Acoustic Counting and Quantity Counting

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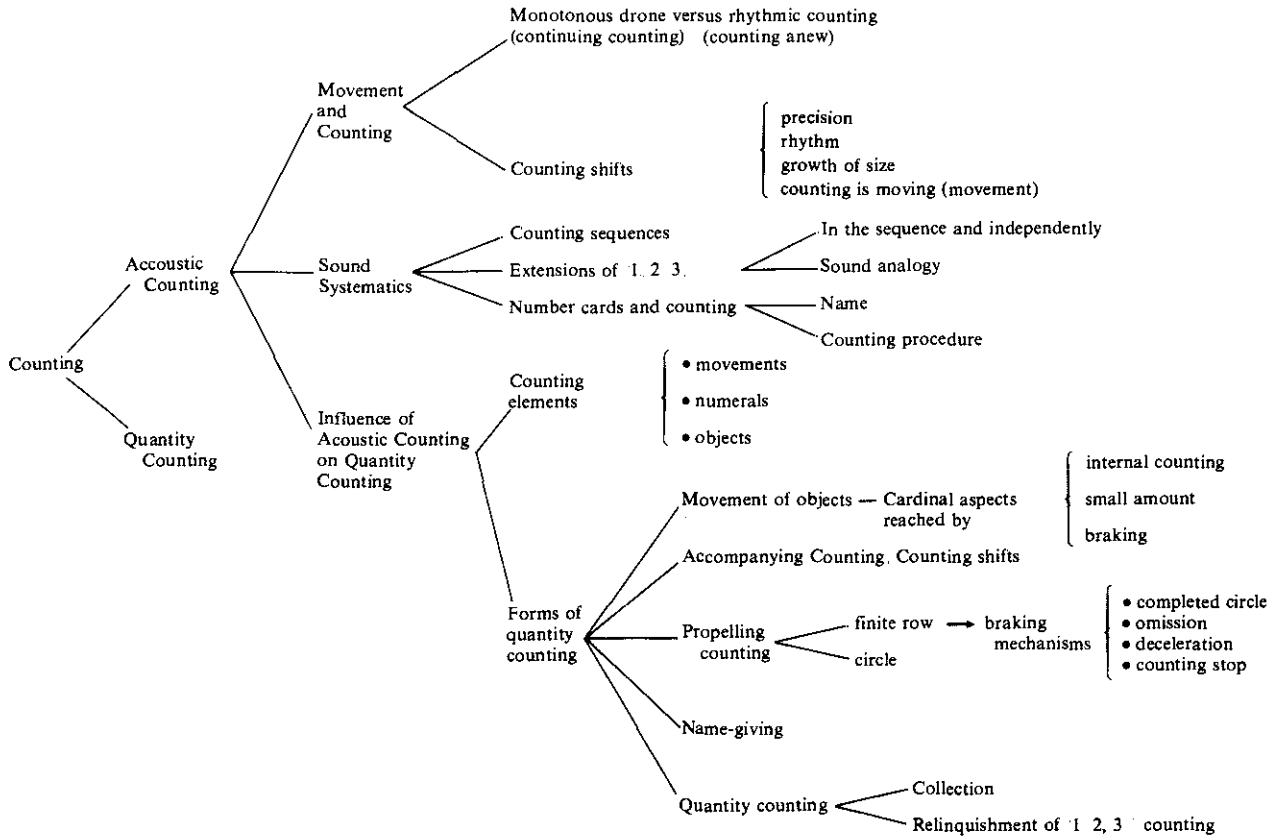
When counting quantities of objects young children (4 - 7 year olds) make mistakes which lead us to believe that these children have ideas about counting different from ours.

Examples:

- Infants are not surprised that a second count of given objects produces a different count to the first.
  They are evidently focused on counting as pure movement, regardless of quantity.
- When asked repeatedly "How many is that?" these children demonstrate the same misconception by counting the objects all over again.
- Sometimes objects are omitted and therefore not counted. This is "wrong" But in counting games, games entailing counting, omission is the rule rather than the exception
- The indication of objects and the accompanying rhythmical recital of numerals does not usually occur synchronically.
  In children there seems to be no essential link between these two activities
- Sporadically it occurs that a child is unable to remember the sequence of numbers
  Improvement can be brought about by the rhythmical use of percussion instruments
- Often the children misuse the acoustical sequence of numbers.
  Instead of counting per word (numeral), they count per syllable. E.g. se-ven means 2 objects, e-lev-en means 3 objects.
- The great difficulty is that children cannot dissociate themselves from counting in the form of 1, 2, 3,... This impedes them from seeing numbers structurally



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Taking all these factors into consideration it would seem that in the first place counting has to do with *movement* and *sound* and only in the second place with *objects*

In this article we advance the opinion that *acoustic counting*, i e. mechanical rhythmical counting *without* objects, is developed prior to counting of quantities of objects In this light, the misunderstandings which these children have about counting quantities become comprehensible.

Within the conception of acoustic counting there are three separate areas of research:

- The mutual influence of movement and counting
- The sound systematics of numerals.
- The influence of acoustic counting on quantity counting.

From this investigation we would like to give some recommendations for the benefit of schools on the subject of counting. These are particularly relevant to the control of counting as movement (slowing down, quickening, stopping)

See the diagram for an overview of the chapters in this article

Counting

Counting is not limited to the counting of quantities It is used by children to accompany movement, games and singing, and just to pass the time. This kind of counting is first and foremost of an acoustic nature The counting of quantities is an application thereof.

Acoustic counting is the recital of a sequence of numbers in a certain order. This is a broad definition as the sequence does not necessarily need to be the natural or "expected" sequence nor does it need to begin with "one". Neither are quantities of objects necessary in order to give the answer to the question: "Can you count?"

Not every researcher defines counting in these terms. Some researchers understand counting exclusively as the counting of quantities [Steffe, et al, 1980, p. 102; Richards, 1981, p. 25; Mierkiewicz and Siegler, 1980, p. 250] But there are also researchers who share the opinion that acoustic recitative counting precedes the more specific counting of quantities. [Gerlach, 1914; Kühnel, 1916, p 37; Fuson, 1980, p. 257; Steffe, et al, 1980, p. 102, 103]

Let us now look into a few differences between acoustic counting and quantity counting.

A The order of numerals does not need to be the natural one of 1, 2, 3,... Children's games actually ascribe a function to the omission of objects, even if this is contrary to the official counting sequence 1, 2, 3, ... Consider the game of hopscotch or skipping songs In Holland there is a skipping song "In *de* tent stond *een* vent" (In *the* tent stood *a* man) and you twirl the rope more at the stressed words and once at the other words. This is one form of omission. Another form is in counting 10, 20, 30, ... where the numerals 1 — 9 are omitted Note that the omission of articles, places or numerals varies In acoustic counting the omission of objects appears more often than the omission of numerals.

On the other hand, in counting quantities, omission is strictly forbidden. The sequence of numerals is fixed and each object must be counted. This is in contrast to the rhythmic acoustic counting which precedes it

B Note that it is not necessary to start acoustic counting with the sound "one" E.g. Paul is three and counts while the other children go and hide: 4, 12, 18, 6, 4, 12, 18, 6, 4, 12, 18, 6, 4..... He starts his counting sequence of sounds "4, 12, 18, 6". And here we notice another phenomenon of acoustic counting.

C The final sound is not fixed. When one is playing "Hide and Seek", one needs a terminal sound, but Paul just goes on For him there is no fixed final sound.

When one is counting quantities, the most important thing is the final sound. When counting acoustically, the final sound is far less important. So it is not difficult to understand that children will have difficulties with the two forms of counting. There are therefore many researchers who have engaged themselves in the study of the establishment of the final sound as number There have been various means suggested to bring the pupil from the counting numeral to the numbers, e.g. by the "interiorisation" of counting, by whispering counting [Karpova, 1972, p. 372 and 379] or working for a long period with small quantities [Kühnel, 1916, p. 35, 36; Bouman en Van Zelm, 1930, p. 84]. In our acoustic investigation we found that a pause in between times was very useful to accentuate the number.

D Double counting (counting one number twice) in counting quantities is not allowed In acoustic counting double countings are in a certain sense even necessary. However, children have continually to repeat the basic sequence of sounds from 1 to 9, of course with the sounds 10, 20, 30, 40, etc., in between (19, 29, 39, 49, etc.)

E Moreover, it is not necessary that there be quantities of visible objects around while the child counts acoustically. We have already said that no visible objects are necessary for the child to answer the question "Can you count?" Here again acoustic counting differs from quantity counting.

Acoustic counting offers great freedom in the choice of sequence, initial sound and final sound. In this respect, quantity counting is a limited particular form of acoustic counting.

Summarizing:

- Sounds may not be omitted in quantity counting After '10' comes '11' and not '20'
- No numeral may be said twice.
- You must begin with the sound 'one'.
- You must go on counting to the end of the row of visible objects.

Acoustic counting differs in these respects

Acoustic counting

The phenomena that arose in our investigation of acoustic counting in 4 to 8 year-old children can be sorted under three headings:

1) The *dynamics* of acoustic counting or the interrelation between counting and movement
2) The *sound systematics* of acoustic counting.
3) The influence of acoustic counting on *quantity counting*.

1) *The dynamics of acoustic counting*
We shall discuss two aspects:

a) monotonous counting as opposed to rhythmic counting
b) count shifting

a) *Monotonous counting versus rhythmic counting*
Some investigators consider that rhythm and measure are derived from physiological processes in man and from movement in space. [Lievegoed, 1939, p. 57 e v.; Bindel, 1982, p. 70] On the other hand, we have been shown that the lexical meaning of the words often escapes young children and that the external appearance of the words, i.e. rhythm and measure, is what gives them significance. [Zaporozhets and Elkonin, 1971, p. 154, 158, 181; van den Brink, 1980, p. 167]

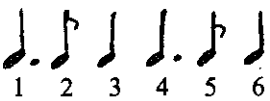
Rhythm and measure have different functions in regard to acoustic counting. Example: I ask 12 infants ((4;0) to (5;5)) individually to copy the following rhythms on the tambourine:

A 

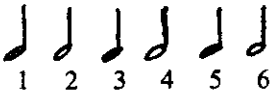
B 

C 

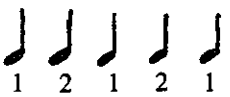
Marieke (5;3) is the only infant in the group who cannot yet count. She imitates rhythm A as two equal beats and does not tap any further. But with rhythm B she won't stop and goes on tapping. Ditto with rhythm C. A week later I do the same thing with the same 12 infants, but now I count while I tap on the tambourine.

A′ 

B′ 

C′ 

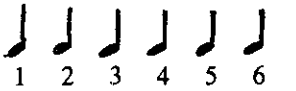
Marieke counts and taps:

A′ 

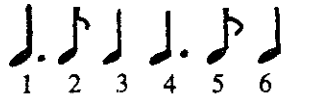
B′ 

C′ 

Evidently the tapping is limited and structured by the counting. I try the last rhythm (C′) once again and now Marieke counts and taps:



This is the first time that she counts correctly. The monotonous tapping on the tambourine stimulates her to go on counting.
It was striking that 1 out of the 12 infants performed:

A′ 

as 

In *rhythmic* tapping there is a tendency among children to *count anew* from 'one' after every period, i.e. at the beginning of each *group* of taps. This is in contrast to monotonous tapping which stimulates them to go on counting and even irons out offered rhythms.

The rhythm 

that we gave 8 infants to tap, without counting, has a "brake" after the 5th tap and before the 6th tap. None of the children, surprisingly, tapped or counted more slowly at the end. Two children actually speeded up at the end.

This "propelling counting" is, according to us, fatal to the awareness of number. The rôle of "braking" is therefore very important.

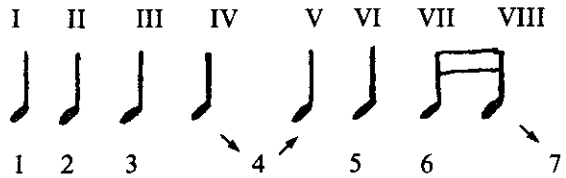
b) *Counting shifts*
Another phenomenon is that counting and movement do not always happen simultaneously. In the past different causes for this phenomenon have been indicated, such as:

- lack of accuracy [Ginsburg, 1977 a, p. 7]
- lack of feeling for rhythm [Baldwin and Stretcher, 1925, p. 141]
- lack of insight into the notion that "counting one further" means "counting one more" [Kühnel, 1916, p. 33], which implies that the link between pointing and counting is absent.

We think that this is caused by the fact that children's counting approximates more to acoustic counting (i.e. counting which is largely dependent on speeds of movement and enunciation) than quantity counting [see also Davydow, 1972, p. 165].

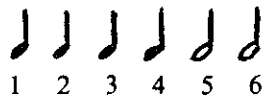
I tap and count the following rhythm:

Sanne (4; 11) copies it at high speed. At the end she drops the drumstick and counts this "drumroll" in. When we analyse the tape, she tapped and counted as follows:

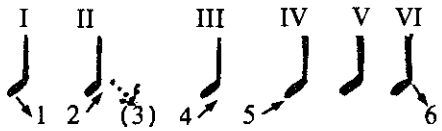


VII and VIII are the drumroll at the end. Note that tap IV is given immediately before saying "four" and tap V immediately after saying "four", which results in two taps for one numeral, which is a *shift*. Similar counting shifts between tapping and counting repeatedly occur.

Brenda (4; 10) does the rhythm



as follows:

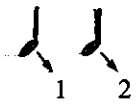


She says the "three" in a soft voice. Tap V is not named. Brenda taps beat 1 and then says "one". Says "two" and taps beat II. Then says "four" and taps III. After that she says "five", taps IV, V and VI and then says 'six'. It is clear that the matching of the sequence of taps and the sequence of numerals is subject to a counting shift.

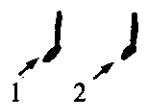
In general there are 3 systems by which one can match counting and tapping:

A: tap and count are simultaneous

B: first tap and then count

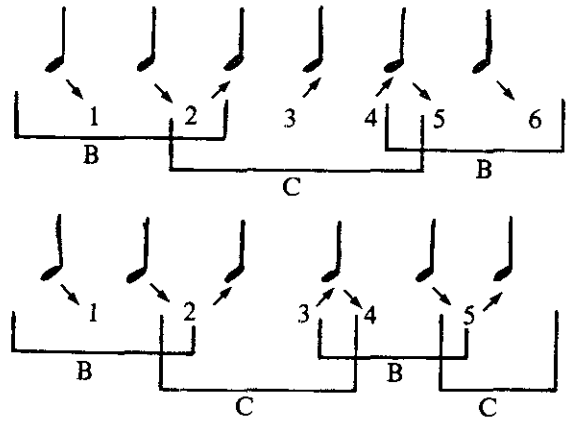


C: first count and then tap



During counting, children can change the system several times, e.g. from system B to system C. This causes a counting shift. Combinations of these counting shifts make it possible yet to arrive at the right number of taps.

Compare:



This makes it even more difficult for children to discover what exactly "counting" means. Demonstrable factors in the origin of counting shifts are:

- The lack of connection between counting and movement. Four year-olds, particularly, must find the rhythm that we give them a complete mystery [see Lievegoed, 1939, p. 60]. At this age there seems to be a broad correlation between counting and movement. The coincidence is more coincidental than systematic, while the numerals are not recognised as separate sounds in the sequence. [Fuson, 1980, p. 258]
- In older children we see other origins:
  - The sequence of numbers is not properly mastered, which causes the pupil to stop reciting for a moment, while his movements continue.
  - Or the sequence of numbers is so well mastered and so quickly recited that the movements lag behind.

  This implies that, in teaching, attention must be paid to braking, acceleration and slowing down in counting.

2) *The sound systematics of acoustic counting*

In this section we shall go further into the meaning of different sequences of numbers (1, 2, 3, ...; 10, 20, 30, ...; 5, 15, 25, ...; 100, 200, 300, ...). We shall look at the attempts of children to extend the number sequence 1, 2, 3, ... and shall discuss the connection between counting and written numbers.

a) *Natural sequence of numerals and other acoustic sequences*

"Counting" for 5 year-olds means exclusively reciting the sequence of natural numerals.

Interviewer: "I count and you tell me afterwards if I did it right or wrong."
Listen:
A 1, 2, 3, 4, 5, 6, 7, 8, 9, 20, 21, 22.
Sylvia : (5; 7) No, you've forgotten the 10.
B *Interviewer* : Again: 10, 9, 8, 7, 6, 5, 4, 3, 2, 1.
Sylvia : No, wrong again. You've forgotten the 1.

C Interviewer : Listen: 1, 2, 3, 4, 8, 9, 10, 11.
Sylvia : No, 8 doesn't belong to 4 and 10 doesn't belong to 8.
D Interviewer : I'll try again: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 1, 2, 3, 4, 5. What do you think?
Sylvia : Wrong You've done the 5 twice
E Interviewer : Now for the last time: 10, 20, 30, 40, 50, 60.
Sylvia : No, you've not said the 1

Sylvia has obviously got the sequence 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ... in her mind. The sequence that *begins* with 1 and does not *end* with 1 (as in *B*) and in which there is a fixed pattern (see *C*) Repetitions are not allowed (see *D*). Counting in tens (10, 20, 30 . . .) is not regarded as "real" counting (see *E*).

In the same way we asked the 5 year-old toddlers what they thought of counting in this way (*A* to *E*).
The distribution was thus:

Sequence	Number of children judged right	judged wrong
A	2	8
B	2	8
C	1	9
D	3	7
E	2	8

For 5 year-olds counting is recital of the fixed sequence 1, 2, 3, . . ., i e. fixed start ("one") and fixed pattern ("8 doesn't belong to 4"). [See also Gelman and Gallisteel, 1978, p 70] They tolerate few deviations.

(Priscilla (5; 7) remarked: You can say "zeuven" instead of "zeven". This pronunciation of the number 7 (zeven) is used in Dutch to distinguish it from the number 9 (negen) as they have identical vowel sounds.)

Therefore the sequences *B* and *E*, counting backwards from 10 and counting in tens, are regarded as wrong

b) *Extension of the number sequence*
I asked 16 toddlers to *predict* how far they could count Then I got them *to count for real*.

Interviewer : How far can you count?
Wendy : (4; 11) I can't.
Interviewer : Yes, you can.
Wendy : To 3.
Interviewer : Do it again.
Wendy : 1, 2, 3, 4.
Interviewer : Can you count any further?
Wendy : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13.

Wendy and I are both surprised at this achievement. 9 out of the 16 toddlers counted further than they had predicted, while the others had made no prediction but immediately started to count to see how far they could go

Two important remarks are relevant to this example On the one hand, it is evident that they are likely to know more numerals, in the context of counting, than outside the context of counting The number sequence is regarded as a whole, out of which the numerals are isolated as separate numerals only as a second thought. [Fuson, 1980; Steffe c.s., 1980, p. 101] On the other hand, they try to extend the number sequence during the game. The children cope in various ways with the task of counting further and they leave off in various states of "confusion".

In the formation of new numerals *sound analogy* plays an important rôle.
Barbara : (5; 7) counts: 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 80. (After 27: "eighty")
Note: in Dutch, 27 is pronounced "seven and twenty".
Bernt : (5; 7) counts from 1 to 29 ("nine and twenty" in Dutch) and continues with "ten and twenty".
Sven : (6; 0) starts to repeat bits: 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 11, 12, 13, 14.
Sylvia : (4; 10) counts the fingers of *one* hand When asked if she can count any further, she looks at *both* hands and counts: 1, 2, 3, 4, 5, and again 1, 2, 3, 4, 5, looking at the second hand.
Emenda : (6;0) counts from 1 to 29 and continues with: "one hundred, two hundred"

From these answers it can be seen that infants find various ways of getting to grips with the task of counting further Some just repeat a section (like Sven and Sylvia), others step over to another "ten" (c.f. "unit") or "hundred" (like Barbara, Bernt and Emenda). Note that after 100: *one* hundred, *two* hundred, *three* hundred, *four* hundred are counted further; analagous to 21, 22, 23, 24 (*In Dutch one and twenty, two and twenty, three and twenty, four and twenty In Dutch 101 is said as "honderd een".*) There is therefore an acoustic analogy between numbers before and after 100

Up to hundred they go on counting in units and after hundred they count in hundreds: 100, 200, 300 instead of 101, 102, 103. This also happens with 1000. They count further in thousands as if they were concrete numbers.

For the benefit of extending the number sequence these acoustic tendencies can be broken through in the course of Class I, not only by using written numbers but also by *acoustic exercises*.

Round about Easter, every Class I pupil masters the number sequence 10, 20, 30 . . . . . , 100. I asked 12 children: Watch how I count. When you've got it, you go further: "5, 15, 25, 35, . . " (in Dutch: five, five teen, five and twenty, five and thirty) 7 out of the 12 children cannot cope. "Another row: 8, 18, 28, . . " (in Dutch: eight, eight teen, eight and twenty, . .) The children cotton on to the structure quite quickly.

Similar sequences of numbers (10, 20, . .; 5, 15, 25, . .; 8, 18, 28, . .) are easily learnt by Class I children because of the sound analogy. But after 100, Class I children get into trouble.

105 (in Dutch "hundred five") is expressed as:
—"five and hundred" by 4 of the 12 children (analogy with "five and twenty" etc.)
—"five hundred" (500) by 3 children
—"hundred five" (the correct pronunciation) by one child
The other three don't attempt to go past 100.

The 12 Class I children are asked the difference between 103 ("hundred three") and 300 ("three hundred") 7 of the

children indicate the difference in the pronunciation, 3 think there is no difference and 2 pupils don't know. Nobody indicates the quantitative difference represented by the two numbers 103 and 300

The changes in the order of units, tens and hundreds when expressing numbers in the Dutch language (in Dutch "237" is expressed as "two hundred seven and thirty") makes counting a profitable occupation. This was evident from the following conversations with 11 children of Class II

First I get the class II children to count from 1 to 25. Then I ask them 3 questions:

A "What comes after 35?" (in Dutch "five and thirty") Out of the 11 Class II children that I asked in October, 8 answered immediately: 36 They counted on, by way of sounds: 1, 2, ... , 23, 24. 25, → 35, 36.

B "What comes after 136?" (in Dutch "hundred six and thirty") 9 out of the 11 children said immediately: 137. Putting a *"hundred"* before the "36" does not pose a problem acoustically

C "And what comes after 237?" (in Dutch "two hundred seven and thirty") Here the children are confronted with a choice: the "2" of two hundred, or the "7", or the "30" has to be changed

Here are a few of the resulting conversations:

Tamara : (7; 4) I don't know.
Interviewer : What comes after 37?
Tamara : 38
Interviewer : And after two hundred seven and thirty?
Tamara : 238

The intermediate step 137 in the series 36→137→238 is confusing

Jessica : (8; 0) 337 (237 is followed by 337)
Interviewer : No
Jessica : 837 (237 is followed by 837)
Interviewer : No, what comes after *two hundred* seven and thirty?" (emphasis on "two hundred")
Jessica : 238

Bob : (7; 4) Yes, after 237 . . that is something with 8 . . . . 237, that must be 238.

Natalie : (7; 11) 3800 (237 is followed by 3800)
Interviewer : Perhaps you mean 138 But what comes after 237?
Natalie : 238

7 out of the 11 Class II children find the answer 238 without help So certainly acoustic counting must be extended with the help of visual aids (number cards, number line) to make the difference in enunciation meaningful.

c) *Counting and written numerals*

With the introduction of written numerals (with two or more figures) the differences in expression emerges. [van den Brink, 1980 A, p. 52; Booker, 1983] In our culture, this discrepancy between the visual and acoustic representation of numbers is an extra difficulty, which possibly does not occur in other cultures. [Hatano, 1982, p. 219; Francis, 1983] In any case, in specialist literature a distinction is made between the acoustic decimal system and the graphical positional system [(Kühnel, 1916, p 41] and there have been voices demanding that the expression or pronunciation of numbers in Dutch should tally with the *reading direction,* i e 23 should be expressed as "twenty three" and not, as in present day Dutch, as "three and twenty".

The children attune the acoustic and visual numeral sequences to each other by means of various activities. For example, the acoustic number sequence is used to find out *the name* of the written numeral and vice versa, written numerals give indications to various counting procedures

Name

If children have forgotten the name of a number card, they will spontaneously start counting in order to find *the name* again. In educational circles, however, this procedure is frowned upon. Children should immediately recognise 5 as "five" For they say, what on earth would happen if the number card "5" was hanging in place "6"? The numbers should be recognised, irrespective of the number (quantity) of number cards, *directly,* from the written symbol

I put 4 separate number cards on the table in front of 12 Class I children. "5", "16", "17" and "28"
Interviewer : Which numbers are they?
At once, 8 pupils give the right names to all the cards. 2 children count to find out the name for "28". But how do they know that they must count on to the sound "twenty eight" if they do not know beforehand that "28" is "twenty eight"?

Ilse (6; 8) counts from "20" to "28"
Interviewer : How did you know that you had to count from "20"?
Ilse : Because there's a 2.
Interviewer : But you didn't know yet that this was "twenty eight"? How did you know you had to go on to 28?
Ilse : Because there's an 8

Counting procedures

Number cards not only function as visual representations of the spoken numerals [Schmidt, 1978, p 158] but also as a pointer to the counting process. They represent, therefore, counting procedures wherein different counting sequences are offered (e.g. 1, 2, 3, . . ; 10, 20, 30, . .)

Bob ((7; 5) end of Class I) does the written problem 34 + 4 = .
Bob : That is 11, because 4 plus 4 plus 3 is 11
Interviewer : But this (34) is thirty four. Bob now counts from 1 to 34 counting with the fingers of *one* hand. After 34 he counts further: "35", "36", "37", "38" He doesn't know the name of "34" and uses the number sequence. [Steffe et al, 1980, p. 102, have the same experience]

Jessica (7; 5) reads the problem 34 + 4 and says: "Three and forty . . . . no . . . four and thirty"
Interviewer : How did you find that?
Jessica : First I counted 10, 20, 30, and then 31, 32, 33, 34 Then I knew it was four and thirty.
She reads 34 + 4 = . and says: Is 38

Interviewer : Why?
Jessica : Leave out the 3, then two fours (4 + 4 =) is 8, and then the 3 back again.

This disposition to seek the name of a written number by the *use* of different familiar sequences of numerals can be used in teaching arithmetic. For example, we can cover *one* of the digits on a number card and ask the pupil which number it will be.

3) *The influence of acoustic counting on quantity counting*
In order to count quantities of objects, a child must combine three elements of counting:

a) The *movements* necessary for pointing out or moving objects
b) The *numerals* that must be expressed
c) The *objects* to be counted
[Mierkiewicz and Siegler, 1980, p 249-256]

In acoustic counting elements a) and b) are central: a) *the dynamics* b) *the sound systematics*.

The counting of quantities of objects is to be regarded as the material appearance of acoustic counting. Acoustic counting is made more specific: you must begin with the sound "1", etc. In the nature of the case the emphasis is now more on visual-graphic counting, on the "seeing" of collections and structures, than on movmeents and sounds. On the other hand, movements and sounds (rhythms and measure, counting shifts, knowledge and use of sequences of numerals, etc) exercise their influence on this specific form of counting (quantity counting). In fact the counting child emphasizes one of these three elements a), b), or c), or uses a combination thereof. And this determines what a certain child at a certain moment understands as "quantity counting"

Theoretically, one can distinguish 7 different combinations of the 3 counting elements (a), b), c)) and therefore 7 different forms of quantity counting, some of which are nearer acoustic counting, (elements a) or b) or a combination of these two) than to quantity counting (a combination of a), b) *and* c)). In our investigation we have subjected five forms of quantity counting to a further scrutiny.

I *Moving* (counting element a) is central)
"Quantity counting" in 4-year olds, for example, is nothing more than the task of moving: moving the eye or the finger, moving quantities of objects. This is accentuated in the counting of non-stationary collections of objects, as when children are counted in the playground

II *Accompanying counting* (counting elements a) and b))
In contrast to III, movement a) can remain primary and counting is regarded as the accompaniment to the pointing (i.e. moving of the finger).

III *Propelling counting* (counting element b) is central)
Here, the pointing movements are counted. The counting propels, as it were, the movements along.

IV *Labelling* (counting elements a) and c))
Counting quantities is giving a name to each object; the name being a numeral which is given to each object separately, not to the whole collection

V *Proper object counting* (a), b) and c))
It is not the counting or the movements which determine and limit the counting itself, but the collection of objects, the structure, visible or invisible *parts* of the collection, etc.

These five forms of counting quantities came to light by bringing children into conflict situations in which a certain aspect of quantity counting became unmanageable

I *Moving*
Célèste (5; 4) can do acoustic counting well. I give her a piece of paper with squares on it. (Figure 1)
Interviewer: How many squares are there?
Célèste counts from 1 to 19 and points them out thus:

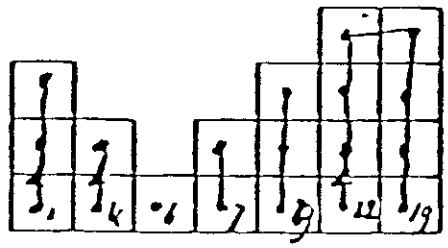


Figure 1

I turn the paper round 90 degrees and Célèste counts again. (Figure 2)

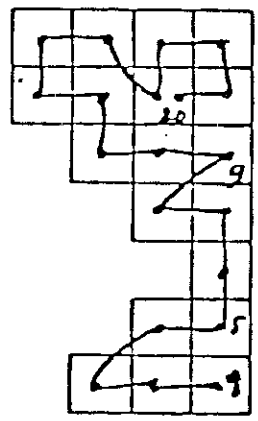


Figure 2

She now reaches 20.
Interviewer: First you had 19 and now you've got 20. How did you manage that?
Célèste : There's an extra one
Célèste does not find it in the least surprising that the number varies if you turn the paper round. She does not look for the fault in pointing. She is evidently not focused on the *number* of squares, but on the *procedure* that we call "counting". Counting is pointing and saying 1, 2, 3, 4, etc..

She is focused on the *method of counting* and it does not matter to her whether she has missed out a square or

counted one square twice. Her laconic attitude to our surprise ("There's an extra one ") does not explain why that has happened. Moreover, her way of counting varies each time, so why shouldn't the number vary? The constancy of the procedure is something quite different from the constancy of the number. The acquisition of the conservation of the number is sometimes defined by the failure to recount. [Fuson, 1980, p. 260]

The concentration on the movement is even more noticeable when I ask Célèste to count the squares in Figure 3. She counts as follows:

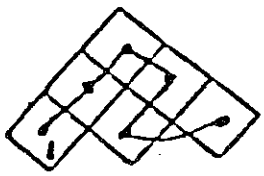


Figure 3

Interviewer: Do it another way.
Célèste counts in the same way, but uses the middle finger.
Interviewer: Do it still another way.
"Count another way" refers to a *bodily way* of counting, as far as Célèste sees it: with her index finger, her middle finger, her ring finger and later on even with her little finger. It is obvious that Célèste interprets counting as an action, a movement, or a moving of something. She does not see it as a means to determine a quantity.

Other investigators have also remarked on this phenonmenon and given instances of its occurrence in various children. [Freudenthal, 1978, p 219; Nicole Nantais-Martin in: Herscovics and Bergeron, 1982, p. 16; Hebbeler, 1977, p. 108; Davydow, 1972, p. 165, 166; Steffe et al, 1980, p. 94] They also give various means of showing children a method of determining a total quantity by counting off objects. We shall name a few

- First out loud, then whispered, and finally internalised counting [Karpova, 1972, p 379]
- First getting the children to work with collections of 4 or 5 elements for a prolonged time in order to accentuate the cardinal number of these small collections. [Kühnel, 1916, p. 35, 36; Bouman and Van Zelm, 1930, p. 84]
- A means that we suggest in this article is the slowing down of the counting *before* the whole collection has been counted, which we call "braking en route". We shall expand on this when we discuss "propelling counting".

Lastly we make mention of the fact that good "counters" in Class I, when they are determining *large* numbers, often relapse into counting out loud and the moving of objects concurrently. It is evidently the surest method (if you compare e.g. internal counting and just looking at objects) and is also regularly used, when counting non-stationary objects (e.g. children in the playground).

II *Accompanying counting*
The preceding form of counting (moving) is here accompanied by numerals. The pointing and moving away or around of the objects is primary; the counting is of secondary importance. This situation occurs with children who do not yet know the counting sequence

I give the infant a box with 31 foam rubber blocks and say: "There are 31, count if that's right?"
Bernt (5; 9): Count the sponges? I can only count to 20. Then he starts counting out loud each time taking a "sponge" out of the box with his right hand, transferring it to his left hand and counting "1, 2, 3, ... 28, 29 (nine and twenty in Dutch) ten and twenty, hey, I don't know what comes after 10?"
Interviewer: "Eleven"
Bernt : "Eleven and twenty." (He has counted the lot.)
Bernt has by this time forgotten the number 31 that I had said at the start of the exercise.

4 of the 5 infants count out loud, one counts to himself, one of the 5 infants (Bernt) achieves the right number; the others omit parts of the number sequence, or repeat numbers. They haven't yet mastered the acoustic number sequence, which means that the enunciation of numbers is no more than a musical accompaniment to the pointing at or moving of the objects.

Movement is still the essence and, as such, this behaviour is in conformity with one of the five types of "counting children" that Steffe et al [1980, p. 94] distinguishes, namely, the "counter of motor unit items". It was noticeable that with the infants who did not know the number sequence properly, the counting lagged behind compared to the moving of objects. In other words a counting shift occurred.

We gave the same task (the 31 foam rubber blocks) to Class I children. The best counting method for them, and it was used most, was: "Move the objects and count out loud while you do it." The Class I children, in contrast to the infants, were masters of the counting sequence. It was conspicuous that they wanted to display this by counting progressively more quickly. Some of them counted acoustically so fast that they could not keep apace with the moving of the blocks. This produced a counting shift. Both faults (too fast or too slow acoustic counting relative to the moving) have already been indicated as causes of counting shifts. This shows that the mastery of the pace of acoustic counting is essential for the proper counting of quantities of objects.

III *Propelling counting*
The mastery of the acoustic sequence of numerals in word and rhythm proceeds *at an increasingly rapid pace*. [Fuson, 1980, p. 257] This causes acoustic counting to produce a propelling effect on the movements (pointing, moving of objects) which are performed in the counting of objects. Counting faults are the direct result of this propulsion. We have already indicated the counting shift, where the child counts more quickly than it can point. Propelling counting also causes problems in the choice of acoustic units to match the quantity of objects to be counted.

27 pencils are lying alongside each on the table for Marije

((6; 7) end of Class I).

Interviewer : "I'm going to count and you point them out to me."

I begin to count and stop at 10. Marije holds onto the eleventh pencil. The counting propels her to the next object.

Interviewer : Where was I?

Marije : At 11.

I count further and after 27 pencils I *keep* counting even though all the pencils have been pointed out.

Marije calls out: "No." and I stop counting at 30.

Interviewer : How many pencils are there?

Marije : I can't remember. There are not 30 pencils on the table.

She now recounts the pencils but at 21 (Dutch: one and twenty) points out 3 pencils for the 3 syllables ("twenty" being regarded as one syllable). Her acoustic counting propels her movements further. She isolates verbal units in the counting sequence: whole numerals, but also *syllables* of numerals. These determine how she points out the objects [Steffe et al, 1980, p. 101, 102] I did this task (the 27 pencils) with 9 Class I children. Here are a few of the children's reactions after I had counted past 27.

Sven : (6; 10) Stop, stop you must stop

Interviewer : How many are there then?

Sven : Well, I'll have to count them first.

And he counts them: 1, 2, ...27.

Evidently the first time he hadn't counted them himself, but had only pointed them out.

Maureen : (6; 4)remarks after 27: Hi, stop, I can't go any further. There aren't any more pencils.

Interviewer : How many are there?

Maureen : 25. *And then she counts the pencils again and answers 27.*

All children lost track of the quantity of pencils because of the "counting past" (each time I counted past 27) and they had to find it by recounting the collection. The *end of the row* of objects is apparently a visual phenomenon which shrinks into insignificance compared to the acoustically propelling counting.

But the *end of the row* determines the quantity, whereas the movements are paramount in acoustic counting, movements which are continued because the counting continues. We can conclude that the braking of acoustic counting *before the end of the row is reached,* is obviously a way of making the number, i.e. the quantity expressed as a number, break through to consciousness.

The problematics of braking

In order to investigate the mechanism of braking, I presented the infants with a *circle* of identical objects, instead of a *row,* while I counted.

I give the infant a string of 48 *grey-blue* beads and say: "I'll count and you'll point to the beads."

Priscilla (5; 7) lets the beads slide one by one through her fingers and counts softly with me to 39. After that she stops counting, but continues to slide the beads.

When I have reached 48, I continue to go on counting, 49, 50.

Priscilla continues to slide the beads, without counting to herself.

Interviewer : 100, 101, 102, 103

Priscilla : Stop

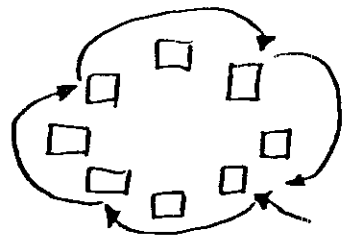
Interviewer : Why?

Priscilla : Because you must only count once, because it's *one bead. You must only count once, otherwise it's wrong.*

How far do the children tend to go on counting? For what reason do they stop counting?

The 7 infants that I tested in this way ended up, respectively, at 47, 48, 69, 74, 100, 103 and 118. Going round the string of beads approximately twice (after counting 96) induces the children to stop. It brakes the propelling counting and focuses the attention on the number. *Double counting* seems to be acceptable to the point that it starts to work as a brake on propelling counting. The *omission* of objects works, on the contrary, as an immediate brake.

I point to 8 wooden blocks, placed in a circle, in the following way and the pupil counts:



Robert (5; 9)said directly: "1, 3, 5, . . .", and omitted the numerals for the omitted blocks.

Priscilla : (5; 4) "You must do this one, too", pointing to the second omitted block.

Most of the infants (5 out of the 8) are of the opinion that omission of blocks is forbidden.

I ask 12 Class I infants to count objects up to 30. When they reach "7" and "27" I call: "Stop a minute. Which number did you say?" 9 out of the 12 infants answer "8", when I stop them at "7". They evidently need a *braking path.*

When I stop them at "27", only 5 out of the 12 infants say "28". With higher numbers there is less need for a braking path because it takes longer to enunciate these numbers. When counting quantities, stops make it possible to direct the children's attention to the quantity which has already been counted, rather than to the activity of counting itself. ("What have you counted so far?")

To sum up: *acoustic counting* propels the pupil to movements with the following characteristics:

- all visible objects are included without exception, nothing is omitted;
- the space after the row of visible objects is divided in units in order not to hinder the movement while the acoustic counting is going on;
- the propelling characteristic of the counting is also evident from the fact that infants quicken or slow down their pointing in accordance with the quickening or slowing down in the counting of the interviewer.

- hitches in rattling off the counting sequence immediately produce counting shifts.

We suggest that number is more strongly coupled to propelling counting than to the quantity of visible objects. Taking the characteristics above into consideration it can be seen that braking plays a large part in converting acoustic propelling counting into a method for determining quantities. Children must be able to control the slowing down. Counting becomes number when it is halted. We mentioned various braking mechanisms:

- if children go on counting round a circle (e.g. string of beads) the children will ultimately come to a halt;
- omission of objects is not acceptable and therefore acts as a brake;
- the acceleration or deceleration of acoustic counting and the accompanying synchronous pointing;
- the cessation of counting before all the objects have been counted leads to the numerosity of the reached subcollection.

IV *Namegiving (Labelling)*

The counting of quantities is sometimes regarded as the naming of objects. The objects become, as it were, visual representations of the acoustic numerals. It is however, still unclear whether the number concept has been acquired.

Eight blocks are lying in a circle. I count and Eelco (4; 5) points them out. When I say "Nine", Eelco responds "No, that was "one" and this is "two" ". And he repeats the numerals while pointing out the blocks.

Here, the counting of quantities is not the "labelling" of a *collection* but of *each object* separately. The name (numeral) is thus identified with one of the objects, so that names and objects are not interchangeable. We have to guard against such a situation, for instance, by means of counting the collection in different directions, or beginning at a different block, so that the block with the name "one" is not always *the* "one", or by propelling counting.

Two out of eight infants reacted like Eelco. I couldn't get them to go on counting. The others regarded the counting as a rhyme to keep the moving going. While they were counting to themselves, they lowered their voices at the end of each round, showing that they knew they were pointing at the objects for the second or third time. In contrast to the "name givers", they were aware of *both* counting forms (rhythmic propelling counting and quantity counting). We are apt to consider these children as better counters than the "name givers".

V *Quantity counting*

In this form of counting the focus is on the *collection* whose number (quantity) must be determined. This collection can vary in nature: the elements can be few or many, visible or invisible, stationary or moving. These aspects of the collection influence the counting. A few objects, all of which are visible, are simply given name-numerals (see IV). This is *one* of the forms of childish counting, which comes quite close to "quantity counting" as understood in teaching, i.e. without omission or repeat-counting. Here the numerals draw the children's attention to omission and repeat-counting.

In the counting of *large* numbers of visible objects (e.g. 48 identical beads on a string) it is not the numerals that draw the children's attention to repeat-counting but the objects (the beads) themselves.

Cindy (5; 3) stops exactly at bead 48. She refuses to go on pointing any further, although I go on counting. "Didn't you see?" she says and points to the knot in the string. "I began at that knot and now I'm finished."

Large collections of *homogeneous* material create counting situations which concentrate the pupil on the coupling of acoustic counting and the numbers of objects (each bead must be counted once—approximately).

In the counting of large collections of *non-homogeneous* material, like the beads on a counting-frame, the visual properties (colour, shape, arrangement) play more tempting rôles than in acoustic counting. Together with collections which are partly or completely *invisible,* these *large* collections have an important function in the *relinquishing* of counting as "1, 2, 3, ...". The children also have to regard sequences like "10, 20, 30, ... ", for example, as counting. In principle, there are at least two means at our disposal for introducing this relinquishment:

- attention to ways of acoustic counting other than "1, 2, 3, "; further attention to *acoustic properties centred on* the enunciation of numbers.
- attention to the *visual properties* of numbers and quantities (positional value, configurations in collection, etc.)

Recommendations for education

Here is a summary of recommendations for the teaching of counting in kindergarten and Class I.

1 The link between *acoustic counting and music* can be worked on in school in many ways.
Here is a mere outline of the possibilities which we have examined:

- *beating time*
  - beating time to unfamiliar songs with varying rhythms;
  - ditto with familiar songs;
  - children miming members of orchestra to recorded music.
- *singing a song*
  - singing to themselves (internal singing) and (external) beating time;
  - guessing the song from the rhythm pattern which is beaten.
    N.B. this can also be the "song": 1, 2, 3, 4, 5, ...
  - counting accompanied by movements (e.g. beating on a drum) will act as a back-up to the child's retention of the counting sequence
- *walking and moving*
  - moving in time to the counting:
    acceleration, deceleration, varying rhythm, stopping and moving in time when counting backwards.
  - ditto when moving objects from one place to another

- teach the sequence "10, 20, 30, . . ." in counting action games (e.g. Hide and Seek) This implies a recognition of the sound when written numbers are introduced later on;
- walking to certain parts of the counting sequence: 1, 2, 3, . . and then 21, 22, 23, . . and then 121, 122, 123, . . (to syllables?)
- get the children to extend acoustic number sequences such as "5, 15, 25, 35, . . ", "18, 28, 38, . . " (Class I);
- give number cards in which one of the two digits has been covered: "which number can it be?";
- arrive at the right place at the right time, as in Dutch songs like "Twee emmertjes water halen", or reach the other side of the classroom using 20 steps.

Accelerations, decelerations etc. are necessary as the counting sequence must be to hand not only as regards numerals, but also as regards variations in pace. After all, both insufficient and full mastery of the number sequence are possible causes of counting shifts

2. *From numeral to number*
Various means are at one's disposal for the accentuation of the final numeral as number.

- out loud, whispered, and then internal counting;
- working for a long time with small collections;
- braking the counting *before* the end of the collection is reached in order to interrupt the propelling counting;
- get the children to count objects both in rows and circles;
- use propelling counting to draw the children's attention to number (keep on counting although there is nothing left to count).

3 *Relinquishing counting*
- Teach the children to relinquish their "1, 2, 3, . . " notion of counting, i.e. they must be weaned from their idea that counting is only 1, 2, 3, . .
Teach *the sequence "10, 20, 30, . . "* in counting action games (e g. Hide and Seek) Counting such as "10, 20, 30, . . " has further important functions. For example it forms the acoustic instrument without which children either cannot or dare not *estimate*.
- Children can be stimulated to relinquish counting by means of *partly covered* collections (e.g players in a puppet theatre);
- or by material which aids counting and which is restricted (the so-called *material lack factor*).
- the use of *basic facts and previous problems* [van den Brink, 1981] which are in the middle of the pupils consciousness (e.g. 8 + 8 = 16 when solving 8 + 7 =).
- models of a context, such as the bus model, conjuring situations and other *box models* in which the quantities inside the boxes are subject to manipulation. [van den Brink, 1980 b]

The aim of the relinquishment is to attain a conception of a more structured number in these children

In Class II we see acoustic counting coming back, e.g. with arithmetical tables. The accentuation by tapping or movement of certain *products*, the variation in rhythms, reciting the tables backwards, the sing-song recital of tables. These are all acoustic means which foster structure in the counting sequence

Many teaching situations can be regarded as acoustic preparations for the counting of quantities and later for *structural cardinal numbers*. We have indicated the movement aspects of counting in particular and posited that the control of acoustic counting, in pace and braking is advantageous to children to have to make the transition *from numeral to number*.

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As soon as attention was fairly diverted to arithmetic for commercial purposes alone, such rational explanation as had been handed down from the writers of the sixteenth century began to disappear, and was finally extinct in the work of Cocker, or Hawkins, as I think I have shown reason for supposing it should be called. From this time began the finished school of teachers, whose pupils ask, when a question is given, what rule it is in, and run away, when they grow up, from any numerical statement, with the declaration that anything may be proved by figures—as it may, to them. Any thing may be unanswerably propounded, by means of figures, to those who cannot think upon number.

A. de Morgan

Arithmetic, at least in the Western world, was always based upon object teaching until about 1500, when the Hindu numerals came into general use. But in the enthusiasm of the first use of these symbols, the Christian schools threw away their abacus and their numerical counters, and launched into the use of the Hindu figures. And while they saw that the old-style objective work was unnecessary for calculation, which is true, they did not see that it was essential as a basis for the comprehension of number and for the development of the elementary tables of operation. Hence it came to pass that a praiseworthy revolution in arithmetic brought with it a blameworthy method of teaching.

D.E. Smith