MATHEMATICAL MEANING MAKING AND TEXTBOOK TASKS

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This article reports from a study of mathematics classrooms [1] It is about the interaction that emerges when students are solving tasks from the textbook and the teacher gives individual assistance. The interest lies in the task and the way in which the teacher-student interaction is influenced by the textbook An analysis of classroom episodes reveals control features of the textbook and shows that a textbook task, in an interrelationship with a teacher, can cause ambiguity as well as generate mathematical discussions

In the late 1970s, Bauersfeld (1979) noticed a shift to a new trend in the research field of mathematics education He stated that the difficulty in generalizing results from studies made in a laboratory context had contributed to an increased interest in doing research studies of real classroom situations. Looking at the current focus of much research in mathematics education, it seems like Bauersfeld's conclusion is correct. Sfard (2005) found that most researchers use carefully recorded classroom interactions as their empirical data. She also noticed that "the last few years have been the *era of the teacher*" (p 409, original emphasis) There had been a change from previous foci that were on the learner and, before that, the curriculum. Sfard commented that we have come a long way since the 1960s and 1970s and

> the era of the curriculum [...] when the main players in the educational game were the developer and the textbook. (*ibid*, p. 409)

There have also been changes to research methods. At present, the dominant type of research is qualitative and what Sfards calls "participationist". This means that learning is conceptualized "as a change in one's participation in a certain type of activity" (p. 398) and involves design experiments and ethnographical studies

Instead of trying to find out "what works in the classroom" the focus is on *how* things work and *what* the alternative possibilities are (Sfard, 2005, p. 410)

Looking at how things work, teaching is an activity that can take place in at least two different locations in the classroom:

- an entirely public interaction with a teacher (or a student) standing in front of the class
- a more private interaction where the teacher walks around the classroom tutoring individual students or groups of students

In Japan the term for this private interaction is *Kikan-Shido*, which means 'between desks instruction' It is a term that describes the time in the lesson when the students, on an individual basis or in groups, are engaged in 'practice' and the teacher walks around the classroom, observing and sometimes interacting with the students. [2] This is a familiar activity, well recognized by researchers and teachers in Sweden. If *Kikan-Shido* refers to an activity of a teacher, what are the students occupied with during that time? Solving tasks from the textbook perhaps? This is probably the most common activity in Sweden, where the textbook seems to define 'school mathematics', including the 'learning paths' for the majority of students. [3]

So, besides the teacher and the students there is a third player in the game of teaching mathematics - the textbook. The book is of course an object and can not play an active role in the interaction. But it is a teaching text, or "a book designed to provide an authoritative pedagogic version of an area of knowledge" (Stray, 1994, p. 2). As such, it mediates a school subject from someone who knows about it to someone who does not know but is supposed to get hold of the subject. [4] The textbook has an authoritarian position, partly because it has been *authorized* by a teacher or whoever decides which textbook to use. It also reveals underlying beliefs of what mathematics is and how it can be learned (*cf.* Johansson, 2005; Luke, C., de Castell, Fraser and Luke, A., 1989)

The influence of textbooks and how textbooks are used in the mathematics classroom are recognized issues in previous research. Some of the studies are mostly quantitative and measure, for example, time allocation (*e.g.*, Fan, 2000). Other studies are more qualitative and focus on how the teachers interact with a new textbook (*e.g*, Remillard, 2000). [5] The purpose of these studies is often to expose changes or difficulties connected to change (*e.g*, Wilson and Goldenberg, 1998)

Curriculum development in Sweden is not associated with 'reform' of mathematics textbooks as is the case in the United States. Consequently, the textbook in this study can not be regarded as 'new', even if it is a new edition. [6] Nevertheless, it is still interesting to study the interrelationships between the textbook and the teacher.

The intention in this article is to explore the role of the textbook in a Swedish classroom under the current state of affairs. It is about the interaction that emerges when students are solving tasks and the teacher gives individual assistance The research question is: In what way does the textbook influence, or not influence, the teacher-student interaction in the *Kikan-Shido* part of the lesson? I want to increase awareness of *how* textbooks influence the teaching and learning of mathematics but also to stress the content issue. When doing research in mathematics education, we should not forget about the curriculum and the textbook. Looking at how things work in the classroom we sometimes notice phenomena that are hard to explain in terms of, for example, the teacher's mathematical knowledge or beliefs about mathematics and teaching of mathematics.

Three theoretical perspectives

Three theoretical perspectives guided my thinking. The first is based on "the third stage of the frame factor theory" (Englund, 1997). [7] It emphasizes the choice of educational content and contextualization of teaching. The fundamental assumption is that different choices can be made, more or less consciously, which have crucial implications for teaching and learning. The student is offered different possibilities to create and construct meaning depending on, for example, what content is chosen and what context the textbook offers.

The second perspective concerns interaction. Following Voigt's (1994) arguments, mathematical meaning is a matter of negotiation, a product of social interactions. Given that a person's beliefs and background knowledge offer meanings for tasks, questions and symbols, using this perspective it is helpful to consider the ambiguity of objects in the mathematics classroom. Interaction, from the view of symbolic interactionism, is about how the participants monitor their actions in accordance with what is assumed to be the other person's background understandings, expectations and intentions. Thus, mathematical meaning is negotiated, even though the participants in the interaction do not explicitly argue from different points of view.

The third perspective relates to the textbook. To begin with, the textbook is designed in a certain way. It is specially made for that purpose to be appropriate to the receiver, often a student, and the educational context. It is also colored by a view of learning, even if this is not stated explicitly. For example, a textbook that focuses on getting right answers to well-defined questions corresponds to ideas of behaviorism. From a constructivist and sociocultural perspective, it would be more important to start from the students' own experiences and create problems that nurture discussions and cooperation. [4] Is it possible to say that textbooks *influence* the teaching and learning? It is of course questionable whether an object like a book or a text really can lead people in a certain direction in a pedagogical process - the issue would probably lead to a fruitless discussion. Instead, it is possible to think about the influence of textbooks as related to peoples' beliefs and values. The influence of textbooks is based on a more or less conscious idea that the book is important. [8]

Combining these three perspectives, I will analyze the choices that a teacher makes in the classroom by looking at the teacher-student interaction. At the same time, the textbook and the role of the textbook in the specific classroom will be taken into account.

Background

A study of Swedish classrooms, part of the CULT-project [9] formed the empirical background for this article. For this article, teacher interviews and video-recordings from four consecutive lessons were chosen. A coding procedure [10] and the computer software, *Videograph* were used to categorize activity in the classroom. The lessons were also transcribed. The results from the coding procedure, together with the associated transcripts, made it possible to make a deeper analysis of some chosen lesson sequences. The focus in this article is on the private interaction between the teacher and the students when the students are solving tasks in the textbook, *i.e.*, the *Kikan-Shido*, as it is defined in the specific classroom. Standard forms of interaction and unexpected and non-prototypic teacher-student interactions are described later in the article. The episodes are chosen to highlight the control features of the textbook. Firstly, here is a description of the teacher.

Description of the teacher

The teacher, Mr. Larsson (fictitious name), is 62 years old. He has a long-standing experience of teaching and a long period of employment at the school, thirty-one years out of a total of thirty-six years of teaching. Besides mathematics, he teaches physics and technology in grade eight and nine (14-15 year old students). On average, almost 60% of the lessons consisted of private interaction where the students were solving tasks in the textbook and the teacher was giving individual guidance. For the rest of the time, in the public interaction part of the lesson, the teacher stood in front of the class. He wrote on the board, presented problems, posed questions and verified or disproved answers. The students' desks were organized in pairs but group work was not observed.

The textbook [11] used in this particular class is in one of the most common textbook series in grade seven to nine (13-15 year old students) in the compulsory school in Sweden. The teacher seemed to adhere closely to the textbook in the private as well as the public parts of the lesson, even if, from time to time, he used examples from outside the book. In one of the interviews, he confirmed his strong reliance on the book. He was asked why he used concrete numbers to show the students how to simplify an expression. He answered:

> It is generally so that I just follow the usual way to do it [...] this is normally how it is done in all books and I have not wondered about it so much, I think it is a system that works.

The class consisted of 22 grade eight (about 14 years old) students. The school practice is tuition in ability groups and the students in this class were identified as high achievers. According to Mr. Larsson, the students are quite homogeneous. He thinks about them as a hard working group that concentrates on mathematics. Non-mathematical activities are rare in Mr. Larsson's classroom. In this sense, one can say that the classroom environment is a comfortable one.

The 'standard' pattern of teacher-student interaction

In a mathematics classroom, an observer can notice certain patterns of interaction where the participants follow hidden rules, which they actually seem to be unaware of. One example of this is the "funnel" pattern (Bauersfeld, 1988) describing an activity where the teacher provides individual guidance through a step-by-step reduction of the demands.

Sometimes this process culminates when one expected word from the student makes the teacher complete the solution by himself. The "funnel pattern of interaction" is a well-known activity in many mathematics classrooms (*cf* Alrø and Skovsmose, 2002) and was also observed in this particular classroom. The following transcript serves as an example of a common type of interaction where the teacher, Mr Larsson, funnels the student until he almost reaches the solution. The task, which the student is trying to solve, is as follows:

> x is an odd number, any one. Write an expression for the two consecutive odd numbers.
>
> *The solution is presented in the answer key:* $x + 2$ and $x + 4$.

Student: [...] odd numbers

Teacher: Odd numbers, yes. Any one, yes ... two consecutive. If you think of an odd number, for example eleven, what is the next odd number then?

Student: Thirteen

Teacher: Yes. How do you get eleven then ... you add ...?

Student: Plus two

Teacher: Yes, and then the next number ... how much should you add then do you think?

Student: Two more

Teacher: Yes of course

Student: Yes, okay

Discussions between Mr. Larsson and his students are not exclusively about specific tasks. Now and then, the comments involve general aspects, conventions or rules of mathematics. These kinds of clarifications can be an answer to a question of a student. For example:

Student: When should you use brackets and when should you not use brackets?

Teacher: Brackets are used in order to ... so you don't have to write the unit two times. If you put brackets, then you can write *kronor* behind

Student: Aha

Teacher: Precisely. I wrote 10 × *kronor* minus 5 *y kronor*, which works as well. But now you are putting brackets and then you write *kronor* after the whole expression there, yes.

The overall picture of the teacher activity in the private-interaction part of the lessons can be described as follows: the teacher interacts with some of the students, mostly individually, in order to help them solve the tasks in the textbook. Sometimes he just checks their answers or gives hints that could help them start working on the task. Sometimes, he helps them all the way through the solution of the task. In principal, the interaction starts from the task in the textbook that the students are working with. Questions about the task, sometimes initiated by the teacher and sometimes by a student, lead to an interaction between the student and the teacher. The kind of guidance that each individual student gets from the teacher differs depending on the task and how far the student has come in his or her efforts to solve it and other factors. The teacher's objectives seemed however to be the same, to arrive at a correct solution to the problem and/or to clarify mathematical properties, rules or conventions.

Critical incident 1

Even if the interaction in the classroom is colored by routines and regularities it is possible to observe phenomena that are interesting to analyze more deeply. This can, for example, be occasions of teacher decision-making in which the inherent learning potential significantly depends on the outcome of that decision. [12] For this article, two critical incidents are chosen. The first incident is chosen because it illustrates discrepancy from the smooth 'standard' pattern of teacher-student-task interaction and highlights the role of the textbook. It starts in the private-interaction part of the lesson when the students are working by themselves solving tasks in the textbook. One of the tasks in the textbook is especially 'challenging' in the sense that many students call for help from the teacher:

> Svante Gruvberg works as a miner. Every second week he works in daytime and every second week he works in the night. Svante earns 118 *SKr* per hour during daytime and 152 *SKr* per hour during nighttime
>
> a. How much does Svante earn in one year with 44 working weeks and 40 hours per week?
>
> b. Write an expression for how much Svante earns in one year with x working weeks if he works y hours per week

The solution to this task is, according to the key at the end of the textbook, the following: a) 237 600 kr, and b) $((x/2) \cdot y \cdot 118 + (x/2) \cdot y \cdot 152)$kr.

During the lesson, the teacher discussed this particular task individually with ten of the students. The first time, he just read *problem a* and checked if the student has arrived at a correct answer. The following discussion comes up the second time he looks at the task:

Teacher: It was good that it was forty-four weeks. If it had been forty-three, could one possibly give an exact answer then?

Student: It depends on what week

Teacher: Exactly

Student: [...] then it can be two different answers

Teacher: It can be that, yes ... surely ... depending if it is twenty-two weeks of nightshift or twenty-one weeks of nightshift. So, one cannot really answer precisely.

The teacher recognized that the task would be quite different if the number of weeks were to be odd and the student seems to follow his reasoning, stating that it can be two different answers.

Later in the same lesson, the teacher discussed *problem b* with another student. He seemed to be a bit puzzled about how the student solved the problem.

Teacher: Now, look here. I don't think I recognize this you know [turns pages in the textbook] ... *y* hours per week ... aha

Student: [...] working weeks

Teacher: Mm ... but you ... did you put *z* here ... where did that come from?

Student: This will be *z*

Teacher: Aha ... well okay ... that is *z* you mean ... yes. But you should rather not write it together then. Eh ... a bit doubtful if that formula will be totally correct there. If you take it like in two steps instead. On the one hand you take what he earns on the dayshift weeks and what he earns on the nightshift weeks ... then you get ... you certainly get a more accurate formula.

It is not clear if the teacher looked at the solution at the end of the textbook when he turned the pages. However, in the interaction with the student, he did not mention that an odd number of weeks gave two different solutions. [13] He left the student before he finished the task. Shortly after this, he approached another student also working with *problem b*.

Teacher: Of course ... then one can write these together ... but you can have it as in two parts ... you put these together wouldn't you? But a rather questionable formula actually. It depends on how ... if it is even or odd and with what he starts with, don't you think?

Student: Mm

Teacher: Thus, it is not certain if the formula is working. It depends a little on how he starts ... if he work more day weeks.

Student: [...]

Teacher: What did you say?

Student: He works half time day and half time night you know

Teacher: Yes, but if it is ... no, he doesn't ... he works whole weeks you know. Then if it is three weeks ... it could be two weeks that he works night and one week that he works day

Student: [...] but if he works night every second week and day every second week

Teacher: Yes, but let us say that he works three ... if you count on a three-weeks period

Student: [...] one year ... that is fifty-two weeks

Teacher: Well, okay ... mm ... I suppose it evens itself out ... sure

Student: [...] forty-four weeks

Teacher: Forty-four working weeks ... then it should work. But if you only work temporarily for three weeks then it cannot ... you cannot know for sure

Once again, the teacher tried to convey the message that there was a problem with the formula. This particular student seemed however to be quite confident with his solution. He did not care about the case of an odd number of weeks and argued "one year ... that is fifty-two weeks". The teacher appears to be unsatisfied and ends the discussion by uttering: "you cannot know for sure".

After that discussion, it seemed like the teacher abandoned his mission to show that an odd number of weeks gave two possible solutions. He helped other students with the same task without mentioning the numbers of weeks. During one of these occasions, the teacher approached a pair of students. He started by saying that "yes, it is quite difficult", then he funnelled the students through the task to the same answer as the textbook, without any sign of hesitation.

Teacher: How many weeks are there ... *x* it is yes ... how many of these will he work days and how many will he work nights?

Student A: Fifty percent

Teacher: Yes, half the time you can say yes. How do you write half of *x*? How do you write that? Wait ... *x* divided by two you can write then ... *x* divided by two or zero point five *x* you can also write. That's the same yes ... a half *x*.

Student B: One hundred and eighty y times zero point five *x*

Teacher: Yes, you can write that ... and then the other thing ... then you have to reckon the number of hours to yes ... so it also is included there. How many hours he works per week ... that is ... this

Student B: [...]

Teacher: Yes, it is y yes ... sure .. yes. If you put y times x half and x half again yes ... then you get a complete formula that works.

At the beginning of the lesson, the teacher was obviously not aware of the difficulty embedded in the particular textbook tasks. When he realized that there was a problem in the textbook he had to decide whether to stick to what he thought was a correct solution or follow the textbook. The teacher-student interaction changed during the lesson and different students encountered different types of suggestions concerning the solution

As an observer, it is difficult to understand why the teacher, in this case, alternates between these two standpoints. One reason could be that he believes that some of the students will have problems understanding the logic; at least one of them did not follow his line of argument pointing at "he works half time day and half time night" and that one year has fifty-two weeks. However, these students are regarded as high-achievers and the teacher is very experienced, in general, showing no hesitation when it comes to correcting mistakes made by the students. So, why this time? The decision is perhaps made more or less unconsciously and one can only speculate about the reasons behind it. One thing is however clear, it would make a difference if the textbook did not present any answers. The result, this time, is that the teacher does not argue against the solution in the textbook, which in this case becomes the authority.

The teacher makes different choices when he interacts with the different students who are working with textbook tasks. I claim that these choices have crucial implications for learning. First of all, from the perspective of an observer of a special incident, it seems as if the teacher's main objective is to help the students to reach a solution, whether it is correct or not. This is a pattern of interaction that could be called "funnelling". Secondly, the teacher chooses to have a passive position in the discussion of the solution of the task. Passive, in the sense that, even if he is aware of the incorrect answer in the textbook he chooses to avoid a discussion, or surrenders after a short time. A discussion could, advantageously, have been carried out in public, for all students, since many of them seem to have problems with the same task. As I see it, the funnelling pattern of interaction and the passive position of the teacher create a special context for the learning of mathematics. When students are working with tasks in the textbook, the focus is on getting the right answer - by looking at the answer key at the end of the textbook, students can be confident about their solution. There is no need to verify in any other way. In such a context, the textbook, and not the teacher, becomes the authority that defines the learning of mathematics.

Critical incident 2

A second example of a critical incident is chosen because it highlights the role of the teacher in a teacher-student-task interaction. The example is taken from a session of public interaction, *i.e.*, the part of the lesson where the teacher stands in front of the class organizing discussions and writing on the board, but it originates from the lesson before, in the *Kikan-Shido* part of the lesson.

Early in the lesson, the teacher talked about division by negative numbers. The topic seemed to be a deviation from what the class was working on in the current chapter of the textbook, which is about formulae. The teacher explained that he wanted to discuss an interesting question that had been raised by one of the students in the previous lesson. The student, John (fictionalised name), was working with the following task in the textbook:

> In the USA and many other countries the temperature is measured in Fahrenheit (°F). The relationship between the scale of Fahrenheit and our scale is the following,
>
> $C = (F - 32)/1.8$, where F = temperature in degrees of Fahrenheit and C = temperature in degrees of Celsius. Use the formula and calculate how many degrees in Celsius corresponds to 212°F b) 32°F c) 14°F
>
> *The solutions, which one can find at the end of the textbook, are the following: a) 100°C; b) 0°C, and c) −10°C.*

In the preceding lesson, John attracted the teacher's attention and asked him if the numbers always get bigger if one divides a negative number

John: When you divide a negative number, they are getting bigger? If you divide minus eighteen you get minus ten.

Teacher: How do you mean?

John: If you divide a negative number it is getting bigger

Teacher: No, it doesn't have to be that way ... it depends .. the important thing is whether the number .. so to speak .. is bigger or smaller than one. If you divide a number that is smaller ... in between zero and one ..

John: [interrupts the teacher] But if it is less than zero .. if it is above one [John probably means "less than minus one"] . .. look . . look here [shows the result on the calculator]

What John discovered when he was working with the task was that $(14 - 32)/1.8 = -18/1.8 = -10$, *i.e.*, that the quotient was bigger than the numerator. He seemed however to be rather annoyed and unsatisfied with the response from the teacher. The teacher, on the other hand, appeared to be a bit puzzled when he walked away from John's desk. About five minutes later, another student called on the teacher. He was working with the same task (*problem c*) and his reaction was that "it is raising". Again, the teacher seemed puzzled but the student repeated "raising". "Yes, but you are dividing a negative number, so ...", the teacher said. He did not finish the sentence; he just walked away from the student's desk. However, at the beginning of the next lesson, the teacher picked up the question again. The teacher started by writing $-18/1.8$ on the blackboard, which was a part of the solution of *problem c*, and asked for the solution Then he wrote 18/2 = 9 on the board and asked the students what happens to the quotient

> Teacher: It's getting smaller yes [writes <]. Nine is smaller than the number that we divide. Nine is smaller than eighteen, isn't it?

The teacher continued writing on the board and asked about the solution of 18/0.5.

The teacher continued in the same manner with -18/2 and -18/0 5, comparing quotient and numerator At the end of this sequence, which lasted just over four minutes, the teacher summarized and generalized the results by saying that there is a reversed result when dividing negative numbers compared to the positive numbers

The episode starts in the previous lesson when the students are working by themselves solving tasks in the textbooks. Some of the students raise questions that the teacher, at first, did not understand But in the lesson that follows, the teacher seems to become conscious of the learning opportunity that the questions could create. He takes the empirical phenomenon that the student discovered ("when you divide a negative number it gets bigger?") as a starting-point when he brings up the questions in front of the whole class. In this case, the teacher and not the textbook becomes the authority, showing that the assumption is not always true. Using this episode as an example, I would like to emphasize the crucial role of the teacher A teacher can decide to deviate from the textbook whenever it is appropriate This is important to have in mind when discussing textbooks and their role in the teaching and learning of mathematics This example, however, also shows that textbook tasks may be deliberately designed for that purpose, can generate mathematical discussions and contribute to mathematical meaning making

What is the role of the task in this case? Obviously, without a response from a teacher, the task will be as any other task in the textbook. It will probably not lead to a general discussion about mathematical properties Nevertheless, the discussion originates from a question that is raised by a student working on a specific task, which makes him think in terms of mathematical generalizations

Discussion and conclusion

In the 'standard' pattern of interaction, this teacher interacts with the students in a confident way in order to help them solve tasks in the textbook Sometimes he checks their answers or gives hints, helping the students to start working on the task. Sometimes he helps them to arrive at a solution.

In the first critical incident, the teacher has to decide whether to stick to his judgment that the formula does not work for an odd number of weeks or accept the solution in the textbook.

In the second critical incident, one of the students makes a statement (or question) that can be interpreted as an attempt to generalize: "When you divide a negative number, they are getting bigger" It is a task, a quite ordinary task in the textbook that triggers him to call for the teacher to discuss this issue. At first, the teacher does not understand what the student means At some point, the teacher decides to leave the book and start a whole class discussion about this subject.

The analysis above shows how the textbook influences the teacher-student interaction First and foremost, the tasks in the textbook guide the activity of the students. Their work is to find a solution to each problem The teacher walks around, looks over shoulders, asks and answers questions that are related to the tasks. The activity is 'framed' by the textbook, which, like a painting, offers a static picture of mathematics. The picture is colored by both pedagogical ideas and traditions. In the 'standard' pattern of interaction, the teacher becomes the guide who explains and clarifies. Both textbook and teacher can be regarded as the authority since it is the textbook that offers the text and the tasks but it is the teacher who selects the tasks and sometimes guides the students (or funnels them) to the correct solution

The situation changes however when there is a discrepancy between the answer in the textbook and what the teacher thinks is a correct solution The teacher becomes ambiguous Instead of arguing against the textbook, and perhaps addressing the question to the whole class, he interacts with the students one by one. The teacher's mathematical knowledge seems to be less important in this case and the result is inconsistent mathematical meaning making. The frame, which is constituted by the textbook, is kept intact and the book becomes the authority But in the second critical incident, Mr. Larsson shows that he can go outside the frame and deviate from the textbook. So, even if the textbook is influential, it is not in charge of everything. This I think is very important to highlight First of all, teachers need to be aware of how they use their textbooks and secondly, they should act as the textbooks' superior.

Furthermore, I would like to discuss the type of mathematical activity, which the textbook seems to encourage In the 'standard' pattern of interaction, there is little room for discussions and exploratory work. The discourse could be described as the *exercise paradigm* (*cf.* Skovsmose, 2001). Most students are occupied with 'silent calculations' To solve the problem at hand is the target for the students as well as the teacher But the problems at hand, in everyday mathematics classroom life, are different from the problems that these young people will have to cope with in the future As adult citizens in a complex world they have to decide upon many issues that are mathematically not as transparent as the textbook tasks. Skovsmose (2001) raises some important questions in this respect:

> How to develop a mathematics education as part of our concern for democracy in a society structured by technologies that include mathematics as a constituting element? How to develop a mathematics education which does not operate as a blind introduction of students to mathematical thinking, but makes students recognize their own mathematical capabilities and makes them aware of the way mathematics may operate in certain technological, military, economic and political structures? (p. 131)

Teaching children mathematics must therefore include more than teaching them to solve textbook problems. I am *not* suggesting that the teachers abandon textbooks in favor of real-life problems and exploratory work. With awareness of the affordances and constraints, I suggest that the textbooks can be used as *one* teaching tool among others.

Finally, the analysis presented in this article can serve as an illustration of the complexity of a mathematics classroom. In order to understand the decision made by the teacher in the first critical incident, it is not enough to consider the teacher's mathematical knowledge or beliefs about teaching and learning mathematics, something else has to be taken into account. I argue that this 'something' can be the textbook, or rather the role of the textbook in the classroom.

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Notes

[1] This article is a revised version of part of my doctoral thesis (2006) *Teaching mathematics with textbooks. a classroom and curricular perspective*, Luleå, Sweden, Luleå University of Technology.

[2] Clarke, D. (2004), 'Kikan-Shido - between desks instruction', paper presented at the annual meeting of the American Educational Research Association, San Diego, USA.

[3] There is a literature in Swedish to support this point, see Skolverket (2003) *Lusten att lära – med fokus på matematik (No. 221)*, Stockholm, Sweden, Statens skolverk.

[4] Paraphrased from, Selander, S. and Skjelbred, D. (2004) *Pedagogiske tekster: för kommunikasjon og læring*, Oslo, Sweden, Universitetsforlaget.

[5] In the last ten to fifteen years, there has been a flood of the latter type of studies, stemming from the so called 'reform' movement in the United States. In 1989, the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards*. The *Standards* called for increased emphasis on mathematical reasoning, understanding, and problem solving. In many school districts, the first step in responding to the call for changes was to adopt a new textbook. Research on teaching raises questions about the effectiveness of this way to implement the reforms (Remillard, 2000).

[6] My Licentiate thesis (2003) *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum*, Luleå, Sewden, Department of Mathematics, Luleå, University of Technology, examines three editions of this particular textbook series (published 1979, 1985, and 2001) and states that the books are in many respects comparable.

[7] The frame factor theory, or model, is the work of the Swedish educationalist Dahllöf and his colleagues in the 1960s. In its early stage, its focus is on how political decisions regarding teaching and education (*e.g.*, time schedules and grouping) influence the pedagogical work.

[8] Paraphrased from Englund, B. (1999) 'Lärobokskunskap, styrning och elevinflytande', *Pedagogisk forskning i Sverige* 4(4), 327-348.

[9] Information about the study can be found on: www.ped.uu.se/kult, accessed, December 2006. The fieldwork and the data collection in the CULT project is based on the research design set out for the Learner's Perspective Study, http://extranet.edfac.unimelb.edu.au/DSME/lps/, accessed, December 2006.

[10] The coding system was adapted from Appendix I in *TIMSS 1999 Video Study, Mathematical video coding manual*, http://nces.ed.gov/ pubs2003/ 2003012_C.pdf, accessed, December 2006.

[11] The textbook is *Matematikboken Y Röd*, Undvall, Olofsson and Forsberg, 2002. According to the authors, it is intended to be used by students who are interested and have good skills in mathematics.

[12] Skott (2001) uses the term *critical incident of practice* as an analytical focal point to highlight lesson episodes in which teacher decision making is critical; to his *School mathematics images*, to the further development of the classroom interaction, and for the students' learning opportunities.

[13] The textbook suggests the following solution in the answer key, (($x/2$) · y · 118 + ($x/2$) · y · 152) *kr*. This formula works fine if the numbers of weeks are even. For a general solution, however, separate the number of working weeks, for example, let x_1 be the number of weeks he works days and x_2 the number of weeks he works nights and write, ($x_1 \cdot y \cdot 118 + x_2 \cdot y \cdot 152$) *kr*.

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