A DIALOGUE ON THE DEEP ETHICS OF MATHEMATICS

PAUL ERNEST

Pat and Alex are two researchers who meet over coffee.

Pat: Hi, Alex. I hear you are interested in the ethics of mathematics. As a working mathematician I'm all in favour of promoting ethical applications of mathematics, to help build a better and fairer world.

Alex: Hi, Pat, that's good to hear! I work in philosophy, and my interest lies in addressing deep issues in an inter-disciplinary way. I'm especially interested in mathematics because it is both about the world we live in, and about its own abstract realm.

Pat: I hear that you have some strong views. What's your take on the ethics of mathematics?

Alex: My view is that mathematics in all its forms is fundamentally ethical. Ethical issues are deeply and inescapably inscribed within it.

Pat: As I said, I'm committed to ethical applications of mathematics. But I don't see how ethics arises, beyond applications. Surely mathematics is an objective science. How we choose to apply it can be for good or for ill, but isn't mathematics itself a neutral tool?

Alex: I want to analyse the ethics of mathematics in a deeper way. My position is that *all* human practices are ethical because they entail obligations and responsibilities. No activity or practice is intrinsically neutral. They are all expressions of our values. I like the way Barry Brummett puts it, "That which is created, whether intentionally or not, is always an ethical product with ethical implications for the kind of living and symbol using that makes the world thus or so" [1]. All our activities, practices and products are ethical, because we need to acknowledge that we are responsible for all that we do and make.

Pat: That's all very general. In what sense do you want to say that *mathematics* is intrinsically ethical?

Alex: I mean that mathematics is wholly and solely embedded and embodied in human practices, so it is shot through, or I might say, *entangled* with human interests, choices and values including ethical values. Mathematics is ethical because we have made it, unleashed it on the world, and we are wholly responsible for it.

Pat: This is not what I mean by ethics. I see it as more to do with the making of moral judgements about whether applications are beneficial or harmful, whether they are good or bad.

Alex: Let me step back for a moment and bring up our ideational outlooks. We all absorb belief systems and conceptual frameworks through our immersion in our culture. We are not just passive recipients of ideologies, for we consider, reflect on, and work out our own belief systems and conceptual frameworks. But a significant part of these belief systems remains dark, hidden and never considered or evaluated by us. They form the underlying basis for our thinking, questioning and reflection.

Pat: This might be interesting, but how is it related to mathematics and its ethics?

Alex: It might seem distant, but it does link with mathematics. Part of the unconscious framework with which we see the world, and through which we structure our thinking and activity, is mathematics. Looking around us we see reflections of shape and number. We see multiple cars on the road, multiple trees in the forest and we see this plurality as something susceptible to numeration and counting.

Pat: Okay, I'll follow your excursion of thought, and trust that it leads back to ethics. So, yes, I can see there are plenty of natural and manufactured things that have a multiplicity about them, that present themselves numerically.

Alex: Well, in trying to look deeper I would argue that our counting of sets of things is neither natural nor neutral. In my view, number is not given in nature; it is something we impose on it.

Pat: But surely when you are walking in the park and two dogs meet two other dogs you have four dogs in the park? $2 + 2 = 4$. Or as Martin Gardner said, even way back in history when two dinosaurs met two other dinosaurs, there were four dinosaurs there [2].

Alex: Let's unpick some of the assumptions behind this. First, you assume there is a natural class of dogs. You assume we can identify different members of this class, whether they be Schnauzers, Terriers, Labradors, Chihuahuas, or whatever, and label them all as dogs. You don't have to identify different breeds, but you need to be able to apply the abstract label 'dog' to them all. So you must have already developed a conceptual framework with the abstract class of 'dog'. More deeply, you are assuming that the members of this class can be taken as equivalent for the purposes of counting. Despite their different sizes and appearances, they can be regarded as the same for the purpose of counting.

Pat: But everyone perceives classes of things, even if some people make finer distinctions than others, and even babies distinguish between small numbers of objects. It comes with being human.

Alex: That's true in part. Most human languages have number words, although some are more limited than others. But as you acquire a spoken language you also acquire a worldview, a way of conceptually cutting up the world and its objects and events according to the nouns, adjectives and verbs. To see $2 + 2 = 4$ dogs in the park you must have an interest, a propensity, as well as the conceptual tools for interpreting your experience in this special way. We *make* the arithmetic sums we see in the world. Consider this. If you were overlooking a busy road intersection you could argue that an unlimited number of number truths are being exemplified before your very eyes, as the cars pass by. Not only might you see 1 + 1 = 2, 1 + 2 = 3, 2 + 3 = 5, 8 + 3 = 11, and so on, but also 3 – 2 = 1, 7 – 4 = 3, and even 2 × 3 = 6, 3 × 5 = 15, *et cetera.* Every potential focus of attention can create an instance of arithmetical truth. But isn't it absurd to say that virtually the whole of arithmetic is flashing before our eyes at every moment, faster than we can count?

Pat: So what you are saying is that number does not reside in the world, we impose it on the world. But surely the objects we count are there in the world, and so are their relationships. We don't construct reality through our language. I know that the Sapir-Whorf thesis claims that language shapes the world of our experience, but I reject it [3].

Alex: I agree that there is a material world not only around us but including ourselves. We are material bodies and beings in a material universe. Some things we experience seem to have boundaries like pebbles, rocks, trees. Some have a softness or a give to them like ferns, wool, the earth. Some are fluid and fleeting or even barely perceptible like water, wine and air. Whatever elements of our experience we group together to make a single object of our attention depends on our interests and intentions at the time, and our capabilities and interests have already been socially shaped. The fact that something has a material presence does not mean that its objecthood, its entity-ness, its unity as a self-subsistent object ready to be counted, is a given. As Tim Ingold argues, the world as given is not unambiguously parcelled up into entities [4]. Only when we have conferred objecthood on a sequence or grouping of newly recognised entities are they available for counting. The 'object' must have distinguishable boundaries or a central feature that we identify as marking it off from its surroundings. In short, we must maintain its identity and distinctness.

Pat: Well, there are two issues here. First, what you are describing is the *difficulty* of counting, not its nature, Second, I really can't see any relevance to ethics!

Alex: I am not just describing the difficulties of counting. I am spelling out the assumptions that must be made about the nature of the entities to be counted before counting can even begin. Counting is not a first act. It must always be secondary to our act of parcelling some part of the world into countables. The world of itself is not countable. We must make it so!

Pat: I'm not sure about this, and you haven't come to the ethical part yet!

Alex: True! The deep ethics I want to talk about is not about telling good from bad, or right from wrong. I want to argue that every encounter with other persons is ethical. Every conceptual or material technology is ethical because it affects the quality of peoples' lives. Teaching all our children that the world is made up of countables, and then teaching them how to count and perform arithmetic, is to impose on them a conceptual framework which will permanently shape their outlooks and activities. This is ethical, whether it be for the good or not. All educated and literate peoples have these deep conceptual foundations. They or rather we ineluctably see the world this way. We have imposed this on them and on ourselves. Making that choice, or maintaining it, is ethical.

Pat: As I said before, I'm not sure that this is what I mean by ethics. Ethics is surely about choosing a course of action and judging whether the outcomes are good or not? Furthermore, don't education and numeracy bring with them great gains? Surely the educated and literate peoples of this world benefit greatly?

Alex: I make a distinction between morality and ethics. Morality is about following the code of right behaviour that is accepted within a social context. It enables you to distinguish what is agreed to be right from wrong. Ethics is about the process of considering the good or human flourishing that follows on from actions, without necessarily having hard and fast rules for making judgements. A consequentialist ethics would not only consider how literacy, numeracy and education bring a better life, and what that means. It should also consider how they enable domination and empire. But I do accept that education in general is largely beneficial. Within our current social system it empowers people.

Pat: Well mathematics is and has always been a central plank in the school and college curriculum, so you are conceding that mathematics is generally beneficial. Overall, education is the foundation of the most complex and sophisticated societies.

Alex: In asserting that mathematics is generally beneficial to humankind you are conceding that it is, in fact, ethical. My claim is even stronger. Mathematics, like every other field of human activity and invention, is deeply ethical, irrespective of its social uses. Humankind is fully responsible for its brainchild, mathematics, both for how it makes us see the world, and for all the ways in which it is or might be employed.

Pat: I notice that you are defining ethics in terms of responsibility. You are claiming that every single human activity, field of knowledge, scientific discovery, and so on, is the responsibility of humanity. And you also deny the neutrality of mathematics itself as a body of knowledge and as a set of conceptual tools.

Alex: Yes, I am saying that deep ethics is about responsibility. Every choice is ethical. Whether you attend to the ethics of what you do, are about to do, or not, you make that choice. Making it is an ethical act. Whenever you participate in a social practice you must accept and assent to the implicit conventions of that practice, so you share responsibility for the outcomes and implications of that practice. Every human is an ethical agent, and you have a duty to question any practice you enter into with respect to the nature of its outcomes, side effects and collateral damage or benefits. Choosing to ignore the ethical consequences of a practice is itself an ethical act.

Pat: Before I return to mathematics let me just say that you are claiming that we individual humans are responsible for everything that we humans do collectively? Even if we are unaware or deceived about the outcomes and implications of our social practices?

Alex: Yes, I am. As a citizen you share responsibility for any wars your country wages. As a voter you are responsible for the political decisions your government makes, even if your preferred candidate did not win. As a consumer and shopper, you share responsibility for waste and the degradation of the environment. Buying or using anything that has been transported or used non-renewable energy in its production makes you complicit in global warming. As human beings, we share responsibility for all that humanity does. We cannot be ethically neutral, and we cannot be ethically pure and unsullied.

Pat: Well this is getting hugely political. I am here to debate the ethics of mathematics not to right the wrongs of the world. I accept your argument that if I, you, or we, are responsible for everything in the world around us, or at least those things that have been impacted on by humanity, then *a fortiori* we are responsible for the applications of mathematics, too. But suppose mathematics is independent of humanity. Suppose Plato was right, as well as Newton, Frege and Gödel. Suppose mathematics and mathematical objects are real and independent of our creation. Then we can only be responsible for their uses and applications and not for their nature and being. A Platonist mathematics is ethics and value-free.

Alex: You are right about this position. The problem is that Platonism and the associated ontological questions about the existence of mathematical objects can never be resolved. They can only be clarified. In this respect philosophy is like theology. I can neither prove nor disprove the existence of God, and all that she entails. For me mathematical objects are real but not independent of humankind.

Pat: So, you too accept the position of mathematical realism. But if mathematics is beyond any one mathematician, and beyond any finite group of mathematicians, surely it is then beyond all mathematicians and exists independently of humankind? And if mathematics exists independently of humankind, what sense does it make to say it is value laden or ethical in any way beyond the fact that it has proved to be useful for humankind?

Alex: Yes, I am a mathematical realist but not a Platonist. Like Reuben Hersh I believe that numbers and indeed all mathematical objects are real [5]. They have an independent existence, but are nevertheless cultural constructs. Every time we name or otherwise use a mathematical object, we are not just using it, we are maintaining and reinforcing its existence. Mathematical objects only exist insofar as we keep using them and maintaining practices in which they are used. Natural numbers are something humans have invented and whose existence and reality we maintain through using them.

Pat: You claim we invented the natural numbers? Invention implies choices and varieties and yet we have no choice but to take the unique natural numbers that nature gives us.

Alex: Here I disagree. The question for me is not why do we invent the natural numbers as they are, and in the only way that now seems possible for them to be? Rather, for me the question is what interests and purposes lie behind the invention of the natural numbers? What social constraints imposed within historical practices shaped the natural numbers and gave them the form that we now regard as essential and inevitable?

Pat: Surely the natural numbers are just for counting things, and then later doing arithmetic?

Alex: Yes, but why do we count things?

Pat: We count to know the number. The early humans came out of their cave or shelter and counted their cows or sheep so that they would know they hadn't lost any.

Alex: Well that's an amusing tale of origins. In my view it is false, although it does contain one important idea, that of invariance. Arithmetic, including counting, is a complex conceptual technology that was not invented so that individuals could keep stock. Farmers and herders usually know their individual animals by sight. Counting and recording the count was invented around five millennia ago in Mesopotamia for the purposes of kingly tribute, tax and for trade. The function was social communication, not individual knowledge.

Pat: Okay, but I'll be interested to hear how social needs give rise to the necessity and uniqueness of the natural numbers, in your account.

Alex: Well, I have said that it was the practices of tribute, tax and trade that needed numeration. These activities go on at two levels, the material and the symbolic. First, the tax or trade involves goods, that is, items of value. These can be sheep, foodstuffs or artefacts such as weapons or jewellery. There is a supply chain between the initial producers of the goods, via transporters, and then on to buyers, acquirers or tax officers, followed by further transporters, delivering the goods to the ultimate owners to use or store. In trade these supply chains are even more complex with a market exchange in the middle where a rate is applied or negotiated. No individuals need accompany the goods all the way from production to consumption. But during the long and complex supply chain and exchange the goods are accompanied by documentation recording the nature and quantity of the items involved [6]. The Mesopotamians used clay envelopes containing tokens to represent the traded or taxed items, which then evolved into inscriptions on clay tablets, according to Schmandt-Besserat [7]. This is where written number symbols come in, to represent the quantities of goods involved.

Pat: Yes, so numbers are used to represent the goods. This is obvious. How does that help your account?

Alex: Don't you see, first comes the goods, whose quantity and materiality must be preserved to satisfy all involved. Number is used to record the quantities, and to serve its purposes it must conserve quantity under material and symbolic actions. Neither traders nor kings want three baskets of grain to become two. They need a symbolic apparatus that preserves quantity. That is, it leaves measures invariant under operations like transporting, recounting, partitioning and recombining. That way the record keeping is not only an accurate description of the goods involved but it also legislates what the quantities must be after the lapse of time or the reordering of the goods. It is these human requirements that impose an ordered and invariant structure on numbers and counting, and that provides the basic format of arithmetic. To record and validate trade the numbers and measures must be determinate, invariant, repeatable and unique.

Pat: But isn't this a backward justification. We know that the natural numbers have these structures and invariants and you have found an analogy in trade which you claim forces this structure on us. But it could not have been otherwise.

Alex: No, I disagree, I have shown how human interests and purposes forced a specific structure on numeration and arithmetic. It is conceivable that other scenarios might exist where a different arithmetic could have emerged. Here's one off the top of my head. Suppose a South Sea Island society is based on yams, and there is a normal rate of growth for yams. So five January yams weighs less than five March yams. If you loan ten January yams and expect repayment with one less yam per month to accommodate growth you would expect eight yams back in March. So ten yams is not invariant, it is time dependent in value. This is a non-standard arithmetic representing social needs and interests. Levi-Strauss has shown how social relations, especially kinship relations have resulted in invariant conceptual structures among the peoples he studied [8]. These differ in structure and purpose from standard arithmetic.

Pat: But I can easily represent this example by representing yams as functions. If Y(m) is the weight of yam according to the month (m), then 8Y(March) = 10Y(January), and invariance is restored. *Quod Eatum Demonstrandum!*

Alex: Very witty! We haven't really got to arithmetical operations as yet in our discussion, such as within Peano arithmetic, in which 1 + 1 = 2. Of course, there are alternatives serving different practical purposes such as Boolean algebra (1 + 1 = 0) or Binary arithmetic (1 + 1 = 10, employing '10' as binary for two), and even 2 + 2 = 1, in modulo 3 arithmetic.

Pat: Well, these examples don't work in showing the contingency of counting, because Boolean algebra and modulo arithmetic don't embody the total ordering we require of the natural numbers. Binary arithmetic of course does, but just with a different numeral system.

Alex: We know little about prehistoric oral arithmetic, but it is plausible that this was developed alongside myths and woven into early religion and the observations of the heavens to predict star movements and seasons. Through such counting, planting and hunting seasons could be regulated as well as religious observances for astronomically significant days. Although there are no direct records of proto-arithmetic we have repeated marks and shapes in cave drawings and notched artefacts like the Ishango bone [9]. It seems likely that counting followed a modular pattern, as repeated events including days and nights, lunar months, significant days of the calendar, seasons and years, all follow cyclic patterns. So it is possible that modular arithmetic precedes the invariant classical arithmetic that you take as fundamental.

Pat: What you describe is just a limited part, the counting part, that comes with finite arithmetic or rather, numeration. Indeed you make my case that standard counting is needed to fix these determinate time cycles and heavenly movements of stars, however it might be recorded. Your oral numerals must form a well-ordered sequence to perform this role.

Alex: You haven't disproved my claim yet, because I am arguing that the total ordering of numbers is necessary and sufficient for formulating the invariant arithmetic that trade and taxes require. The corollary is that if our interest was not in preserving the underlying invariant quantities of the goods counted, we could have a different arithmetic.

Pat: Even then, I claim that any new arithmetic you can come up with, assuming you can construct one that works, whatever the motivation for its quirky non-standard structure, can be translated into standard arithmetic, as with your yam example.

Alex: Well that's because modern mathematics is a marvellously abstract and highly generalised set of theories that can accommodate almost every imaginable structure. Where new structures emerge as exceptions to current rules and constraints we take them and adopt them as new theories with different rules alongside established theories. But none of this contradicts my fundamental claim, which is that we defined arithmetic the way it is so that it would serve the purposes of tribute, taxes and trade, allowing material resources to be regulated at a distance so that they are quantitatively invariant. Hence human interests and values underpin the formulation of arithmetic. Since we have chosen to construct it in this way, admittedly for good reasons, arithmetic is the outcome of an ethical process. There is a deep ethics underlying arithmetic. Saying this is just an acknowledgement that, like everything else invented by humankind, we are all responsible for the resulting symbolic technology of number and its presence throughout our lives. Arithmetic, and more generally mathematics, are not neutral and value free. Arithmetic has been constructed to serve and express certain human interests, most notably the control of resources, humanity and the world. Such purposes are inescapably ethical.

Pat: I'm still not satisfied that we could have invented and defined arithmetic and indeed mathematics differently. Surely there is only one mathematics, admittedly with new theories added on and incorporated as they are developed?

Alex: Well, how would I identity a different mathematics? Would it have to have different concepts, different axioms, different rules of proof, different theorems, all combining to give rise to a different body of knowledge?

Pat: Oh yes, something with these features sounds like a completely different mathematics. And no, these are not impossible demands, if you truly think an alternative mathematics is possible.

Alex: Well, then I give you Intuitionist mathematics [10]. This differs from Classical mathematics in its concepts, axioms, rules of logic and proof, theorems, and gives rise to a different body of knowledge. According to Intuitionism, the laws of the excluded middle and double negation do not hold, the continuum is countable, and there are results like the Fan Theorem and the Bar Theorem unique to Intuitionist mathematics with no counterpart in classical mathematics.

Pat: Well I'll grant you that Intuitionist mathematics differs from Classical mathematics. But we can translate their every relationship, rule, procedure and theorem into classical mathematics in a truth preserving way.

Alex: Yes, technically you can subsume all of Intuitionist mathematics into Classical mathematics by some translation like a sentence 'S' is converted into 's is a proof of S', but in so doing you preserve truth at the cost of meaning. You do not maintain the concepts and their meanings, only the eviscerated skeletons of their formal relationships. You subvert a theory designed to avoid completed infinities by subsuming it into theory that admits them. So, your translated Intuitionist theory is just a poor and contrived shadow of the original.

Pat: But once again this shows that mathematics is unique, and *a fortiori,* arithmetic is unique.

Alex: But listen. I can translate every known language into English, admittedly with some distortions of meaning. Just like we can translate Intuitionism into Classical mathematics. But this doesn't mean that English is the one and only fundamental and universal language. We could just as well translate everything into French or Farsi.

Pat: But Classical mathematics is unique. It is the one and only fundamental and universal language. And because it is unique, our choices play no part in its foundations. We are not responsible for how it is, just how we apply it. Ethics cannot enter into mathematics, only its applications.

Alex: What you are stating is a belief commitment, an ideology. When I say there is a deep ethics underpinning arithmetic because we have chosen arithmetic as our framework for numerating our world you reject this. To you there is no choice, for it is the numeration system inscribed in the very nature of existence, part of the very essence of being. We have no choice as whether we want to adopt it. We can be blind to it, or even turn our back on it, but if we want to start enumerating the world there is only one way to do it.

Pat: Yes, I believe there is only one fundamental and underlying arithmetical system, and I believe that exceptions and alternatives are not possible except in contrived ways that fall away when probed. Putting non-standard arithmetics to one side as different kinds of beasts, there is one and only one true arithmetic, and Peano is its name! While I can point to historical episodes when many if not all mathematical concepts, signs, methods, problems and theories were invented, I don't think these were ever arbitrary inventions. We can choose to invent unlimited new theories, with their own concepts, and so on. But they must be consistent with the underlying truths and logic of mathematics, those of Classical mathematics. Or if not consistent (such as paraconsistent logics) then they must be kept in their own sealed theory-space, away from mainstream mathematics. I suppose I agree with Kronecker that God made the integers, all else is the work of humankind [11].

Alex: Well given this commitment it is clear that I cannot persuade you of my point of view. Just to make it clear, my central claims are the following, where the word 'we' stands for us humans.

We choose to parcel up selected aspects of the world we experience into separate but equivalent units, thus making what we perceive as 'countables'. These must remain unchanged during counting, conceptually if not materially.

We create a counting procedure whereby we can enumerate any designated collection to give a specific and unique number count.

We develop a numeral system that records counts, both encoding the number and preserving it over time.

We extend and develop this counting procedure into a number system that treats abstract quantities (numbers) and has a superstructure of operations, definitions, and procedures that can be extended if and only if they preserve number (count value).

We end up with an arithmetic theory that enables transformations of compound numeral terms if and only if value is preserved, that is, there is number invariance. In consequence number operations are, and must be, replicable, commutative, reversible, associative, and distributive.

My point is that we collectively make choices about how to see the world, how to count, and how to record number. Only after we have made these choices does arithmetic became necessary and unique.

Pat: I think this is misleading. No one ever 'chooses' how they see the world. It presents itself to us through our senses. Likewise, we do not 'choose' our counting system or arithmetic, we simply learn arithmetic.

Alex: Well you are largely right. As individuals we are not fully free to choose how to see the world. As individuals we do not choose to make our counting value invariant. Nor do we choose how to record number, although in psychological studies by Martin Hughes, young children have invented their own simple numeral systems [12].

Pat: But just as no individual ever 'chooses' how they see the world, no social group has ever 'chosen' their counting system or chosen not to have one.

Alex: Yes, you are right. Generally children do not actively choose to study arithmetic, although many throw themselves into it with great enthusiasm. We induct them into number early on with counting and number games and activities. I am not saying this is bad, but we should acknowledge that it is an ethical decision to induct children into this outlook, and it has a deep impact. We can never know what outlooks children would develop or what alternatives we could offer them if we did not engage in this indoctrination. We can see the analogous impacts of the imposition of minute measures of time into people's lives. All the impatience and anxiety about lateness, all the ennui of clock-watching are modern impositions due to the universalisation of strict timekeeping.

Pat: Well I think that teaching arithmetic at home and at school, and regulating time everywhere in the world bring vast benefits at every level, the home, street, town, region, nation and the world. Modern travel would be impossible without timetables.

Alex: I won't challenge you except to point out that what you are saying is that the imposition of a numerical outlook and standard time keeping are very useful and have valuable benefits. This is a justification through consequentionalist ethics, like utilitarianism, so you are making my point that there an underlying ethics behind their imposition.

Let me tell you a story. Kathy Crawford was teaching a numeracy programme to a group of Pitjantjatjara students in the outback of Australia in the 1970s [13]. She kept being asked "Why do you need to compare things all the time and need to know how many there are of everything?" They then explained that comparisons, especially about people, were highly censured in their community. One Anangu woman said, "If we study these things we won't be Anangu anymore". The mathematical concepts and practices were incompatible with their community values.

Pat: Well, of course it is possible to turn one's back on modern concepts or technologies, as the Amish communities do. But this means rejecting the link with modern government, lifestyle and all of benefits they bring like reducing the labour of farming, and providing healthcare and educational opportunity, and enabling travel and communication technologies.

Alex: But don't you see; you are acknowledging that one can choose not to follow the path of arithmetic for ethical reasons. Just as the vegan choice not to use animal products is ethical. So if you follow the normal practices of studying arithmetic, or letting your children study it, you have made a choice, a decision. This is true even if it is an implicit choice made by just going along with the crowd and received expectations. By this I mean you are responsible for your action, not that it is necessarily good.

Pat: I still maintain that your personal or community choices do not affect the nature and being of mathematics. Love it, hate it, or ignore it, mathematics is unmoved by your actions. Your choices are subjective, and possibly ethical. But neutral mathematics is impervious to your choices. I don't see that these examples link with the ethics of mathematics.

Alex: Well I've been talking about deep ethics, which is primarily about responsibility. Since we chose the foundations of arithmetic, and I would argue the same for geometry and algebra, we are responsible for what it was, and for what it has become. It is an abstract technology that permeates all our lives and channels our thinking. I reject the widespread view that ethics does not enter mathematics itself in any way. But I understand that this view is legitimate, and acknowledge that it is socially and philosophically valid to hold it, without conceding that it is correct.

Pat: Well you are right to say this. Debate hinges on respecting opponents and their opinions. But I am still far from convinced about this so-called deep ethics of yours, let alone that it applies to mathematics. And to throw your Anangu quote back at you, "If we study these things we won't be mathematicians anymore".

Alex: I disagree. A new generation of mathematicians is emerging that see ethics as integral to mathematics. Even Plato, father of Platonism, saw the connection between ethics and mathematics, arguing that studying mathematics teaches you the fundamental ideas of ethics. But let me end by thanking you for a pleasant and respectful debate, even if we see things differently, for now. Let's talk again.

Acknowledgements
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Notes
[1] See Brummett (2015), p. 383.
[2] See Gardner (1981).
[3] See Whorf (1956).
[4] See Ingold (2012).
[5] See Hersh (1997).
[6] The modern equivalents are Blockchain technologies which record and validate virtual currencies, contracts, titled ownership, and every form of human transactions. These raise significant epistemological and ontological issues just as mathematics does.
[7] See Schmandt-Besserat (1997) and pages 24 & 41 in this issue.
[8] See Levi-Strauss (1969).
[9] The Ishango bone from around 20,000 BCE, has several groups of notches, possibly with mathematical or calendrical significance. See page 8 in this issue and https://en.wikipedia.org/wiki/Ishango_bone.
[10] See Heyting (1956).
[11] See https://en.wikipedia.org/wiki/Leopold_Kronecker.
[12] See Hughes (1986)
[13] See Crawford (1996), p. 135.

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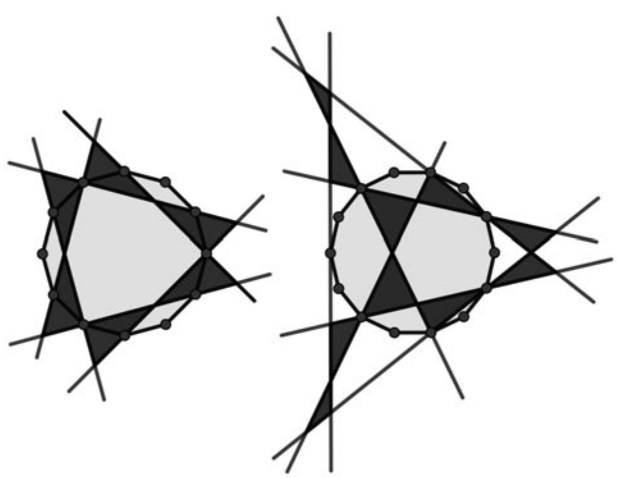

Figure 2, see article on facing page. The configuration by Füredi and Palásti for n = 6 and n = 7 lines. It consists of an adequate choice of the lines containing the diagonals of a regular polygon of 2n sides. These are not optimal, since K(6)=7, K(7)=11.