Mathematics and Society: Ethnomathematics and a Public Educator Curriculum

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Our primary purpose in this paper is to generate a discussion about the potential role of a "Mathematics and Society" curriculum in formal education. The paper is probably most relevant to formal education systems in Western industrialised countries though not exclusively so. The development of such a curriculum immediately raises questions about the sort of educational philosophies underpinning the curriculum. What might be the purpose of such a curriculum? Implicitly, a "Mathematics and Society" curriculum suggests that the study of the relationship between mathematics and society will be involved. Furthermore, questions concerning both the nature of mathematics and the nature of society need to be addressed. Thus a "Mathematics and Society" curriculum needs to be informed by models/theories of how mathematics is socially produced and of the role of mathematics in society as well as an underlying educational philosophy.

Much can be learned from debates about the purpose of mathematics education, both past and present. In fact, school *science* educators were the first to grapple with the problem of how the nature of the subject (in this case science) is related to educational philosophy and vice-versa. The nineteenth-centry advocates of the "science-of-common-things" curriculum have much in common with the proponents of modern day ethnomathematics, for instance.

Our aim is to provide a conceptual picture of the elements required for a "Mathematic and Society" curriculum in terms of values, philosophy of mathematics, and social theory.[1] The conceptual picture should be seen as the essential product of the paper. Though it is an abstract product, it is arrived at by considering concrete curriculum debates. Also the product should be seen as a tool which can be applied to a variety of concrete mathematics curriculum development situations.

1. The curriculum debate

In nineteenth-century Britain, the growth of democracy and industry with concomitant radical changes in types of work led to a fierce debate about the purpose of education. Williams [1961] has provided a useful description of the protagonists in this debate. He refers us to three lobbies, namely, the "industrial trainers", the "old humanists", and the "public educators". An expanding and dynamic economy, on the one hand, and the development of an organised working class demanding education on the other, framed the struggle over the purpose of education. The industrial trainers pressed for a curriculum which would serve the perceived desires of industry within the economy whilst the public educators proposed an education for democratic citizenship. By contrast, the old humanists can be seen as a conservative influence who regarded the curriculum of the *status quo* as intrinsically justified. School subjects would, thus, continue to be studied for their own sake and to be geared to the interests of an elite.

In an in-depth study Layton [1973] gives a vivid account of how the old humanist, industrial trainer, and public educator lobbies influenced the development of the British school science curriculum in the nineteenth-century. In the 1850s advocates of the "science of common things" curriculum, such as Dawes and Mosely, felt that science teaching should draw upon and reflect the children's experiences of "everyday living". Their teaching materials thus included problems of cottage ventilation, personal hygiene, family nutrition, manual competences, and agricultural improvement.

Dawes argued that science must be made to bear upon "practical life" which he maintained changed according to the dominating problems of each society and the immediate concerns of particular learners. In particular, scientific knowledge was to be adapted to the needs of working class children for whom science had previously been considered of no relevance. Thus the movement for the teaching of science-of-common-things reflected, in the main, a public educator perspective.

However, it also drew on arguments closely related to those of the old humanists and industrial trainers. For example, Dawes argued that the science-of-common-things would "raise them (pupils) in the scale of thinking beings".[2] This is reminiscent of the old humanist view that science for its own sake refined and elevated every human feeling. Simultaneously, industrial usefulness of scientific knowledge was stressed. Moseley had a history of interests in industrial education and according to Layton supported opportunities to apply to companies such as the East India Company for financial support for the curriculum.

As Layton explains, the science-of-common-things curriculum was opposed on several grounds. Firstly, to some in politically powerful positions, the idea of giving the masses access to knowledge which the upper classes would not also have, was threatening. The fear was that it might lead to social instability and subversion of the pre-existing social order. Secondly, liberal educators argued that a curriculum selected because of its immediate utility to a particular social group (namely the working class) might

For the Learning of Mathematics **8**, 2 (June 1988)
FLM Publishing Association Montreal, Quebec, Canada

lead to a "ghetto curriculum" in which pupils were discouraged from looking beyond their own environment, thus frustrating the liberal ideal of social mobility. Thirdly, modern science and its industrial applications were thought to be best served by the application of mathematics to scientific problems, especially physics. The mathematization of science was the antithesis of the science-of-common-things curriculum. Ultimately, according to Layton, these arguments were enough to lead to the demise of the science-of-common-things curriculum movement

Some one hundred and fifty years later a similar discussion is growing with respect to the mathematics curriculum. Faith in the old humanist perspective that mathematics education is a rigorous training of the general faculties of the mind has diminished, and currently an expanding forum of debate regarding the purposes of mathematics education is developing.

One of the major justifications of the old humanists for mathematics as a crucial staple in the curriculum is that it offers a rigorous training in rational thought. On this argument the study of mathematics is supposed to extend logical thinking, critical thinking and problem-solving abilities Essentially, then, mathematics in itself, and for its own sake, engenders the educated person Amongst educators there is now considerable scepticism about this old humanist perspective in its pure form, though institutionally it remains a powerful lobby.

Further criticism of the old humanist view of mathematics education has also come from the industrial trainers They argue that the study of mathematics for its own sake cannot be relied upon to deliver the skilled manpower for work in the economy. Mathematics education, according to the industrial trainers lobby, should be oriented towards applying mathematics to industrial problems (Thwaites [1961]) In contrast to the old humanists, this view shows little concern for the development of the general faculties. It is characterised by an emphasis on society's declared need for specific pragmatic and instrumentalist thinkers.

What, then, can be said about the relationship between mathematics and society as viewed through the perspectives of these three educational philosophies? For the old humanists, mathematics has no apparent relationship with society at all. For the industrial trainers, mathematics has a narrow one-way relationship in which mathematics is required to provide the techniques and expertise demanded by particular interests created through technological change and industrialisation Neither group sees mathematics and society as having an interactive relationship. The public educator perspective, however, takes a somewhat more interactive approach in that the kind of mathematics which is seen as appropriate for the curriculum is built on a view of society which takes account of different constituencies of interests, incuding the cultural interests of the learner.

2. Ethnomathematics and self-generated mathematics

Recently, the importance of cultural context has formed a central theme in a number of mathematics education research projects. As with the movement for teaching the science of common things, a common element of these projects is that the legitimation of learners' experiences is recognised as being of fundamental pedagogical importance. In short there is a focus on "ethnomathematics", a term coined by D'Ambrosio (1985)to refer to "mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labour groups and so on" (D'Ambrosio [1985] p.45)

D'Ambrosio's research programme is designed to identify cases of ethnomathematics and relate them to their historical origins and patterns of reasoning. Clearly this has implications for education since it allows the possibility of a redefinition of legitimate mathematical knowledge and practises. The vision of the ethnomathematics perspective is that the "psychological blockage" often associated with the learning of academic mathematics might be avoided (Gerdes [1986] p 21)

Gerdes [1986] provides one of the most striking examples of ethnomathematics in Third World countries His demonstration of the geometrical thinking involved in Mozambican weaving illustrates indigenous "frozen" mathematics. Gerdes' argument is that the weavers, through their activities, already engage in complex mathematical thinking Winter [1987] makes a similar point about the "metaphorical" thinking of young children playing number games in informal contexts He suggests that infants and young children already possess the conceptual understanding of mathematical notions such as infinity or probability They lack only the agreed conventions to articulate them in academic mathematical terms. Winter maintains it is the *sophisticated format* of academic mathematics rather than the underlying concepts which children do not readily engage with

Winter's analysis closely parallels that of Hoines [1986] who emphasises the significance of children's own language in their development of mathematical concepts She explains how unconventional units of distance derived from the pupils' labelling (e.g. "Asmundchord") aid them in measurement [3] Again the implication is that non-academic experiences involve mathematical thinking and that these experiences in the form of modes of thought, jargon, interests or myths can be used as liberating tools (in a psychological sense). That is, they can be part of mathematics education but simultaneously liberate learners from the tyranny of conventions and formats in academic mathematics. They can combat the "mathophobia" and "psychological blockage" to which Winter and D'Ambrosia respectively refer. We suggest that many of these pedagogical perceptions contained within the ethnomathematics perspective would be of considerable significance in a Mathematics and Society curriculum

Cobb [1986] spells out some other consequences of opting for ethnomathematics (what he calls "self-generated mathematics") in preference to academic mathematics. Self-generated mathematics, he argues, is individualistic and anarchistic The criterion of acceptability for self-generated methods is pragmatic, i.e the methods function to enable children to attain their goals. By contrast, academic mathematics requires children "to play the academic mathematics game when he or she is introduced to standard formalisms, typically in first grade." (Cobb [1986] p. 7) Cobb charges academic mathematics with authoritarianism and totalitarianism and says:

> The child's overall goal might then become to satisfy the demands of the authority rather than to learn academic mathematics *per se* (Cobb [1986] p 7)

Given the similarities between ethnomathematics and the science-of-common-things, it would not be surprising if criticisms of the latter applied to ethnomathematics. In effect, might not giving legitimation to alternative (or self-generated) mathematical thinking in "everyday life" lead to a "ghettoising" of the curriculum? In the case of the science-of-common-things curriculum, liberal educators voiced this concern because they wanted to maximise social mobility. Ethnomathematics has already faced analogous difficulties. For example, Mellin-Olsen [1986] comments on how the Norwegian Social Democratic government resisted ethnomathematics in the curriculum on the grounds that it contravened the principle of equality of opportunity in the form of equal curricular content for all

Our own reservations about ethnomathematics are rather different. However, they are also related to the problem of a "ghettoising" curriculum. We want to consider this problem under the two related themes of ethnocentricity and critical intervention.

ETHNOCENTRICITY

Unlike Gerdes, we do not want to define mathematics as ethnomathematics but neither do we wish to define it as academic mathematics. Mathematics is more than either of these. Mathematics is produced not only through "everyday experiences" untouched by academic influence, but also through the organised activity of particular social groups whose mathematical problems arise because of an historical conjuncture between the groups' structural role in society and the predominant mathematical paradigms of the time. We shall call this socially organised mathematical activity the *social institution of mathematics*. Indeed it is precisely due to an appreciation of the importance of the social institution of mathematics that the industrial trainers have lobbied and continue to lobby for an industrially-oriented mathematics curriculum. Mathematics education should involve some individual and group generation of mathematical problems — this is the great insight of ethnomathematics. But we do not believe that this is sufficient for a mathematics education: in addition we wish to include that part of the public educator perspective which emphasises *democratic citizenship* rather than the liberal promises of *social mobility* or *equality of opportunity*. In our view democratic citizenship with respect to mathematics means not only having the skills to generate one's own mathematical problems but also having some understanding of how and why other pervasive mathematical problems are generated and maintained along with their most important consequences for democracy and citizenship. In short, having some understanding of the social institution of mathematics.

CRITICAL INTERVENTION

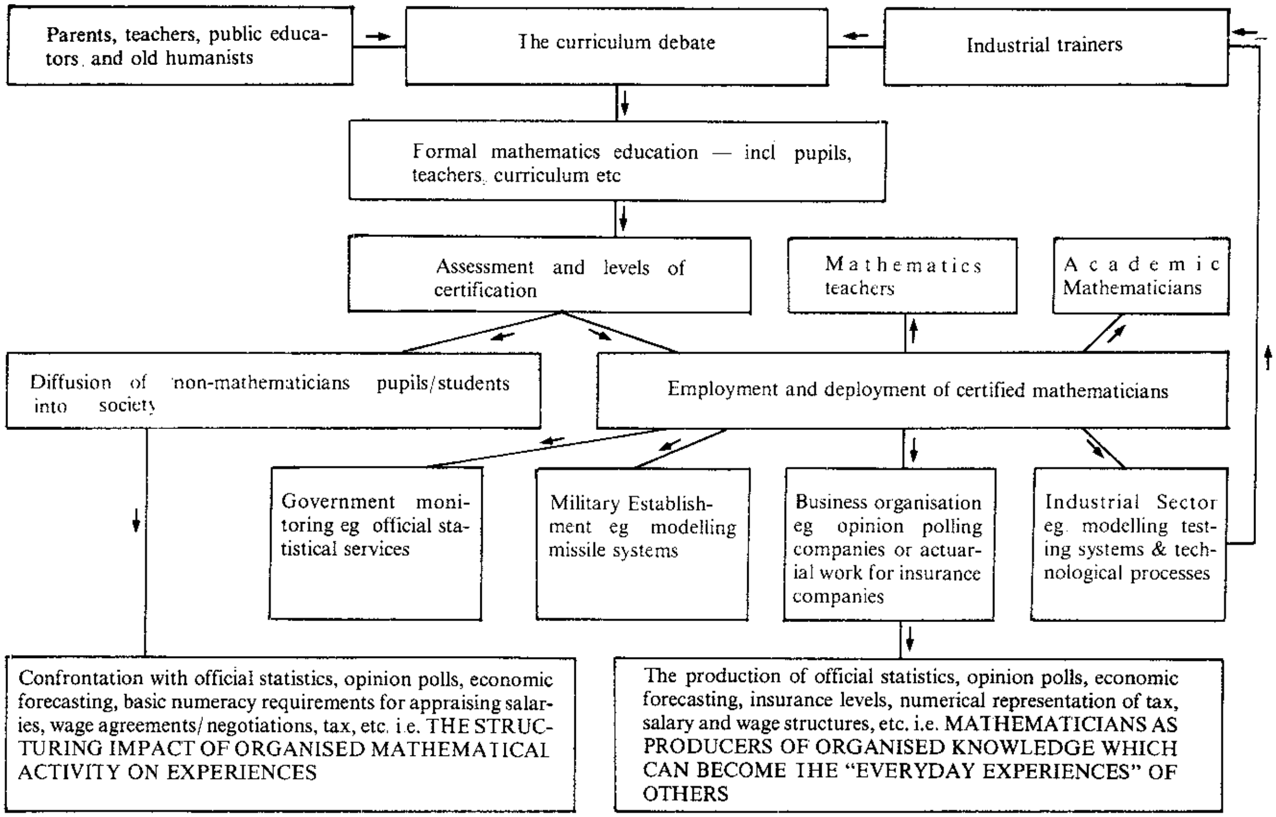
We presume that proponents of ethnomathematics would argue for the development of mathematics as a cultural resource — i.e. something which a group or subculture/culture can use as readily as speakers use their own language. However, this requires more than a mathematics curriculum which legitimises everyday encounters with mathematics. Such legitimation is necessary, but we argue not sufficient, for the development of mathematics as a cultural resource. To achieve this objective, surely practitioners must be empowered to either use, or reject the use of, mathematical techniques by reference to their cultural value system? This implies that practitioners possess skills which enable them to make value-judgements. It seems to us, therefore, that a mathematics education which seeks to develop mathematics as a cultural resource should not only relate to learners' experiences but also contain a critical dimension orientated towards making judgements about experiences on the basis of an understanding of how context influences those experiences. This might involve, for example, students critically comparing their non-formal types of mathematical thinking with the "official" version of mathematics presented to them

To clarify these arguments it is necessary to consider in more detail how critical intervention and the social institution of mathematics relate to mathematics education

3. The social institution of mathematics

Sociologists, historians and others have already studied many aspects of the social institution of mathematics (e.g. Mackenzie [1981], Mehrtens *et al* [1981], Grattan-Guinness [1981], and the Government Statisticians' Collective [1979]). For our general and simplified view of the social institution of mathematics (largely informed by the aforementioned works and others) see Figure 1. Sociologists of education such as Cooper [1985] and mathematics educators such as Ernest [1987] have also studied some of the institutional relationships between mathematicians, mathematics educators and the curriculum. Nevertheless all these works and the understandings gained from them have remained the province of history, sociology, educational studies or some other discipline.

What we are suggesting is that *mathematics* cannot be completely understood without some understanding of the social institution of mathematics. This means having an understanding of the human actions and commitments that give rise to major developments in mathematics. It means having an understanding of the role mathematics plays in structuring our experiences and judgements. This point is a crucial complement to the ethnomathematics perspective because it addresses the question of how context affects and structures experience. The ethnomathematics perspective leaves this question unasked since "everyday experiences" are taken as the starting point. We believe that this is not sufficient because mathematics which we do not directly experience can still have important consequences for our lives and from the public educator perspective especially those aspects of our lives which



The Social Institution of Mathematics [4]
Figure 1

depend on democratic citizenship. We propose that the study of the social institution of mathematics becomes part of mathematics education.

After all, mathematicians experience this social institution, mathematics educators are products of it, mathematics teachers are products of it, and so on. Hence, a Mathematics and Society curriculum should bring the study of the social institution of mathematics *into the curriculum* as well as embrace the insights from ethnomathematics

4. Critical thinking and conscientization

According to McPeck [1981] most people interested in educational issues tend to see the ability to think critically as a desirable human trait The old humanists supported this view with respect to mathematics education except that they regarded it as an automatic appendage to thinking-through academic mathematical problems This old humanist perspective is concisely countered by McPeck [1981, p 21] when he says that "the requirement for assessing a problem critically are epistemological, not logical, in character". It follows that expert manipulation of logical relations within the paradigms of academic mathematics gives no guarantee of, or even cognitive basis for, critical thinking.

Wellington and Wellington [1960] go further by asserting that critical thinking arises out of problem-generating and problem-solving activity The definition of the problem is not the teacher's but rather it is considered to arise from the anxieties [5] of the students in areas of interest to both teacher(s) and students

> The process of planning and sharing by teacher and students helps to produce coalescence, the basic method of critical thinking (Wellington and Wellington [1960] p.85)

This model of critical thinking has some similarities to the ethnomathematics perspective but it can be contrasted with Cobb's self-generated mathematics in the important sense that critical thinking is considered to be *cooperative*, rather than *individualistic* Problems are *shared* by individuals rather than *possessed* by them.

It is in the work of Harris [1981] for UNESCO and Freire [1972] that conceptions of critical thinking best bring together the ethnomathematics and public educator perspectives Harris's UNESCO report lists the need for active and critical participation in the democratic process as one purpose of mathematics education Similarly Freire [1972] is concerned to promote "free critical citizenship" through the educational process of conscientization [6] But what do we mean by the Freirian concept "conscientization"? Mellin-Olsen [1986] gives the concise interpretation that conscientization is the process by which people are made aware of their culture

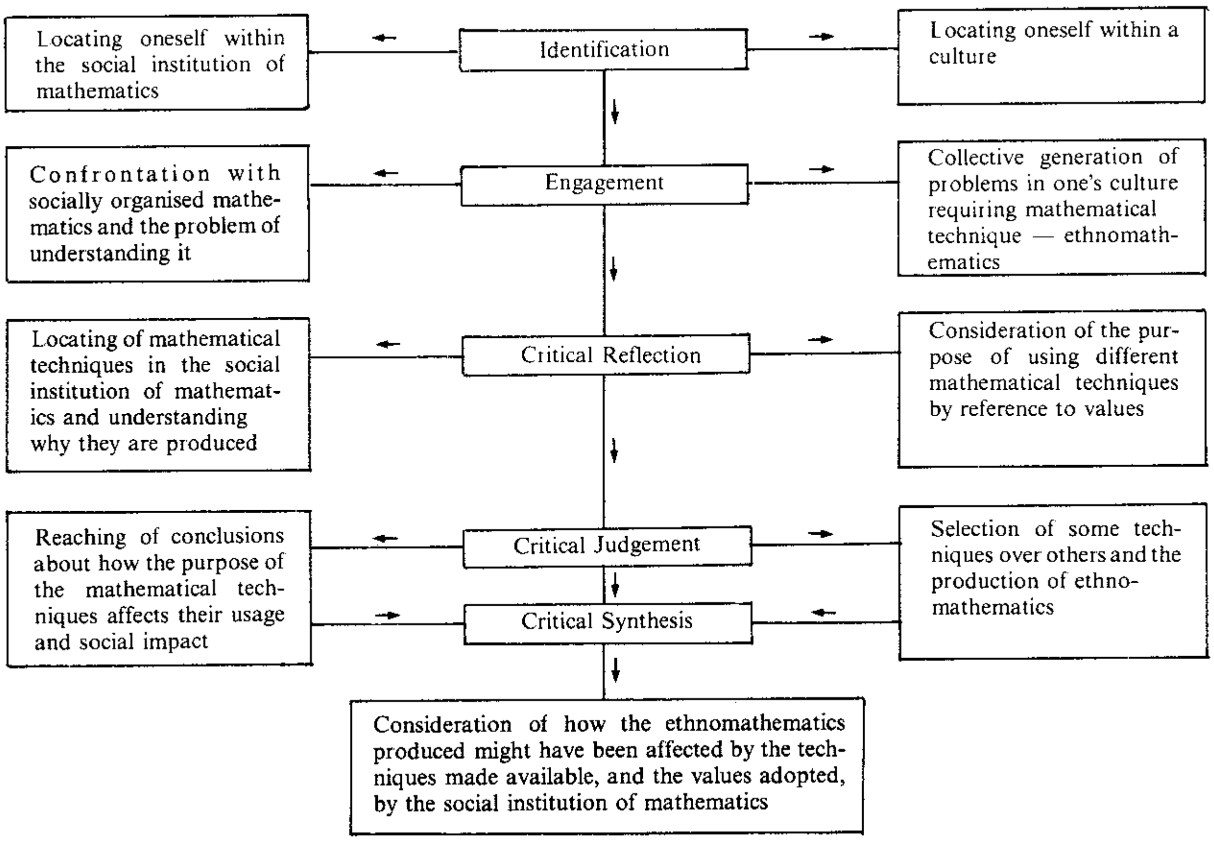
To be more precise, conscientization can be understood as the process by which people become aware of how their experiences are structured and conditioned. This awareness enables people to make critical choices about actions. As Freire puts it:

> Conscientization is viable only because men's [7] consciousness, although conditioned, can recognize that it is conditioned. This "critical" dimension of consciousness accounts for the goals men assign to their transforming acts upon the world. (Freire [1985] pp. 69-70)

For a Mathematics and Society curriculum conscientization is the crucial process by which the relationships between mathematics and society (especially the social institution of mathematics) are related to the personal development/situation of the pupils or students. The process involves the learner in a number of stages. Firstly, engagement with some form of organised mathematical activity. For pupils/students of mathematics this is immediate. [8] Secondly, objectification of some mathematical problem, i.e. the distancing of oneself from the problem so that it is seen clearly as the object of study. Thirdly, critical reflection upon the purpose and consequences of studying this problem in relation to wider values.

As informed by the ethnomathematics and public educator perspectives, the aim of the Mathematics and Society curriculum would be to continually link knowledge of relationships between mathematics and society to students personal and collective development. For example, the curriculum would encourage the explanation of how controversial issues can be discussed and sometimes resolved with the aim of enabling students/pupils to develop frameworks, concepts and approaches to guide action. Teachers would try to go beyond a pedagogy which merely presented a range of alternative experiences and opinions because this would provide no insight into how to arrive at conclusions critically or make choices. In this respect teachers would try to generate an awareness of the social responsibilities of mathematicians as opposed to "an isolationist view which divorces mathematics from its social and political context". (Ernest [1986] p. 17)

We are now in a position to consider the *conceptual* scheme underpinning the Mathematics and Society curriculum being proposed. Mathematics exists in society as a socially organized activity (the social institution of mathematics) and as *ad hoc* experiences. The former structures our mathematical experiences in well-defined ways which have been established through the historical devlopment of certain forms of organization. The latter may be used as



The Process of Conscientisation in a Public Educator Mathematics Curriculum

Figure 2

everyday resources to generate ethnomathematics But structured mathematical experiences can also impact upon the generation of ethnomathematics since they too are usually part of a subculture's mathematical resources. Finally, if and when ethenomathematics becomes established it can then become a structuring agent of the mathematical experiences of those outside the subculture which generated it.

Figure 2 is a diagrammatical representation of the role of conscientization in such a curriculum. Essentially our approach differs from the ethnomathematics perspective in the sense that we wish to see a mathematics education which, in part, aims to enable students/pupils to understand how knowledge is established (including, and sometimes especially, in those sphere of social activity of which they have no immediate experience) and critically relate this understanding to their own experiences

5. Examples of Mathematics and Society curricula

The idea of relating mathematics to society is not new. One of the most impressive and exciting curriculum developments in this area is the international Mathematics in Society Project (MISP) It is being developed in the UK, USA and Australia mainly for the secondary school level This project is based around eight themes, namely (a) political development, (b) economic development, (c) the natural world, (d) science and technology, (e) art, (f) sport and recreation, (g) structure of modern society, and (h) what people do. The project started in 1980 and is currently being introduced into some secondary schools

Rogerson [1986] outlines much of the thinking behind the MISP project. He argues that "mathematics as used in society" (what he labels M2) is not the same as "school mathematics" (what he labels M1). M1, says Rogerson, takes the form of standard syllabus lists so that the "living body" of mathematics as evidenced by its use in society (i e M2) is reduced to a skeleton Consequently, as Rogerson points out, school mathematics is more analogous to a dictionary than to a language. Just as we have argued for the conceptualisation of a Mathematics and Society curriculum, Rogerson maintains that in order to see how M2 is different from M1 it is necessary to ask: "What is mathematics?" One of the important insights of MISP is that it tries to answer this question by using an "extensive definition", i e. one which is made up from the uses of mathematics in society. [91]

Inchley [1985] emphasises how social change (e.g. the introduction of calculators and micro computers or changes in currency) provides a major rationale for studying mathematics in society. Simultaneously, she argues for social change to be used as a resource for MISP curriculum developments Similarly, Romberg [1985] suggests that the history of mathematics should be a resource which provides students with a "story" of mathematics. In general, MISP protagonists are keen to relate mathematics to the "real world" even though the nature of the "real world" is rarely debated. [10]

MISP is certainly a progressive movement within mathematics education and may be the most significant one with respect to Mathematics and Society curricula to date. We share Rogerson's concerns about the "skeletonising" of mathematics via traditional school mathematics. [11] However, there are, in our view, some problems in adopting MISP as a model for the development of Mathematics and Society curricula.

In the first instance MISP is justified on the grounds that (i) it is likely to provide "motivation" for pupils studying mathematics, thereby reducing the frequency with which it is found to be "difficult", and (ii) mathematics is widely used and utilised in society The problem is that neither of these justifications can be an educational rationale. Motivation *per se* cannot be an educational rationale because opposing rationales can take different stances with respect to the same kind of motivation For example, Freirian approaches to mathematics education might oppose a curriculum designed to motivate students to practice mathematical techniques in the absence of understanding the purpose of the activity whilst the old humanist philosophy might equally want to encourage this kind of motivation. Merely the fact that mathematics is widely used in society cannot be an educational rationale either because several opposing philosophies can equally use this fact as support for their case — e.g public educators and industrial trainers The fact that mathematics is widely used is merely an observation and no more Since neither motivation *per se* nor the pervasiveness of mathematics can form an educational rationale singly or together, it is difficult to see how they provide the justification for MISP that Rogerson claims they do This difficulty highlights the importance of trying to relocate the current debates about the purpose of mathematics education in relation to debates about educational philosophy

A second problem with MISP is the absence of a model of social relations. This problem arises most vividly in the MISP distinction between topics according to the two headings "Maths content" and "Society content" The MISP method for formulating questions for the curriculum seems to be to extract the "maths contents" from the topic and ask questions about the "maths content" but in the context of the "society content". (Rogerson [1982]) Many questions for the curriculum can be generated by this method However, this method reveals little about *society*. The student learns little about society, and mathematics *as part of society* This is because society is represented as fragments of, and fragmented, activities within which mathematics "hangs around" and gets used The result of this is two-fold:

(a) the nature of society is apparently considered unproblematic and, therefore, some of the MISP questions carry profound sociopolitical assumptions which are not presented as open to discussion (e.g maximization of profit)

(b) the process of conscientization tends to be dislocated because there is no framework to guide critical judgements made on the basis of cultural values. This framework is absent because rather than the role of mathematics in society being

> studied in terms of its human activity *as understood within society* it is resurrected and presented within a range of apparently unrelated value-free activities However, a refreshing exception to these criticisms is the example of modelling the energy crisis presented by Fishman [1985].

We would suggest that some of these problems could be addressed by the incorporation of a model of mathematics in society such as the "social institution of mathematics" model Having said this a great deal of innovative work has been completed through MISP. Some of the ideas are very encouraging and have brought forward the possibility for further work in the area.

Another less extensive example of a Mathematics and Society curriculum is the Norwegian "Mathematics, Nature and Society" (MNS) project developed for secondary schools in the country of Hedmark in Eastern Norway. (Kvammen [1986]) This project emphasises the critical selection of mathematical techniques to help in the management of environmental and social problems In so doing it raises questions about the nature of these problems — for instance, the problem of how energy resources are allocated and how they can be conserved, and of what agricultural resources are needed to sustain a certain level of production and how these should be organised to reduce long term starvation.

The MNS project appears to give more emphasis to how context affects experiences and the environment than does the MISP There also seems to be a greater tendency to relate the purpose of the project work in the curriculum to explicit value positions, e g uncontrolled exploitation of the environment at the expense of agricultural land is undesirable because such land is required to produce food, therefore it is worthwhile to use mathematical techniques to monitor the extent of this exploitation with a view to informing future discussion and action on the issue The one major difference between MNS and the Mathematics and Society curriculum which we propose is that the pupils studying MNS are not given a framework through which to judge the impact of their mathematical competences on the wider aspects of their future lives This is because of the absence of any engagement with the social institution of mathematics in the MNS project.

MISP and MNS involve students/pupils in project work and the assessment is project oriented. However, it is possible to find Mathematics and Society issues raised in traditional style examination papers In 1986 the Secondary Mathematics Individualised Learning Experiment (SMILE) under the English London Regional Examining Board included a question in its Certificate for Secondary Examination on quantitative estimates of military spending based on figures published by the Swedish International Research Institute (SIPRI) Among other things, candidates were asked to compare the average growth of military expenditure per year in the USA with that of the USSR for the period 1980 — 1984. They were also asked to calculate military spending per head of population in the two countries in 1980 and to complete a graph using the SIPRI table of data At the end of the question candidates were asked to comment on their results

This is a considerable improvement on many tables of data in mathematics questions in that the source was actually noted — implying at least that somebody *produced* the data Indeed, in some quarters the question has been celebrated. (Anon. [1987]) However, we consider the question to illustrate a number of problems relevant to the teaching of controversial issues in a Mathematics and Society curriculum For us the main problem with the question is that the data tends to be treated as the final facts of the matter and, therefore, does not guarantee the development of a critical perspective on the data No rival data were presented and thus the ways in which military spending may be controversial were not conveyed in the question Different groups with different institutional interests make different claims about military spending and they use different measurements and data sets to support these claims. No doubt many of the question's deficiencies are related to examination constraints.

In the absence of examination contraints we would suggest the following alternative which is based on a public educator perspective embracing the notion of conscientization. Rival sources of data could be provided and following an analysis of the differences between the data sets there might be a discussion of why these difference have arisen This discussion might involve the development of a framework which describes the different institutional interests involved and the way these can affect the production of statistics (i e again, how context influences experiences). Following this, students/pupils would be asked to make critical judgements about the likely reliability of the different data sets and to reach some conclusions about the controversy with a view to guiding their future actions This process would probably involve the use of some further mathematics as well as the use of the understanding of this part of the social institution of mathematics generated by the previous discussion.

Clearly this kind of alternative is not a possibility in traditional style examination questions. As an examination question the SMILE contribution is a welcome shift from the "unreality" of examples often found in so-called "relevant mathematics" questions. (Howson [1983])

Our view is that a concerted effort to develop a Mathematics and Society curriculum of the sort we envisage can, in principle, make a crucial contribution to mathematics education and improve on some of the important developments already achieved However, it is helpful to contrast two kinds of arguments in favour of a Mathematics and Society curriculum — the "strong case" and the "weak case"

The strong case holds that mathematics can only be understood when its social and historical origins are also understood Under the strong case the mathematics of a computer programme, for example, can only be understood if the reasons for the design and production of the programme are also understood On the other hand, the weak case holds that mathematics cannot be completely understood unless that understanding partially involves an awareness of how mathematics is socially organised, produced, and maintained throughout history, and in the context of cultural influences. In the absence of such an awareness, under the weak case, the mathematics of a computer programme is understood but only partially, and possibly in a way which is socially dislocated. This kind of incomplete understanding is similar to the notion of "semi-intransitive consciousness" which characterises a state of being in which one has only a fragmented, localised awareness of one's situation. (Frankenstein [1983], p. 318) We support the weak case and this can have some implications for the policy analysis, although it does not prescribe one policy or another.

6. Policy issues for a public educator mathematics curriculum

No doubt there are many who do not share our educational perspective on mathematics or a Mathematics and Society curriculum. We do not assume that our arguments have created a consensus in support of our perspective. Nevertheless, leaving fundamental differences aside for the moment, we would like to consider the many practical problems of introducing such a curriculum into the formal education system.

We would suggest that at least five distinct policy problems arise in considering the development of a public educator mathematics curriculum. These are (i) political ideology (ii) subject maintenance (iii) teacher education (iv) student expectations and the examination system (v) differentiation and ability stereotyping.

POLITICAL IDEOLOGY We should not underestimate the extent to which particular political interests would attempt to undermine a curriculum which sought to critically debate the relationship between mathematics and society. Critical debate is threatening to those interest groups who wish mathematics education to *serve* their interests directly or indirectly. The examination question which we discussed earlier could hardly be accused of bringing critical debate into the examination room. At most it implied, though not explicitly, that some kind of social issue was reflected in the SIPRI data. Yet the response in the more conservative British press was definitively hostile. For example, *The Daily Mail* headline read "Six thousand pupils take the "propaganda test " and asked the question: "What has arms spending to do with a maths exam?" *The Sun,* another British newspaper, described it as "sinister" and concluded that political propaganda, Left or Right, has no place in the classroom. The Hillgate Group, in its *Radical manifesto,* referred to this particular examination question as "downright propaganda" and complained that "even mathematics and music are to be given a "peace" or "global emphasis" resulting in "a gradual pollution of the whole curriculum by practices which are profoundly diseducational." (Cox *et al* [1987]) As a result of the conservative disapproval of this examination question it was decided that an examination board should vet future mathematics papers for political content. (Brown [1986])

On the basis of this response to one examination question it is reasonable to conjecture that a widespread curriculum which sought to relate mathematics to society, including controversial political issues, would confront ideological opposition. The best suggestion that we can make to counter this is to pre-empt such ideology in the proposals and negotiations for curriculum change. There is little point in pretending that this ideological opposition does not exist and hoping for the best.

SUBJECT MAINTENANCE It has been argued that subject areas are the product of social forces which, in practice, legitimate certain aspects of curriculum content and constrain others. (e.g. Fensham [1980], Goodson [1983], and Young [1976]) For example, university mathematicians and professional mathematicians, in general, may have an interest in maintaining the present discipline of mathematics. Given our adherence to the weak case for a Mathematics and Society curriculum, one policy which might avoid such resistance could be to develop Mathematics and Society as a separate and distinct subject. There are at least two problems with this, however. Firstly, the separate subject might be marginalised from the rest of the mathematics curriculum. Secondly, in many countries, especially Britain, the curriculum is already "overcrowded" (O'Connor [1987]). Cramming a new subject into the present system faces obvious difficulties in this respect.

TEACHER EDUCATION The public educator curriculum we have proposed has radical implications for pedagogy. One powerful obstacle to the development and sustaining of such a pedagogy is the authoritarian "custodial pupil control ideology" that many teachers bring to the learning situation (Denscombe [1982]). There is now considerable research indicating that, in the UK, USA and Australia, teacher training fails to equip teachers with the capacity to construct and/or sustain an alternative to the custodial control approach in the face of practical and professional pressure to be seen as a competent teacher.

Teachers are central agents in the educational process and this means that teacher education must give a much more bpowerful and empowering legitimation to alternative dialogical pedagogies if a public educator curriculum is to really function.

STUDENTS' EXPECTATIONS AND THE EXAMINATION SYSTEM Even if teachers and mathematicians can be persuaded of the value of a public educator mathematics curriculum, students might not be convinced. Insofar as our public educator curriculum includes an ethnomathematics perspective, student resistance should not, in principle, be a problem since the mathematical problems are generated from their own experience. But if this curriculum is to have any impact on schools, its developers have to take account of certain pre-existing features of schools.

For example, the status of school mathematics is such that high attaining students gain the opportunity to enter high status universities. Under these conditions there is a tendency to promote what Holt [1969] refers to as "producer thinker strategies". According to Holt a "producer" is a student who is only interested in getting right answers and who makes more or less uncritical use of formulae to get them. In this respect the comments of Reid [1984] are worth bearing in mind:

> Students as rational consumers, are less concerned with knowing than with the status that comes from categorical membership and the future promise that this implies

It is precisely this mismatch between the expectation of students as influenced by the competitive credit-collecting examination system and the educational ideals of public educators that leads to "pupil resistance to curriculum innovation in mathematics" (Spradbery [1976]) This "diploma disease" is particularly marked in some Third World countries (e.g. Sri Lanka) and constructing policies to counteract it is extremely difficult. (Dore [1976])

We suggest that one strategy would be to hive off entirely part of the mathematics curriculum from the examination system and have this taught and assessed by other means. The idea is that this would be the Mathematics and Society part of the curriculum. This is consistent with the policy option considered in (ii) and is, again, based on the weak case for a Mathematics and Society curriculum

DIFFERENTIATION AND ABILITY STEREOTYPING We know from sociological research that differentiating pupils according to their "ability" can lead to "polarisation" effects amongst the student population. (Lacey [1970], Hargreaves [1967], Ball [1981])

Recent research also indicates that mathematics teachers, in particular, see a direct correspondence between the "ability" with which they label students and the hierarchical structure of mathematical knowledge with which they label the subject. (Ruthven [1987]) To avoid these difficulties students of Mathematics and Society should not be ordered according to some external ability criteria. This, of course, necessitates other policies such as (ii) and (iv) also being implemented Students would be encouraged to learn cooperatively and collectively without differentiation. Again this needs to be appreciated in unison with a teacher education policy which tries to combat the polarising effects of ability stereotyping and labelling teachers

It is not appropriate for us to propose any precise policies in this paper. Ultimately particular policies depend on specific contexts and situations which affect what is possible for various protagonists. Instead we have offered general guidelines for a public educator mathematics curriculum policy. The changes which we have identified as desirable for policy implementation are extremely challenging, some would say impossible. But irrespective of the possibilities for implementation we would suggest that our proposals provide a reference framework for the development of a Mathematics and Society curriculum Even if some diversions from what is suggested here are required for implementation, a refererence point still remains by which to judge the extent of the success of that implementation On the other hand, those who support our suggestions might argue that it is the education system which should be changed rather than these ideas about the curriculum

Acknowledgements

This paper was first presented at the "Research into Social Perspectives on Mathematics Education" conference at the Institute of Education, University of London, on 7th July 1987. We are very grateful to all the participants of that conference for their comments. We would also like to thank Barry Cooper and Jeff Evans for additional helpful responses.

Notes

[1] It is implicit in this statement that we do not believe education can be value free.

[2] Quoted from Layton [1973] p. 187

[3] "Asmundchord" is the name which the class gave to a chord made by Asmund, one of the boys in the class

[4] By "non-mathematicians" we do not mean pupils/students who do not study mathematics at any time but rather we mean pupils/students who have not for whatever reason become employed in an institution in which they will be engaged in mathematical activity and production

[5] It is important to distinguish between the sense in which Wellington and Wellington [1980] use the word anxiety and the neurotic response sometimes associated with mathematics learning and numeracy.

[6] We are not the first to relate the Freirian notion of conscientization to mathematics education: see Abraham [1982], Frankenstein [1981], Frankenstein [1983], and Mellin-Olsen [1984]

[7] We assume that Freire's "men" should be read as "human" or "people" — for us his ideas apply as much to women as men.

[8] N.B. Engagement is not the same as "motivation" True motivation can come, surely, only after critical reflection.

[9] Rogerson contrasts an "extensive definition" with an "intensive definition" An "intensive definition", he explains, "seeks to produce in a small number of words an understanding or concept of mathematics" (Rogerson [1986] p 613)

[10] For more detailed information about MISP, readers might wish to consult *Mathematics in Society The Real Way to Apply Mathematics?* or *1985 MISP Report*

[11] See Abraham and Bibby, "Human Agency: The Black Box of Mathematics in the Curriculum" Forthcoming

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Under the present dominance of formalism, one is tempted to paraphrase Kant: the history of mathematics, lacking the guidance of philosophy, has become *blind,* while the philosophy of mathematics, turning its back on the most intriguing phenomena in the history of mathematics, has become *empty*.

Imre Lakatos