The Ideology of Certainty in Mathematics Education

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Mathematical results and statistical figures are constantly referred to during the ongoing debates in society They form part of the structure of argumentation. In this way, mathematics is used to frame the political debate. But not only this. It becomes part of the language in which political, technological and administrative suggestions are presented. Mathematics becomes part of the language of power. In this article, we present one aspect which makes mathematics the final word in many discussions. The power attributed to mathematics to comprise the definitive argument is supported by what we will call an 'ideology of certainty' (Borba, 1992)

Many authors have argued that mathematics has political dimensions Mellin-Olsen (1987) and Volmink (1989) argue that students who do not learn mathematics will be disadvantaged, since they will not be able to deal with the complexity of present society. Frankenstein (1989) points out that misuse of mathematical information leads to racist, sexist and socio-economic discrimination in society, and proposes using mathematical problems embedded in social situations as a way of empowering students with mathematical tools that will enable them to have a critical approach to the world. And Frankenstein and Powell (1994) argue for a society in which minority and female students will not be mathematically disempowered by the social, political and economical filters of the U.S.A. which screen them out because of their mathematical skills. These authors emphasise the (mis-)use of mathematics and the effects on those who do not have access to mathematics education.

It might be reasonable to assume that these disempowered students will be able to become more critical actors in society if they have access to mathematics. But, on the other hand, gaining access to mathematics education without being critical of the ideology of certainty may reinforce the *status quo*

1. The nature of the ideology

In general, we conceive of an ideology as a system of beliefs which tends to hide, or disguise, or filter a range of questions connected to a problematic situation for social groups. An ideology might fudge or soften this situation in the sense of obstructing possibilities for identifying and discussing the nature of the 'crisis' of this situation. To struggle to make explicit this ideology represents a critical attitude towards this situation and the ideology that covers it up.

We see the ideology of certainty as a general and fundamental frame of interpretation for an increasing number of issues which transform mathematics in a 'language of power'. This view of mathematics - as a perfect system, as pure, as an infallible tool if well used - contributes to political control.

Such a view of mathematics has its basis neither in the debate about the foundations of mathematics (Snapper, 1979), nor in the discussion about the social and cultural aspects of mathematics (Borba, 1987; D'Ambrosio, 1994). However, this is the view that is used by television programmes about science, by newspapers and by schools and universities In these settings, mathematics is often portrayed as a stable and unquestionable instrument/structure in a very unstable world. Phrases such as 'it was mathematically proved', 'the numbers express the truth', 'the numbers speak for themselves', 'the equations show/assure that ...' are frequently used in the media and in schools. These phrases seem to express a view of mathematics as an 'above-all' referee, as a 'judge', one that is above humans, as a non-human device that can control human imperfection. [1]

We find it necessary to struggle against this myth, if our ethical goals are to construct a pedagogy that fights oppression in society, since this view of mathematics corroborates the notion that mathematics is free of human influence and above humans. In schools in particular, this belief is expressed in a special sense. The usual mathematics curriculum adopted deals with problems with one and only one correct solution, a fact that reinforces the idea that mathematics is free of human bias.

A mathematical problem can be contextualised, but the 'contextualising' might introduce an artificial world. Say a word problem has to do with shopping: 'What is the price of the food needed to follow a given recipe (for four persons), when nine persons are expected for the party?' As usual, the necessary information is given, and no other information The students can then work out this problem as an exercise in proportionality

Imagine, however, a comment like this: "I know a slightly different recipe, and if we use some extra carrots, we do not need so much of this; I think it might even taste better. In fact, I like carrots ...." Normally, such a way of thinking does not lead anywhere, and certainly not to the correct answer. The context of the mathematics classroom shapes one's experience in a far different way than other situations do (Lave, 1988); the recipe mentioned in the mathematical exercise is not the recipe from the kitchen We have to deal with pseudo-problems, with a world where the true-false paradigm dominates

The ideology of certainty is hidden: it is implicitly connected to powerful mathematical tools. This discourse about mathematics promotes its overwhelming power in applications, but it does not tell much about the assumptions that have to be made in order to use mathematics The basis of the ideology which underlies this discourse can be summarised by the following ideas.

> (1) Mathematics is perfect, pure and general in the sense that the truth of a mathematical statement does not rely on any empirical investigation. The truth of mathematics cannot be influenced by any social, political or ideological interest.
>
> (2) Mathematics is relevant and reliable because it can be applied to all sorts of real problems The applications of mathematics have no limit, since it is always possible to mathematise a problem.

The first statement deals with the purity and generality of mathematics; the second with the endless applications of it. The ideology of certainty wraps these two statements together and concludes that mathematics can be applied everywhere and that its results are necessarily better than ones achieved without mathematics. An argument based on mathematics is therefore reliable in solving real problems

This appears logical on the surface Believing in applications of mathematics is not a problem in itself; the problem arises when one believes that by applying 'a perfect body of knowledge' to a problem one will have '*the* solution'. The problem gets more complicated if one thinks that the application of mathematics (or any other body of knowledge) is neutral and does not help to format both problem and solution, as we will discuss later in this article

By assuming (1) and (2), a whole range of crucial questions concerning the reliability and consequences of applications of mathematics becomes hidden. Therefore, we find that reliance on (1) and (2) constitutes an ideology. We find that humans always have to use judgement when using mathematics Mathematics can be applied to problems only if they are 'cut' in an appropriate way to fit mathematics, and mathematics is 'perfect' only when we construct a context adequate enough for this purpose.

An application of mathematics involves an 'interaction' between mathematics, seen as a sort of formal device, and a context. By means of mathematics we can talk about a 'piece of reality'; we can use mathematics as a basis for a decision; we can refer to mathematics when we, as technicians, are involved in a process of technological construction, etc. However, as part of the simplified problem-solving process, we are placed in a magical world where the grammar of mathematics fits the Platonic world which we are talking about Here we have the necessary information; we calculate; and the calculation becomes either right or wrong.

The ideology of certainty is spread thoughout society, and not only by means of sentences such as 'the numbers *show* ...', which appear every day on the news. In schools, the fantasy about the super-powers of the application of mathematics can become even stronger, since most problems students deal with there are designed to have mathematics fitting nicely into them

When applications of mathematics fit the true-false paradigm, this may reinforce (and students may learn more about) the belief that applying mathematics is 'the best way' to proceed. They learn how to handle word problems in a school context But, simultaneously, they are introduced to a belief which is dangerous. This belief gives the impression that questions and difficulties in real applications of mathematics are 'similar' to the questions and difficulties they face when dealing with word problems. This belief eliminates a whole range of problems related to applications of mathematics outside schools. Therefore, once again, we talk about an ideology, and because it refers to the true-false paradigm, we call it the ideology of certainty.

Mathematics educators with a critical perspective should try to teach mathematics in a way which shows:

(a) that this 'body of knowledge' is just one among others;
(b) the short-cuts made in the process of mathematising.

Students should therefore be talked out of ideas such as: a mathematical argument is the end of the story; a mathematical argument is superior by its very nature; or arguments such as 'the numbers said such and such' We think that mathematics could become simply one possible way of looking at phenomena, and not *the* way

2. The ideology as part of a general interpretation of technology

The ideology of certainty becomes paramount for our society as the discussion between mathematics and technology becomes central to the on-going political debate (Keitel, Kotzmann and Skovsmose, 1993). Technology cannot be seen as a simple tool by means of which humanity tries to 'survive' in its struggle with nature Such an interpretation has been most common in the philosophy of technology, but this does not make sense any longer Technology has become a basic condition for human life. Our living conditions are located in technology and by means of technology Therefore, it makes little sense to talk about 'nature' as an autonomous category. Humanity is embedded in a 'techno-nature' Technology in techno-nature has become a double-edged sword. It is a main source for solving problems, but also a cause of problems (D'Ambrosio, 1994; Skovsmose, 1994).

This paradox of technology is also a challenge to mathematics education Education has developed as a democratic concern [2] and mathematics education can support this tendency as well (Skovsmose, 1990, 1992) However, technological development does reveal undemocratic features, as, for example, when it causes critical situations such as ecological catastrophes. These consequences of technological development also have to be dealt with as part of mathematics education (which, on the other hand, also constitutes a necessary condition for the technological development itself).

That mathematics is playing a vital role in bringing about this techno-nature is, perhaps, a rather new phenomenon. Definitely, it is a phenomenon that has only been recently realised The role of mathematics in the technological society can be stated as mathematics has a *formatting power* (Skovsmose, 1994). This refers to the fact that parts of our world are organised according to mathematics. Mathematics can be used in a prescriptive way, and in this way it becomes a principle for (technological) design (see Davis and Hersh, 1988)

By drawing attention to the formatting power of mathematics, a different discussion of applications of mathematics is introduced Applications of mathematics have often been conceived from the perspective of description. 'Reality' is 'given', so to say, and a mathematical model of this 'reality' can be compared with a map which can be more or less accurate A key concept in evaluating a mathematical model then becomes 'approximation'.

This, however, only applies to a certain (and limited) domain of mathematical models By means of mathematical models, we also become able to 'design' a part of what becomes reality. We make decisions based on mathematical models and, in this way, mathematics shapes reality; and therefore we cannot restrict the discussion of mathematical models to 'approximations'. More fundamental questions must be raised If mathematics, in a certain modelling context, exercises a formatting power, then we must ask: 'What is done by means of this modelling?' Which social and technological actions are accomplished? What are the social, political and environmental implications of these actions?

The fact that we are situated in a techno-nature implies that the general attitude towards this techno-nature becomes important How do we establish general conditions for evaluating this techno-nature? Do we perceive this background as a 'fact'? The ideology of certainty supports the assumption that a technical problem has one 'optimal' solution.

Therefore, this ideology may provide a 'block' to a serious discussion of the applications of mathematics The ideology of certainty can limit the range of questions which can be asked about the applications of mathematics. If mathematics can be applied 'everywhere', then the fundamental question concerns how accurately the mathematical model in fact 'fits' reality. This question is not fundamental to the discussion of the applications of mathematics – only the ideology of certainty establishes it as paramount.

When mathematics becomes a part of technology, an attitude towards mathematics can easily be transferred to technology in general. The ideology of certainty supports the assumption of the existence of an optimal solution As a consequence, a suggested technological solution can only be questioned by (other) technological means. Identifying the 'optimal' solution becomes a technological task. Optimal solutions as a rule can therefore become a fundamental part of what we are calling the ideology of certainty

3. How does the ideology emerge?

While many sources outside the classroom do exist, here we take a look at classroom sources for this ideology. One is related to the structure of communication and especially to the way 'mistakes' are treated in the classroom.

Many observations show that teachers, in their communication with students, focus either on the algorithmic procedure or on the results of the students' investigations (Alrø and Skovsmose, 1996). The 'result' of the students' mathematical activity becomes the focus of the corrections, and not what the students had in mind when they made their calculations.

The teacher, the textbook and the answer book make up a united authority which hides the background of the correction. It becomes unnecessary for the teacher to specify the authority that is behind different types of corrections The students are not met with argumentation but with references to a seemingly uniform and consistent authority, even though the basis for the corrections might be very different Some rest upon mathematical features, while others rest upon practical matters of organising the educational process All mistakes, however, are treated as absolute, irrespective of what they have to do with the results or with the fact that the students may have written an algorithm down incorrectly.

By making corrections in an absolute form, the teacher influences the world-view of the students. The language – into which the students are pushed by the corrections of the teacher – fits the world-view of the true–false paradigm and can help generate an absolutist view of mathematics

The ideology of certainty is confirmed and re-confirmed and, certainly, this ideology is functioning in school. When tests and examinations play a crucial role, then the results must be correct. It is exactly the correctness of the results which is tested. To focus on a different aspect would mean that the teacher breaks the 'contract' with the students. Therefore, the ideology of certainty applies to the tasks which face the students as part of their preparation for upcoming tests.

One question might arise: since communication between teacher and students and the corrections made by the teacher are two sources of the ideology of certainty, should we then blame the teachers for the emergence of this ideology? Our answer is: "No" Teachers are part of a network which contributes to the spread of the ideology: it also includes parents, business, funding agencies, professors, and so on. In addition to this, teachers themselves were often educated by mathematicians who are not, in general, interested in educational or philosophical issues about uncertainty in mathematics.

4. Can the ideology be challenged?

From these remarks, it could be concluded that if we give courses on communication issues to teachers, or have teachers learn new philosophical approaches to mathematics [3] or change the structures of curricula to incorporate project work, or emphasise the need of students to choose their own problems as the basis for modelling situations, then we could challenge this ideology of certainty We do believe these may be ways to challenge the ideology but, in this article, we propose another way of approaching the problem

We want to propose a topic which may help to break the style of communication and the elements of absolutism which we find are sources of the ideology of certainty We do not, however, propose this example as a formula for how to challenge this ideology. The topic serves as an example of a variety of topics, which we shall try to characterise, all of which can be considered when the ideology of certainty is going to be challenged.

In *voting theory*, mathematics is clearly related to social issues and there is no optimal solution for problems. As reported in Garfunkel (1991), mathematicians have known for over 40 years that there is no perfect voting system. A lawyer generated parts of voting theory as a response to a problem posed by the United States' Supreme Court (Banzhaf, 1965, 1966, 1968a, 1968b), and part of his findings have been formalised by mathematicians (e.g. Lucas, 1974). Among other problems, Banzhaf tackled the issue of having the apportionment of voters in different districts be consonant with the notion that every person's vote should weigh the same at the moment that a decision is to be taken by a body of elected representatives.

Voting theory can be used to analyse the distribution of seats in, say, the Brazilian Congress. Many politicians all over the world, including Luis Inácio 'Lula' da Silva, the president of the Workers Party in Brazil from 1980 to 1995, have complained about unfair division of seats among the states. Lula has complained that small states from Northeast Brazil have more seats than they should have when compared with the densely-populated states of the Southeast. Although Lula, and many others from the left, may have a point in complaining, since historically the dictatorship in Brazil (1964–1984) increased the numbers of seats from the Northeast as a means of keeping its political control, he might have been trapped by the ideology of certainty.

Basic ideas in voting theory can be brought to the classroom with no need for lengthy introductory work, pre-requisites or fancy mathematical notations. This has been tried successfully in an engineering class at Cornell University, U.S.A. (Borba, 1992). When he presented a table with the population of each Brazilian state in one column and the number of representatives per state in another to engineering students, and asked them whether the distribution of seats was fair or not, they reacted similarly to Lula: they used 'proportional reasoning' to justify the answer 'no', because the number of seats per state was *not* proportional to the population of each state.

In an attempt to challenge the assumptions of this mathematically pure solution, a simpler version of this problem, inspired by both Banzhaf's work and Garfunkel's (1991) book, was presented to the students.

> Districts A, B and C have, respectively, populations of 50,000, 20,000 and 10,000. Each elects one representative with the same power for the state house of representatives of Saint Lawrence. Many people from different districts have complained about this situation; they think it is not fair. How many representatives should each district have, if the total number of representatives should be kept to a minimum?

In having these students, other students or professors solve such an 'easy' problem, we found an overwhelming compulsion to use simple proportion. The reasoning behind the answer would be: if the populations of the districts have different sizes, it is possible to correct the above situation easily by assigning proportional numbers of seats to each district. Therefore, district A should have five representatives, B two, and C one. One 'equivalent' solution was also offered by those who were more familiar with congressional politics: they argued that just one representative should be elected by each district ("it saves taxes"), but each representative's vote should have different weights. According to this reasoning, representatives of district A, B and C would have, respectively, five, two and one vote(s).

The inclination to solve the problem in one of these two equivalent ways is understandable, and we experienced the same compulsion ourselves when thinking about the "Brazilian issue", especially because of the politics involved in it (which are beyond the scope of this article).

Based on a reading of Banzhaf's work, however, we think that there is a questionable assumption in the previous reasoning. It is taken for granted that fairness is to have the same ratio of representatives per inhabitant. Banzhaf's work, however, suggests that fairness should instead be related to the 'power' each representative has in the legislative house, which is not necessarily the same.

Let us analyse the solution that considers weighted votes: a representative from district A has a vote which weighs 5; B's weighs 3; and C's weighs 1. According to Banzhaf's reasoning, or to what later became the Banzhaf power index, the ratio of power between the representatives of, say, district A and C is not necessarily the same as the ratio of representatives. In this case, the ratio of representatives is 5:1, while the ratio of power is 8:0, since no matter what the representative of district C votes for, the vote of district A's representative will decide the vote. The power ratio of A to C is defined as "the number of voting combinations in which A casts a decisive vote" compared with "the number of voting combinations in which C casts a decisive vote". As the number of parties in this case is 3, and the decisions always follow the decision of A, the number of voting combinations in which A casts the decisive vote is $2^3 = 8$. Therefore, the power ratio of A to C is 8:0. And even though B has three times the votes of C, the power ratio of A to B is 8:0 as well. A is called a *dictator* and B and C are called *dummies* in a legislature composed of the three of them in which simple majority is required to pass a law [4].

But what does this example tell us? Our interpretation is that the compulsion toward the 'proportional' solution is caused by the perception of objectivity of arithmetic that many of us have. Through assuming that arithmetic is objective, it becomes a decontextualised tool, that is one apparently built on no assumptions, one which can be detached into a context vacuum and therefore can be used everywhere as a neutral instrument.

However, there are many different solutions to the problem of representativeness of the Brazilian Congress, or to the simplified version of it in the Saint Lawrence example, or in the reapportionment issue that Banzhaf helped the United States' Supreme Court to deal with. There is, therefore, no objective solution to such problems, "even if mathematics is applied". Mathematics in this case can be used to build arguments in the same way that sociology can.

In both cases, neither mathematics nor sociology is assured of the 'final word' There is no final decision which can be given 'by the numbers' or 'by the historical facts'.

It must be noted that the use of voting theory in the classroom is not immune to the temptation to turn mathematics into an objective body of knowledge, even though it seems apparent that voting theory is not 'pure'. Arithmetic has helped to shape phenomena in American society and has become 'objective', as discussed in a historical fashion by Cohen (1982) The same can happen to voting theory In this way, the use of voting theory in the classroom can be recaptured by the ideology of certainty.

For example, Banzhaf's ideas in particular were developed to solve a problem proposed by the Supreme Court of the U.S.A. Later they were formalised and Banzhaf's index was created. It is not difficult to imagine that a formalistic caricature of the whole approach could be developed. Voting theory can certainly be taught in a form which contains all the rhetoric of classical mathematics teaching: definitions, maybe proofs; more definitions, maybe more proofs; to be followed by exercises, naturally having one and only one correct answer. To pay attention to voting theory does not guarantee that the ideology of certainty is overthrown

In 'decontextualising' mathematical ideas, the 'magic' takes place: human beings and the world disappear, and 'academic mathematics' is created. If we understand context in the way Lave (1988) suggests, as a synthesis of a dialectic between the activity of the knower and the arena in which the knower is acting, it can be argued that this so-called 'decontextualisation' is also made by human beings, in a given setting, within socio-historical forces. In other words, this 'decontextualisation' is contextually bounded (Borba 1990).

Moreover, the general knowledge produced is powerful only when different circumstances are fulfilled (Borba, 1990, 1994; Lave and Wenger, 1991). It should be part of a critical mathematics curriculum to show the powers and limits of mathematics and the discussion of the political dimension emphasised in this article: the ideology of certainty.

The ideology of certainty may be challenged by an open-ended approach to voting theory. Voting theory, for us, illustrates how important it is to work with ambiguity. But there is no claim in this article that students who work with a curriculum based on uncertainty will not use mathematics for fabricating disastrous technologies. There is, however, a claim that students who work within this approach will be more likely to see themselves as actors in the process of mathematical construction and less likely to see:

> mathematics as omnipresent (context neutral), omniscient (the final truth), and omnipotent (it works everywhere).
>
> (Borba, 1992, p. 333)

5. Different landscapes of discussion

Classroom structures of communication and absolutist elements in the working philosophy of mathematics may produce an ideology of certainty. These absolutist elements need not be simply analysed as features of the mathematics teacher's conception of mathematics. They might instead be thought of as part of a broader picture which includes exercises given to the students, tests, exams, national exams, unquestionably correct answers, etc.

One conclusion seems to be: in order to challenge the ideology of certainty, one has to challenge the sources of the ideology. Therefore, one could suggest that the teacher change his or her strategy of communication and then try to convince the teacher that absolutism need not be the only possible philosophy of mathematics. This, however, is not our strategy, although we find it relevant.

We recommend intervention directly in classroom practice in order to challenge the ideology of certainty. The communication in the classroom as well as the actual working philosophy of mathematics are related to what we call the *landscape of discussion*. By this, we understand the possible set of references which the students and the teacher might consider when they discuss and try to solve the tasks that they are confronted with in classroom practice.

The landscape of discussion can be of a varying nature.

> (1) *The empty and rocky landscape* includes objects only which are relevant to the logical construction of the mathematical concepts, as these are conceived by structuralism.
>
> (2) *The cultivated landscape* makes up a pre-structured reality. Mathematics can be applied to a variety of problems, and a certain problem-context can be presented to students A mainstream of post-structuralism has invited students to travel around in such organised landscapes.
>
> (3) *The Amazonian jungle* represents the chaotic and unorganised landscape for discussion. Here, references to reality are not pre-structured by any simple mathematical purpose. We think of the broader thematic approaches as well as many forms of project-based mathematics education as examples of students trying to find their way through such a jungle.

Our first suggestion, then, is to base classroom practice on a 'chaotic' landscape of discussion. In the Amazonian jungle, it is difficult to preserve true–false-based communication, and an absolutism seems impossible to uphold. This represents a challenge to the ideology of certainty

This strategy, however, is not simple and straightforward A counter-argument might emphasise that the reason that it is impossible to preserve true–false communication is simply that the topic for discussion has nothing to do with mathematics. The Amazonian jungle may simply be judged external to mathematics The question, therefore, is whether it is possible to introduce a chaotic landscape for discussion which cannot be labelled 'external' – at least not in any simple sense of 'external'.

Not every change in the landscape of discussion has the same effectiveness in challenging the ideology of certainty. Many examples of project work (Skovsmose, 1994; Borba, 1987, 1994) in mathematics education illustrate the 'chaotic' landscape, but they might appear to raise questions 'outside' mathematics, so they do not exemplify a strong attack on the ideology of certainty itself. Moreover, the mathematics involved does not suggest that uncertainty is present within mathematics. We have, therefore, been looking for a challenge which emerges from what can be conceived as 'proper' mathematics

Thus our interest in voting theory The application of voting theory cannot be characterised as an application of pre-established mathematical formulas. The landscape for discussion concerns voting and democratic processes. It concerns the notions of fairness and power, it concerns mathematics, and it emphasises the lack of 'perfect mathematical solutions' to a set of problems Nevertheless, as we have pointed out before, the challenge to the ideology of certainty is not guaranteed by any context.

6. The formatting power of mathematics, voting theory and the classroom

The formatting power of mathematics is different from (and, from a sociological point of view, stronger than) the descriptive potential of mathematics Description raises the issue of accuracy while formatting emphasises the actions taken in order to frame phenomena The locus of the discussion of descriptive powers is different from the locus of the discussion of formatting powers.

Voting theory provides a paradigmatic example of mathematics used as (part of) a principle for design. Voting theory is not simply applied in the sense that some part of reality is described up to a certain degree of approximation by means of some mathematical formalism Instead, with reference to voting theory, we become prepared to do something to a social structure. We become equipped to make a decision and to intervene in social and political reality We get into the process of design; in this case, a design of a (more-or-less) democratic procedure. Naturally, the development of voting theory need not be accompanied by any action; and naturally actual actions need not be based on mathematics. When we talk about the formatting power of mathematics, we first of all think of applied mathematics, together with other 'instruments', as a structure of support for action and design

The formatting power of mathematics is a common phenomenon Technological design, from the most advanced constructions to the design of the queue in the baker's shop, exemplify the formatting power of mathematics. In fact, the design of a queue is a common phenomenon. For instance, how are we to determine priorities in hospitals? Which operations are to be carried out first? This discussion can be carried out with heavy references to the cost for the individual, the cost for society, etc. In cost-benefit analysis, mathematics plays a crucial role Mathematics is part of economic super-structures, and this phenomenon can be illustrated on a small scale as well.

In Skovsmose (1994), the example "Family support in a micro-society" is discussed. Here, secondary school students are given the task of designing a way to distribute family support in a small community. Although based on some social and political principles, formulas are needed in order to calculate how much money the individual families will in fact get. In this way, mathematics becomes part of the technique of distributing social welfare in the micro-society.

Similarly, Borba (1994) discusses the experience of children from a shantytown in Brazil as they use mathematics to solve problems which seem to be of paramount importance to their lives. In the same way, mathematics is part of those economic techniques by means of which society operates on its citizens.

Questions raised by mathematical formattings make it impossible to rely on the ideology of certainty It becomes obvious that the two basic assumptions of the ideology of certainty must be challenged. It is no longer possible to maintain that the truth of mathematics cannot be influenced by social and political interests. Furthermore, when we take the formatting power of mathematics into account, the notion of truth is not the fundamental category any longer.

A description can be accurate or not, and in this sense we still make (implicit) reference to the notion of truth. But when we take formatting into account, the discussion has to do with social functions of technology, and the fundamental question concerns the value of what we are doing. In this case, mathematics is not always relevant and reliable. When focusing on the formatting power of mathematics, the two basic assumptions comprising the ideology of certainty become challenged.

What is interesting about voting theory is that it exemplifies the formatting power of mathematics in a direct way Our point is that a way of challenging the ideology of certainty is to change classroom practice by introducing a landscape of discussion of a chaotic nature, where relativity, provisional starting points, different points of view and uncertainty are valued. To challenge this ideology also becomes a challenge to the formatting power of mathematics itself

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Notes

[1] Machado (1990) has discussed similar assumptions about mathematics (mathematics is abstract, exact, inate,) to make his point that mathematics education and one s mother language education should be much more similar than they actually are

[2] This is certainly the case for critical education, as it was developed in Europe partly in reaction to the Second World War, which was based on the presumption that education should also be part of the struggle against the 'authoritarian personality'. Many other developments in education have had similar concerns.

[3] For instance, teachers could study the fallibilistic approach to mathematics as advocated by Ernest (1991)

[4] Banzhaf defines the ratio of power between two legislators, X and Y, as follows:

> [I]n a case in which there are N legislators, each acting independently and each capable of influencing the outcome only by means of his vote, the ratio of the power of legislator X to the power of legislator Y is the same as the ratio of the number of possible voting combinations of the entire legislature in which X can alter the outcome by changing his vote to the number of combinations in which Y can alter the outcome by changing his vote. (Banzhaf, 1965, p 331)

Banzhaf's analysis goes further: he analyses the voting power of each individual manifested in the case that his or her elected representative would cast a decisive vote on the legislature, and he makes generalisations for larger populations and larger legislative bodies, and so on.

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