phases of the image":

1. It is the reflection of a basic reality.
2. It masks and perverts a basic reality.
3. It masks the *absence* of a basic reality.
4. It bears no relation to any reality whatever: it is its own pure simulacrum. (Baudrillard 1981/2001, p. 173)

A simple example of this progression relates to cultural meanings of money. If a gold coin is an example of a kind of reality, since the gold has intrinsic value, then a paper note that can be exchanged at any time for a lump of gold is a *reflection of a basic reality*; a counterfeit version of such a note would *mask and pervert a basic reality*, while leaving that reality intact. A system in which the 'gold standard' is removed but the paper money remains *masks the absence of a basic reality*, and a system in which electronic pulses travelling globally by satellite change numbers in electronically tallied accounts may *bear no relation to any reality whatever* insofar as gold is concerned.

What would constitute an image of Baudrillard's first type, a *reflection of a basic reality* within a mathematics education context? Perhaps an accurate map or diagram of an actual, physically existing place or object would qualify (for example, the plan of a house or a schematic diagram of a machine), or an accurate table of measured data (the number of cars passing a particular corner at rush hour, or the results of an opinion survey actually carried out).

An example of Baudrillard's second type of representation, which *masks and perverts a basic reality*, would be a diagram, chart or table of data which *purports* to represent a real, known situation but knowingly lies about this data. Such deliberately counterfeit representations are rarely seen in mathematics education; we are more likely to find them in politics and the news media, where a power advantage may be gained by misleading people.

Word problems belong to the third type of representation, which *masks the absence of a basic reality*. Word problems cannot be considered transparent, simple simulations of "real-world situations". When they appear to refer to actual situations, word problems mask the fact that they represent situations impossible in real life, and possible only in the conceptual 'world' of mathematical relationships. Word problems are constitutionally, generically unable to be faithful emulations of real-life tasks.

If, with Palm, we demand that word problems represent the everyday experience of the real world, we will be sorely disappointed in their lack of effectiveness. The three word problems that Palm has used as examples – the dissection of a Swiss roll, the 269 people waiting for a lift that only carries 14, and the 540 Little League members who are planning bus transportation – are all far from representative of the contingencies and purposes of everyday situations, as Palm himself has pointed out. Tellingly, Palm has not provided any examples of word problems which "could be considered simulations with high representativeness" (Palm, 2006, p. 46).

In a recent conference presentation, Kavousian [1] gave a good example of the problems that arise when a literal, real-life interpretation is made of the referential intentions of a mathematical word problem. Kavousian gave this word problem to her college students on a test:

> How many ways can ten people sit around a circular table?

In her role as a mathematics instructor, Kavousian meant the roundness of the table to signal that the permutations were invariant under rotation, so that the number of permutations would be 9! rather than 10! However, one of her students insisted that the answer ought to be 10! The student interpreted the table as a real one, in a real room, with windows on one side, bookshelves on another and so on. In a real room, there would certainly be 10! ways that 10 people could sit at a round table, since it would be different to sit near the window, the bookshelves or the blackboard. Kavousian acknowledged the real-life sensibleness of the student's interpretation, even though it short-circuited the mathematical question she had meant to ask.

Kavousian referred to the mathematical intentions of word problems as a "secret language" that students are expected to know, without having had any explicit instruction or attention paid to it. This "secret language" is a hidden set of meanings and references that live within the world of formal mathematical relationships. A novice cannot be expected to know these references while in the process of acquiring them; it is only in retrospect, once one is familiar with the mathematical structures in question, that the referential universe of the word problem and its answer can make any sense. If one assumes that the word problem refers to real objects and situations, the word problem masks the absence of such situations as table seatings invariant under rotation. Like the example of paper money without the gold standard, which operates on our collective faith in the illusion alone, word problems pretend to refer to a reality which does not actually exist, which survives only by our collective acceptance of the illusions built into the genre.

As for Baudrillard's fourth type of image, one which *bears no relation to any reality whatever*, I believe that this level of simulacrum goes beyond the capacities of word problems on the printed page. Examples of "pure simulacra", perhaps as yet unrealized in mathematics education specifically, might include computer gaming, interactive and networked text, image, sound, video, even tactile and CAD interfaces in which there is fluid movement between various virtual and actual representations. In such a system of simulation, the image would precede and possibly pre-empt reality; actual physical artefacts would be no more than instantiations of online patterns and designs (see Sterling, 2005; Gerofsky [2] for a further discussion of such potential systems of simulations).

Alternative models to think with

In Palm's article, he hints at several alternate ways to consider the problematic relationship between word problems and 'real life'. Although Palm mentions these in passing, and often introduces them only to reject them, the following seem most interesting and fruitful to me:

High fidelity: Palm refers to word problems as simulating real life with low or higher fidelity (Palm, 2006, p. 46). In its more common usage, 'high fidelity' referred to hi-fi stereo systems in the 1960s. Contrary to the most extravagant advertising claims of the time, these systems were not really expected to fool anyone into thinking that there was an orchestra or pop singer in their living room – they were not meant to be taken as transparent representations of an existing reality. In fact, it was at this time that innovative highly-engineered studio-produced record albums used multi- tracking to produce synthetic performances that could never take place in a live performance – an early example of Baudrillard's simulacra.

Because the term 'high fidelity' has these associations of a higher, but never possibly complete, faithfulness to some aspects of an originating reality, while at the same time invoking new possibilities of the synthetic and artificial (which represent things that never existed), I think it holds promise in the exploration of word problems and other representational modes.

Semi-reality: Palm cites Alrø and Skovsmose (2002) writing about different kinds of "worlds" related to different learning tasks. The concept of "semi-reality" is an interesting way to describe the referential world of word problems, "a world that is fully described by the text of the task and in which all measurements are exact" (Palm, 2006, p. 43). It is rather mysterious but also fascinating to try to conceive of the tidy world of the semi-real. How would a semi-real world compare to a virtually real world, for example? (see, for instance, Sismondo, 1997). Are there other learning tasks aside from word problems that deal in semi-reality? (Alrø and Skovsmose (2002, p. 47) give some other examples within the realm of mathematics education.) How do learners and teachers engage differently with semi-real tasks?

Problem solving in role: Palm writes, "In many word problems, of which Example 3 is one, it is not known in what role the students are solving the task." (Palm, 2006, p. 45) I am intrigued by the idea of mathematical problem solving *in role*, especially as it relates to Heathcote and Bolton's work on drama in education (Wagner, 1989; Bolton, 2003). Heathcote, Bolton and others have developed ways of teaching across academic disciplines using whole group improvised drama, with teacher and students in role. I have experimented in using this kind of drama in education for teaching mathematics, and have found it particularly effective as a way to incorporate emotion and a sense of purposefulness in problem-solving. I would like to hear more about the ways that Palm would involve students working on a task in role, and whether he would include fictional roles as part of the learning and teaching repertoire.

Reality and imagination: Palm quotes Van den Heuvel-Panhuizen (2005) writing about context and 'reality' in word problems from the perspective of the Dutch theory of *Realistic Mathematics Education* (RME):

> The task context is suitable for mathematization – the students are able to imagine the situation or event so that they can make use of their own experiences and knowledge […] The fantasy world of fairy tales and even the formal world of mathematics can provide suitable contexts for a problem, as long as they are real in the students' minds and they can experience them as real for themselves. (quoted in Palm, 2006, p. 43)

Palm rejects Van den Heuvel-Panhuizen's characterization of "realistic" (*i.e.*, vividly imaginable) contexts as insufficient to "facilitate an experience of mathematics as useful in real life beyond school" (Palm, 2006, p. 43). However, from the perspective of Baudrillard's theorization, Van den Heuvel-Panhuizen's sense of reality as the imaginable is the most appropriate one for our age in which the boundaries between the virtual and the real are dissolving. I am much more concerned with finding vividly imaginable, emotionally engaging modes and far less concerned than Palm with the simulation of out-of-school 'real life' situations in mathematics education.

Note

[1] Kavousian, S. (2006) Presentation in the panel discussion 'Obstacles to understanding', chaired by Dubiel, M., conducted at the Pacific Institute of Mathematical Studies conference *Changing the culture*, Vancouver, BC. Contact e-address: skavousi@langara.bc.ca.

[2] Gerofsky, S. (in press) 'Moving fluidly among worlds: multisensory math software', poster presentation, *Proceedings of the 30th Annual Conference of the International Group for the Psychology of Mathematics Education*, PME30, Prague, Czech Republic.

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What is required now is that educators of all kinds make themselves vulnerable to the awareness of awareness, and to mathematization, rather than to the historical content of mathematics.

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