COMMUNICATING 'NOTS': A JOURNEY IN MATHEMATICS EDUCATION

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This article is about communicating what is *not* said when something *is* said or what is *not* observed when something *is* observed by an observer. I have come to believe it is powerful and particularly relevant to mathematics education to engage in "seeing 'nots'" (Brown, 2015, p. 194) and to communicate about them.

I have collaborated with Laurinda Brown since 1996, in overlapping phases of work: being a research subject; collaboration within a classroom; her supporting my practitioner research (as a Master's and then PhD student); co-researching on funded projects; co-observing prospective teachers; co-teaching on a teacher education course; co-supervising PhD students. Laurinda describes herself as an observer of nots in all these contexts. Only slowly have I come to realise what it might mean to attend to what is *not* being said by a learner and what this might afford (see Culligan & Wagner, 2018 for a related discussion). And so, in this article, I firstly set out a theoretical position about some features of communication. I then offer three illustrative stories of communicating nots, chosen from across as wide a span of mathematics education as I have been involved with: a mathematics classroom; working with a prospective mathematics teacher; working with in-service mathematics teachers on using video.

Communication

My perspective on communication is influenced by ideas of enactivism (Maturana & Varela, 1987) and the notion of 'distinction' (Spencer-Brown, 1972). Spencer-Brown noted that any observation entails a prior distinction between the observer and the observed. Another way of saying this is that any observation is an observation of an observer (Maturana & Varela, 1987) or that there is a frame (Goffman, 1975) within which any observation takes place. Self-observation is possible, via distinguishing ourselves from ourselves (see Mason's discussion of the two birds of awareness, 1989, p.4). Spencer-Brown's terminology was that an observation takes place within a 'marked space', which implies the existence (and features) of a distinction, namely a boundary and an 'unmarked space'. To indicate something is, by necessity, to ignore something else. In marking something with a label, for instance, I necessarily exclude other things from that label. It is not always easy to notice what is *not* being said or *not* being observed when something is being said or being observed.

In the next two sections, I attempt to draw on enactivist ideas about cognition and living (namely, 'organisation' and 'boundary') and apply them to communication. The sociol-

ogist Niklas Luhmann (2002) theorised about social systems, using Maturana and Varela's (biological) concept of autopoiesis. ('Autopoiesis' literally means self-creation and was the enactivist answer to the question of what characterises living beings.) The debate continues today about whether extending ideas from biology to social systems is valid (Urrestarazu, 2014), and Maturana and Luhmann famously disagreed about this. Luhmann argued that the analogue of the biological processes inside cells that reproduce themselves are the communications and decisions between humans in a social system. He suggested that it is not humans who communicate, but rather communications which communicate (p. 169). While I broadly agree with Luhmann's view on communication, it is not my intention to draw tight parallels between processes of living and communication. Rather, I hope to side-step debates around autopoiesis and social systems by focusing on concepts that can be observed more directly when groups communicate.

Communications have an organisation

Bateson (1972) proposed all communication takes place within frames that define the *kinds* of communication that are expected or allowable:

> the frame is involved in the evaluation of the messages which it contains […] the frame merely assists the mind in understanding the contained messages by reminding the thinker that these messages are mutually relevant and the messages outside the frame may be ignored […] a frame is metacommunicative […] Every metacommunicative or metalinguistic message defines, either explicitly or implicitly, the set of messages about which it communicates. (p. 188)

The notion of frame here has a sense of a container of communications. I propose the 'organisation' of a group of communications to mean the set of relations between those communications that characterise them as exhibiting a certain frame. In Bateson and Goffman's conception, 'frames of talk' remain static while communications shift from frame to frame. The organisation of communication is a slightly different way of conceptualising the ever-present metacommunicative features of the communications within a context. Rather than thinking about communications shifting frames, the organisation of communication within a group evolves over time. Or, more accurate could be to say the organisation 'co-evolves', pointing to how communications within one group exist within (and evolve along with) the web of communications of all other groups.

When a group meets (once or repeatedly over time), their communications have an organisation that can be described, independent of the actors taking part. For example, communications within one teacher's classroom are likely to share particular patterns of organisation, independent of the individual students (see Brown, 2015). This is not to say patterns of communication with different students will be identical, but rather that relations of communications, with an experienced or expert teacher, are likely to have common features over time. For instance, one classic pattern of organisation that has been reported from some classrooms is Initiation–Response–Feedback (IRF) (Sinclair and Coulthard, 1975). I take the IRF pattern as an aspect of the organisation of communication in some educational contexts.

Some communications establish a boundary within an organisation of communications

Relating to communication, I take a 'boundary' to mean a distinction between what is and what is not relevant (or allowed to be said) and/or between who is and who is not allowed to say relevant things at relevant times (of course, a boundary can sometimes be crossed). The creation of boundaries is, I take it, the mechanism through which metacommunicative frames are established. There will be many boundaries that make up a specific framing (and within a particular organisation of communications). My experience is that some communications, within a context, are effective at establishing boundaries. This is not a causal claim and there is no certainty involved, but effectiveness can be observed in retrospect. I offer four instances below.

Bateson (1972, p. 217) relates the story of a mother saying to her son, "Don't you love me any more?" after her son had tried to hug her, she had stiffened, and he had withdrawn from his attempted hug. The son blushed and the mother then said, "Dear, you must not be so easily embarrassed and afraid of your feelings". The mother's words proved highly effective at creating a boundary around the organisation of her communications with her son, such that her ambivalence towards her son could not be mentioned. (Bateson cites the example as one that created schizophrenia in the son.)

A less pathological example might be a mathematics teacher who, in the first lesson with a new class, asks students only questions with right/wrong answers and takes the position of evaluating the correctness of responses (a classic, prolonged IRF sequence where the feedback is explicitly evaluative). The teacher responses, without making anything explicit, are likely to establish a boundary that signal, within the organisation of communication of this classroom, the teacher (and not the students) is the (sole) arbiter of truth.

In my doctoral study (see Coles, 2013), I observed a mathematics teacher who, in the second lesson of the year with a class, set up a task with multiple possible lines of enquiry and used the word 'conjecture' thirty-six times in a twenty-minute discussion. My observations were that a boundary was established relating to the organisation of communications (though not made explicit), that in this classroom the students (not the teacher) were the ones who made conjectures.

A prospective teacher might tell the students in a class not to speak when the teacher is speaking. I have observed many classes where this explicit statement is ineffective at establishing the desired boundary about who speaks when. In general, the boundaries around communication are not made explicit, but rather are enacted (and, potentially, not always consciously). These examples indicate that stating who can speak when and what is relevant within a group's communications is not guaranteed to establish those boundaries and, conversely, that boundaries can be established effectively via communications that do not make that boundary explicit.

The overall argument of this article can be summed up by the following claim: communicating *nots* can act as an effective mechanism for establishing a boundary within an organisation of communications. I provide some evidence for this claim with three stories, below.

A story of stories

One of the first co-authored pieces of writing Laurinda and I worked on was called *The story of Sarah* (Brown & Coles, 1997) and, at the time, we were warned off using the word 'story' in the title by a colleague, in case it was interpreted as indicating we were using fiction (which we were not). We linked 'story' at that time (as I still do) to a notion from Bateson (1979) that a "story is a little knot or complex of that species of connectedness which we call relevance" (p. 12). In other words, it is via story that something comes to be relevant to something else. This image of story is not exclusive to a full narrative, but rather to moments of awareness or connection. Since 1997, narrative approaches to researching education have become more recognised (see Olive Chapman's contribution to this monograph). In creating the stories and narratives that follow, I have focused on generating "brief-but-vivid" (Mason, 1996, pp. 25–26) accounts, where the account *of* (*e.g.* an articulation of observable features of experience) is distinguished from the account *for* (*e.g.* connections, links, explanations, justifications, evaluations) those events (Mason, 2002). These stories were chosen as ones that illustrate the communication of *nots*, across the journey of my own engagement in mathematics education, from the classroom to working with teachers.

Story 1: Mathematics classroom example of communicating a not

The account below is a reconstruction of a familiar lesson start to me, that I would use in school, most often with students of age 12, but also with primary age classes and with adults.

Account of

I draw two shapes (see Figure 1) on the board.

"These are both *eight*-dot shapes. Someone come and draw me another, different eight dot shape". A student comes up and draws a new shape. I "look" for several seconds and then draw next to it ☺ (see Figure 2).

"Thank you, I want another different eight-dot shape." I pass the pen to another student.

The next student draws a circle. "One rule here is that we are only using straight lines. And corners must be on the dots." I get the student to rub out the circle. "So,
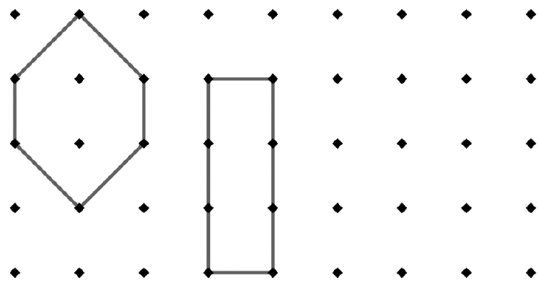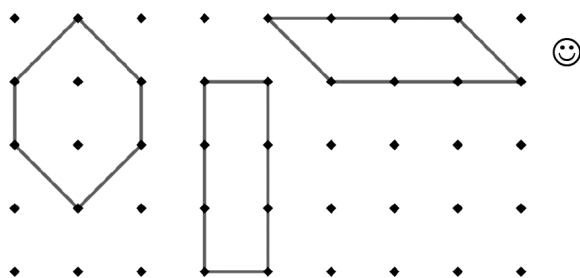
Figure1.   A starting image.

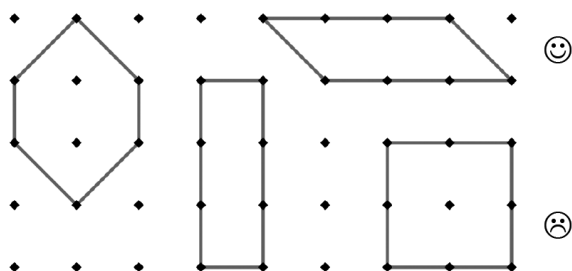*Figure 2.   A different eight-dot shape.*

Figure 3.   A further nine-dot shape.

can someone draw me another, different eight-dot shape?"

A student draws a 2 by 2 square (Figure 3). I draw a ☹. "This is a nine-dot shape. Can someone draw me a different *eight*-dot shape?"

Account for

This phase of offering feedback (with a ☺ or ☹) typically continues until I feel confident most students are able to draw what I mean by an eight-dot shape. At some point, I will ask for an articulation from a student of what defines an eight-dot shape. The task proceeds from there, with students working on the relationships between the number of dots inside a shape, the number of dots on its perimeter, and the area (for a further write up of the task, see Banwell, Saunders and Tahta, 1986; see also Brodie, 2000).

The *not* here is a deliberate setting up of the context and a decision as a teacher that I will attend to when a shape offered is not an eight-dot shape. When a student draws a circle, my feedback is that they are not operating with the distinction around 'allowed' shapes that I need them to use (the circle does not get a face). Drawing a square (Figure 3) receives a ☹, since I am reflecting back that they have chosen something incorrect, but nonetheless from the set of allowable shapes. My feedback sets the context for the different ways in which something is and is not allowed.

There is something perhaps peculiar to mathematics in any problem or theorem having highly specific constraints related to its domain of application. The feedback above supports students acting within the boundaries of considering shapes with straight edges and corners on dots, and in using the definition of 'dotted-ness' within the task. I do not give a definition, but let students know when what they draw does not fit what it means for a shape to be eight-dotted. My feedback occasions the organisation of communication linked to the task to have these desired features.

All students are, relatively soon, 'let in on the rule' as a definition of dotted-ness is made explicit. However, compared with an alternative of, say, a teacher starting off by stating to the class a definition and explaining why the two shapes in Figure 1 are called 'eight-dotted', there is some mystery about the start, something for students to attend to, a connection to be made (allowing the possibility of a small 'Ah-ha!' moment). Moreover, I gain accurate information about the meaning-making of students by being able to observe their productions of shapes.

The focus is on what students do; inviting communication about what they do comes once that doing appears smooth, for most. My noticing of a not, in this instance, is planned and prepared. It is also worth noting that this part of the lesson is in what might be imagined as a 'hosepipe' phase, a structured beginning, leading to shared communications (in this case, about how we label shapes) that allows a later release of diversity and creativity (students typically would then begin exploring patterns in the areas of different kinds of shape and be invited to formulate questions they then work on), but within constraints that mean students hit up against the mathematics (in this case, Pick's Theorem).

Story 2: A prospective mathematics teacher and noticing a not

The incident below (adapted from Brown, Helliwell & Coles, 2018) involved a prospective mathematics teacher who was enrolled on a one-year, post-graduate teacher education course. The prospective teacher had emailed me (as his University tutor) to say he was having some difficulties at his placement school (he was part-way through a twelve-week school practice) and he asked if we could meet. I wrote up this account of soon after the meeting (some details are fictionalised to preserve anonymity).

Account of

We met soon after the email and I invited the prospective teacher to talk to me about the difficulties he was experiencing. This provoked a number of stories of incidents in school. I was aware of listening with a

sense of what I might be able to offer. I suspect I made little comment in between the description of incidents. I have lost the details of these stories except for the one that provoked a response in me. This story concerned an incident with another mathematics teacher in the staff room and what the prospective teacher expressed, as being on the receiving end of a social rudeness. With this, an awareness crystallised: in all the stories of incidents, one thing he was not talking about was the students he was teaching. I expressed this awareness with the suggestion, it was as if his concerns were all centred around his relationships with the other teachers. I advised that he place energy and attention in his relationships with the students he teaches and forget about how he thinks the other teachers are reacting to him. Soon after expressing this awareness, our meeting ended. (Brown, Helliwell & Coles, 2018, p.18)

Account for

This prospective teacher went on to complete the teacher education course successfully, something that was not clear would happen at the time of the conversation. I do not want to suggest any direct causality, but rather to point to this conversation having a feel (to me) that something significant took place. The experience of offering my awareness of a *not* had a quality of intensity for me and, I sensed, for the prospective teacher. Some evidence for the experience having a certain vividness is that, in reflecting back on the course (in conversation with me several months later), he mentioned this incident. What occurred, for me, during this conversation can be seen in three chronological parts: slowly dawning awareness that there was something 'off' about the kind of conversation we were having; a recognition that what was unusual was that there was no mention of students; articulating the awareness of this *not*.

My comment about what was not taking place in conversation afforded a shift in the organisation of communication between the prospective teacher and myself and the creation of a boundary (we talk about students, not staff). In essence, I was suggesting that his worries about other staff were not relevant to the kinds of communication he needed to be having at school. Of course, there might be instances when the substance of a teacher's concerns needs to be given full priority: I judged that not to be the case in this instance. The evidence of his successful completion of the course suggests that the shift (or co-evolution) in the organisation of our communications afforded a shift in the organisation of other communications for that future teacher (perhaps, but I cannot know, with a boundary meaning there was a focus on student learning). The incident has been significant in the subsequent organisation of communications for me also, both in terms of encouraging me to explore my awareness of discomfort about the kinds of communication I am involved in and in terms of my conviction that communicating this discomfort might, in certain contexts, be an important thing to do.

Story 3: Working with mathematics teachers on video and using nots

The story below is taken from Coles (2019) and is in the context of working with teachers as part of a video club in the UK. In a parallel to the distinction between *accounts of* and *accounts for* events, the way of working of this club was to watch a short clip of a lesson and, in discussing it, to insist on a distinction between an account of what took place on the video and an interpretation or judgment. Discussion proceeds with a period of time trying to reconstruct the events, before moving to interpret them. The transcript below was taken from straight after the end of watching the clip of video when the invitation is to only offer observations.

Account of [1]

P   I couldn't stop watching, thinking of you *[P looks at J]* and your independent children *[Alf raises his hand towards P]* and unfortunately all

//the children that weren't paying attention//

Alf   //so, so, so//

J   // Yeah, yeah//

Alf   That's an interpretation. So, at this stage, the invitation is to say what you saw, what you observed *[pause]* so *[pause]* how did it begin?

[3 minutes later]

G   He said that it wouldn't work if your one whole was ten?

J   Yeah, I think he was talking more on the discrete nature of number, he was thinking about things being discrete

Alf   So, try to avoid interpreting what you think he was saying *[Alf laughs]* try and stay with *[pause]* so, what did you hear him say?

Account for

In these transcripts, I can be seen intervening in the discussion to point to a distinction not being made. In both instances, I point out to the speakers that they have offered an interpretation of the events on the video, when the way of working is to begin with a reconstruction, *i.e.,* with what happened, or an account of what was observed. What I am doing, therefore, is sharing my awareness of the distinction—account of versus interpretation—and an awareness this distinction does not seem to be shared with participants. In other words, having set up that we begin work on video by sharing accounts of, if a teacher moves into what I see as an interpretation, I know the organisation of our communications, in that moment, is not what I need it to be. In pointing out to teachers when I hear an interpretation or judgment (*e.g.,* P's comment about children not paying attention), I suggest this is not the kind of comment required at this time. I do not engage in a discussion about whether the children are or are not paying attention. My comment is not about the content of P's initial statement, but rather about the organisation of our communications.

As a facilitator, I take responsibility for imposing a boundary on the initial stages of communication, in the belief that if

teachers are allowed to expand on initial judgments or reactions (*e.g.,* children not paying attention), then little will be gained from discussion, as communications will cycle around existing ways of seeing and acting. As I report in Coles (2019), three instances of making the kind of intervention above, to point to the difference between interpretations and accounts of in the first session of the video club, are sufficient to mean this distinction is not made explicitly again, *i.e.,* there is evidence that the organisation of communications (co-)evolves to the point where discussion during the reconstruction phase remains within the space of accounts of, for the rest of the working of the club over five meetings. At the end of the video club, when I asked participants to reflect on what they had learnt, all of them referred to the distinction between the detail of their observations (accounts of) and their judgments and how this distinction in itself was provoking shifts in the way they listened to students in their classrooms or conducted lesson observations of other teachers in school. There is evidence here, perhaps, of how the organisation of communications in one setting co-evolves with the organisation of communications in others.

Communicating nots and learning

One commonality across the three stories above is that the intervention of reflecting back to others a not (this is *not* an eight-dot shape; you are *not* talking about students; you are *not* offering an account of) brings attention to the organisation of communications within the group. There appear to be some common elements of what happens next, when a *not* is pointed out. A strong sense I have is that the experience (for all parties) is an unusually vivid one; *i.e.,* the experience is often memorable. Where there is some evidence from opportunities for reflection (in Stories 2 and 3), the instances of noticing a not are ones that are mentioned. They are also moments or events that I remember, from among the myriad of interactions in those contexts.

There is some evidence that pointing out a not is an effective way of establishing boundaries within the organisation of communication. In Story 3, just a few instances of noticing a not and making this explicit, are enough to establish a particular way of interacting with video with this group of teachers. Other studies suggest that desired discussion norms around video, when offered to teachers in terms of things they should try to follow, can take a long time to become established (*e.g.,* van Es & Sherin, 2008).

As the teacher in the three stories above, I take responsibility for making a decision about elements of the organisation of communication that (I hope) will be productive for participants. In Story 1, I know from experience of working on Pick's Theorem with different classes (over two decades), that establishing the idea of eight-dot shapes as a way (initially) of organising and planning work on the problem means that students will be forced into considering shapes that are challenging in terms of finding their area (working on 'area' tended to be one purpose for me in offering this task). The arising of complex shapes comes about because organising around eight-dot shapes, as a starting point, inevitably leads to questions of what different kinds of eight-dot shape there are (8 dots on the perimeter, 0 inside; 7 on the perimeter, 1 inside; … ; can you have 3 on the perimeter,

5 inside?). The fact that all students use the notion of dottedness determines certain implications or features of the Theorem will be prominent.

However, there is also a second level at which it is possible to interpret the context and organisation of communication. In setting up that we all work with one definition of dottedness, there is also a determination that will become apparent to students over time, that in working on mathematics, this is the kind of thing mathematicians do. In other words, the consistent application of a definition determines a context of deductive explanations and steps that follow from those definitions and this is a specifically mathematical space. The imposition of a boundary on communications can be interpreted as taking responsibility for students' communications having features of organisation that are recognisably mathematical.

In the case of the prospective teacher, focusing on student learning is one of the principles of the whole teacher education course on which I work (*i.e.,* we do not specify how we want people to teach, but we do offer feedback and expect adaptation in relation to what students are learning, or not learning, in their classroom). A focus on students' learning entails a particular way of engaging in the world that typically means prospective teachers do not get caught up in issues about whether what is offered at university conflicts with what they see in school, or whether teachers at school might have a different philosophical outlook to them. The incidents being related to me, by the prospective teacher in Story 2, sounded like the kinds of things I do not usually hear and, for a while, I felt a discomfort, but could not point to why. Again, there is a sense, as a teacher educator, of having taken responsibility for a decision about a particular boundary within the organisation of communications, relating to a conviction about the kinds of communications that will help prospective teachers learn. This boundary is not usually brought into the conversation, but what occurs is the offer of a not, to occasion the possibility of new kinds of future interactions.

And, lastly, the video club (Story 3) can be interpreted as my having made a prior decision to take responsibility for organising communications via a distinction between account of and interpretations/judgments. This decision is not opened to question and is made on the conviction that it will support productive discussion of video. In other words, the imposition of a boundary that constrains communication (initially) into reconstruction determines particular ways in which participants are able to talk about (and hence, to some extent, experience) a video clip. A sense of the learning that follows comes from the teacher feedback about the significance of the distinction.

Inside–outside

In Story 1, there is the imposition of a rule on students, namely that only communications about eight-dot shapes are allowed. Paradoxically, in order to establish this boundary, there are communications about rules but not about eight-dot shapes (on my part). In Story 2, I aim to impose a boundary, that we will communicate only about student learning; but to do this, I do not talk about student learning, I talk about the distinction between focusing on students and focusing on

staff. In Story 3, I impose a distinction between accounts of and judgment, saying that only accounts of video are allowed. In other words, I do not offer an account of the video, while imposing a boundary that the only allowable communications are accounts of the video (see Coles, 2019).

In all these contexts there is perhaps a feature of the organisation of communications which is that I am the one (as teacher) who comments about this organisation; over time, for instance in my own classroom, students might occasionally take on the role of commenting about the organisation of communications taking place (for example, by saying, "Now might be a good time for a class discussion"). Communicating a not momentarily brings in to focus the context and organisation (of desired communications) with the hope of constraining future communications and occasioning a co-evolution of the organisation of communications. A boundary is offered, in each case, in the belief that the constraint will occasion freedoms and learning in future communications.

Crucially, the imposition of a boundary around communications itself does not do certain things. For instance, in offering to students what is not an eight-dot shape, there is an avoidance of the likelihood of the funnelling of student responses via teacher prompts until a desired response is given (the topaze effect (Brousseau, 1997, p.25)). Rather, communicating only what is not an eight-dot shape means the students are the ones who have to generate the examples, which the teacher can then assess; the examples students offer give insight for the teacher into their awareness. Likewise, in the work with teachers (both prospective and in-service), communications are monitored for whether they are inside or outside a desired organisation.

As facilitator, I do not see my task as being required to judge the usefulness or otherwise of the content of particular comments, in the faith that, so long as communications exhibit particular relations, the participants will be interacting in a way that means they will learn. And, in this learning, they will both be learning about the specifics at hand and also learning (through enacting) a way of being in the world, operating within particular boundaries (as mathematicians, or as teachers of mathematics). In other words, pointing out nots is a mechanism that can occasion learning about the organisation of communication, as well as being effective at shifting that organisation.

I have only alluded, through this writing, to the awarenesses needed to notice nots in order to communicate about them, whether as the teacher or facilitator. In some cases, it is possible to plan and prepare to communicate a particular not (Stories 1 and 3). In other cases (Story 2), communicating a not depends on an awareness in the moment. Experiencing how the organisation of communications can and does shift in some contexts (and sometimes rapidly) has been powerful for me, in terms of working on myself to become aware of opportunities to occasion shifts in other contexts. My own experience is that developing an aware-ness of nots has been the slow work of decades of communication about communication, often with Laurinda. In the articles in this monograph, it is possible to interpret ways that she has supported different individuals. This support is never at the content level (as evidenced by the range of different areas of focus), but rather at the level of the organisation of communications that might be useful; and, my conjecture is, offered through a consistent noticing and communicating of what is not being observed when something is being observed.

Note
[1] // indicates overlapping speech.

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