Mathematics and its social context: a dialogue in the staff room, with historical episodes*

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Introduction

The idea for this lecture arose out of a discussion that I had with Ivan Tafteberg Jakobson, one of the organisers of this conference He suggested that I should talk about the use of mathematics in society and social influences on mathematics. He also mentioned a number of questions that often arise when these topics are discussed among teachers. Thinking about these questions I decided that the best way to approach the subject would be to present a kind of inventory of current opinions and arguments concerning the social context of mathematics, and to discuss some episodes from the history of mathematics which are relevant to these opinions and arguments.

This plan suggested a kind of division of roles: I would relate the episodes, but who would express the opinions and formulate the arguments? This made me decide to use as the setting for my story an imaginary school with imaginary teachers. I should say in advance that I do not teach in a school myself, so part of this setting is pure fantasy But I hope that some of the situations and arguments will be familiar

The lesson

A mathematics teacher is in the classroom, giving a difficult lesson about differentiation and the determination of extreme values towards the end of the lesson one of the pupils asks: "But what's the *use* of all this?"

"Ah," says the teacher, "what I'm teaching you can be *very* useful, in science, in economics, in engineering, etc I'll give you an example from *traffic control*. Say, there is a narrow bit of road along which many cars have to pass. Someone might want to know what is the best speed for these cars. In other words, at what speed should the cars be made to travel so that as many as possible pass through? Should the speed be high? No, because drivers of cars travelling at high speed should keep plenty of space between their car and the car in front, otherwise there'll be a collision if one of the cars suddenly has to stop. We'll call the distance between the cars r, the length of the cars l, the speed v and the number of cars passing per second f (see Figure 1) The police tell us that r should be kept proportional to the square of the velocity, $r \sim v^2$." And so the model is set up, the values of the constants are introduced and the teacher arrives at a formula for f:

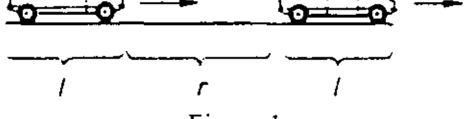

Figure 1

$$f = \frac{8v}{32 + v^2},$$

and draws the graph (see Figure 2).

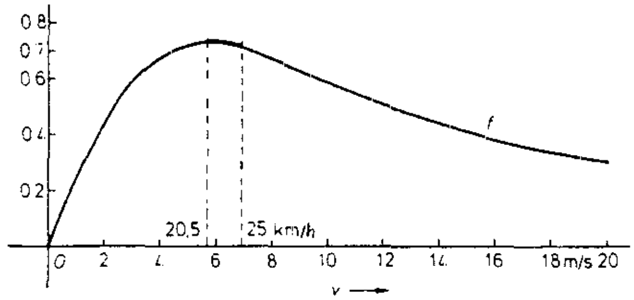

Figure 2

The optimum speed can now be determined by differentiating; it turns out to be about 20 km/h At this point the bell rings, the lesson is over, and the teacher leaves the classroom

In the corridor

The teacher—I shall call him or her A, because there will be more teachers around soon—feels rather pleased It was a nice example he used And he knows that there are many more good examples, indeed whole books of them, written by mathematicians such as Noble, Sharron, Bell or Steur [1] (the traffic example comes from Steur's book). Some of these books have very handy bibliographies. So you can really collect a large number of easy-to-understand applications in order to convince pupils that mathematics *can* be used

But half-way to the staff room A begins to have doubts, for he realizes that all these examples relate to fairly senior and specialized jobs The man who calculated the optimum speed was not an ordinary policeman. How many pupils will get jobs where they have to deal with such applications? And: "What mathematics will the average pupil need in later life?"

At this point A meets a colleague, mathematics teacher B, and tells him what he has been thinking about. "Yes,"

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says B, "I've often asked myself that question too, and I've discovered that very little is known about it." He goes on to say that he has recently been reading the *Cockcroft report* [2] of 1982 In his view that report is the first one to give the results of serious research on the questions of what mathematics is actually used, in jobs, in professions, and in "life " The report contains a section on the "mathematical needs of adult life," the author lists these needs as follows:

- to read numbers and count,
- to tell the time,
- to pay for purchases and give change,
- to understand weights and measures,
- to understand timetables, simple graphs, charts, and carry out necessary calculations connected with these,
- some ability to estimate and approximate,
- confidence that one can do these things.

That isn't much, you would say; but the report is rather pessimistic about the success of mathematics teaching in passing on these skills

In the staff room

B's remarks only increase the doubts that A had already They enter the staff room and talk to colleagues about the problems they've been discussing. They get very mixed reactions.

C, another mathematics teacher, says: "Who says applications are important? Mathematics is a science, regardless of what people use it for. Mathematics is independent of society, it is eternal truth. At school children should learn about 'real mathematics,' it does them good " He goes on to quote Hardy's *A mathematician's apology*:

> The "real" mathematics of the "real" mathematicians, the mathematics of Fermat and Euler and Gauss and Abel and Riemann, is almost wholly "useless".... It is not possible to justify the life of any genuine professional mathematician on the ground of the "utility" of his work [3]

This provokes a comment from D, who teaches classics and history. He says: "What I hate about these "real" mathematicians is that they are so addicted to their beautiful mathematics that they don't want to see any connections with the outside world I don't know whether applications are important or not, but at least there should be some cultural connections. If I want the children to learn something about science and mathematics in Greek and Roman culture I could ask C to talk about Greek mathematics. And they'd hear all about Euclid and Archimedes, but nothing about what mathematics had to do with the rest of Greek culture. So what are the children likely to think? They'll come to the conclusion that mathematics has always been weird and difficult and irrelevant!"

E, another mathematics teacher, disagrees: "Nonsense, mathematics has everything to do with society. Just think, if Greece had not been a society in which production was based on slave labour, there would have been no "beautiful" Greek mathematics at all, no Euclid, no Archimedes, no axiomatic Euclidean geometry with theorems and proofs and all that. The Greeks could do all those things because they had slaves to do the work for them." (He exaggerates and oversimplifies an argument which is used by some historians [4] to explain the emergence in classical Greece of a science of mathematics which was concerned with proof and deductive structure of theory.)

This is too much for C, who says: "Who is the one who's talking nonsense here? Listen! seven is a prime number, isn't it? And the Greeks knew that and they kept slaves; now we have Western democracy and seven is still a prime number And if you go to Russia you'll find a different social system, but they also have seven as a prime number. Mathematics is about truths that are totally independent of society!" (As you can see, C and E get on each other's nerves.)

Finally, there is F, yet another mathematics teacher He raises a new point: "I'm not so sure about this dependence on society, but I do know that talking about the use or applications of mathematics is not enough The real question is: are the applications *good*? These books in which A finds his examples, I've looked at them and, you know, they use examples from all over: the army, the environment, the economy, medicine, etc But they don't discuss whether these applications are socially *acceptable*. I think that mathematicians ought to feel responsible that their subject is used in a good way So the real question is: what about *responsibility* in using mathematics?"

The visitor

Here they stop and look at a visitor who happens to be in the staff room The visitor is a historian of mathematics, that is, he is a story-teller So he says: "Well now, I don't think I can really say much about these very big issues, but I can tell a story which somehow links up with what D and E have just said about mathematics in antiquity " And the visitor starts to tell his first story, or rather his first episode.

First episode: harmony

The slavery argument is controversial, but it does point to the role played by mathematics in ideology and philosophy—which is an example of the social use of mathematics and which the teacher could also study with the pupils. In Greece (as everywhere) ideology and philosophy were closely connected They dealt with major questions such as: What is goodness? What is morally correct behaviour? What makes a good ruler? Philosophy, ordered thinking about questions such as these, was considered to be a suitable training for young men of the ruling class So there was a market for philosophy, and there was an opportunity for philosophers to set up schools. One philosopher who did this was Plato He created a school about 380 BC, which came to be called the *Academy*; it was named after the demi-god Akademos to whom the gardens where Plato held his meetings were dedicated. Tradition has it that above the entrance to the Academy was written "Let no man unversed in geometry enter " Whether true or not, the tradition points to the importance of geometry, that is, of mathematics, in Plato's philosophy and in his teaching

Why should mathematics be important? Because, in the view of Plato and his followers, the study of abstract mathematics is the best training to bring man into contact with "true reality." True reality does not concern the world of natural phenomena that are subject to movement and change but it concerns the world of immaterial, eternal ideas and forms, the highest idea being that of goodness

Geometrical forms were also important in Plato's understanding of the universe The regular polyhedra, for instance, were connected with the elements from which, according to Plato, God had made the universe; the tetrahedron was the form of the element fire, the octahedron that of air, the icosahedron that of water, and the cube that of earth. On the subject of the fifth solid, the dodecahedron, Plato only remarks that God used it "for arranging the constellations of the whole heaven." [5] Later Greek thinkers supposed the existence of a fifth element, the ether, from which the heavenly bodies were made; this element corresponded to the dodecahedron.

The mathematical theory of these five solids, later called "Platonic solids," was already well developed in Plato's time. We find it expounded in the thirteenth and last book of the *Elements* of Euclid (ca. 300 BC). No text has come down to us in which Euclid explained why he structured the *Elements* in the way he did, but some later Greek writers took the view that all the theories in the book were preparatory to an understanding of the five Platonic solids. This view was stressed particularly by Proclus (410-485 A.D.), one of the most important commentators on the *Elements*. Proclus belonged to a school of philosophy that has come to be called Neo-platonism, a mixture of Plato's philosophy, religion, mysticism, and very clear and precise argument

Mathematics, according to Neo-platonist ideas, studies entities (numbers, geometric figures) that are not themselves the highest form of reality, or the true ideas or forms, but they come very near to being so Proclus says: mathematics is "in the entrance hall of the true forms " [6] Thus mathematics offers the best training for the contemplation of the true forms; it is part of a practice to arrive at a clear and direct vision of truth

Proclus' ideas, and Neo-platonism in general, had an important influence on the development of science in the Renaissance and in the seventeenth century. There were three traditions in that period which contributed each in its own way to the "scientific revolution," that is, to the emergence of a natural science which is essentially the science we know now One was the mechanical tradition, which viewed the world basically as a machine, and tried to understand the workings of nature as effectuated by very complicated mechanisms. The second was the organistic tradition, for which the world was a large organism and nature consisted of many separate organisms each with its own purpose and function. Thirdly there was the magical tradition, which saw the world as a work of art, created according to hidden laws of harmony and beauty. These laws were mathematical, and so mathematics, in this tradition, was considered as the key to understanding the universe. [7] Neo-platonism, with its stress on mathematics, fitted well into this tradition, and the great things that Neo-platonists expected from mathematics led not only to lively interest in such things as number-mysticism, numerology, cryptography, but also to the study of theories of harmony and proportion in music and astronomy.

Many mathematicians were inspired by these ideas. This is especially clear in the work of Johann Kepler (1571-1631) His mathematics is largely inspired by the search for harmony, harmony in mathematics (as in the Platonic solids), harmony in music and harmony in the structure of the universe. His Neo-platonic interest is best illustrated by his famous hypothesis about the relative distances of the planets from the sun Convinced that there whould be a hidden harmony in these seemingly random distances, he considered a "nesting" of planetary spheres and Platonic solids (see Figure 3) Successive spheres are inscribed and circumscribed on the solids. Choosing the order as follows: sphere of Mercury—octahedron—Venus—icosahedron—Earth—dodecahedron—Mars—tetrahedron—Jupiter—cube—Saturn, he found a good approximation to the proportions as they were then known. [8]

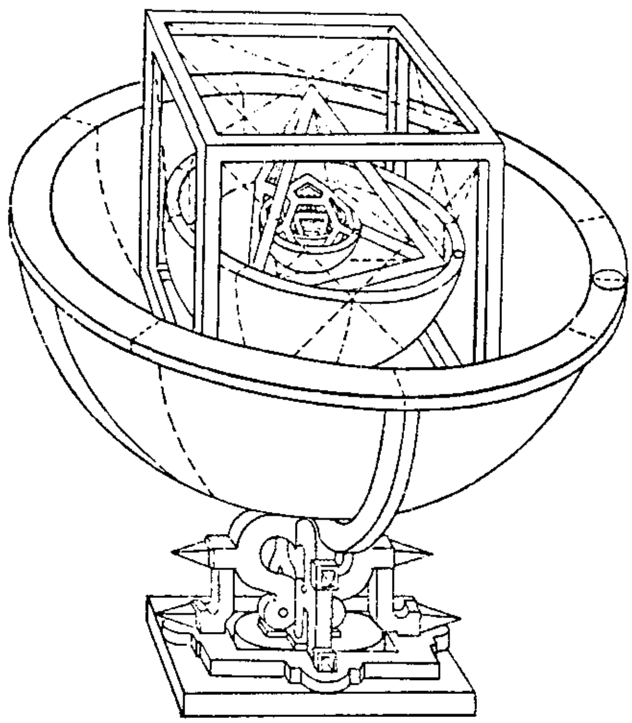

Figure 3

Other mathematicians were also inspired by these ideas. So here we have an example of a link between mathematics and the great movements in the history of philosophy and of culture in general. In the case of Neo-platonism and the magical tradition, the link had a considerable influence upon the development of mathematics; great things were expected from mathematics. This suggested new directions for research and gave the subject greater prestige.

Reaction: the economic context

Here the visitor ends his first episode. There is an immediate reaction from E, who says: "This is not what I meant at all. The essential thing is the economic context, the ways of production; all cultural and philosophical movements and all ideologies are derivatives of the economic context and can only be understood in terms of changes in the production process. Take for instance the scientific revolution which you mentioned..."

And E explains that the sixteenth and seventeenth centuries saw the rise of merchant capitalism which explains the emergence of science. What were the problems in early merchant capitalism? They were: transport, especially ships, on canals, rivers and at sea, industry, in particular mining, and way. Closely connected with these there were scientific problems: in hydrostatics the stability of floating bodies (ships), the construction of locks and sluices and the construction of pumps to remove water from mines; in hydrodynamics the resistance of ships when moving in water; in astronomy and navigational science the problem of determining a ship's position at sea (especially the geographical longitude); in mechanics the construction of machines and the trajectories of projectiles fired from guns; in aerodynamics air resistance in the flight of projectiles and the flow of air through mine-shafts. E points out that all the subjects which Newton treats in his famous *Principia* of 1687, the book that so to speak consolidated the results of the scientific revolution in the science of mechanics, have something to do with these problems. Newton deals with motion in a resisting medium, with hydrostatics, with mechanics and with celestial mechanics. The fact that Newton's *Principia* was so obviously rooted in society and economics—and E here concludes his argument—shows that science, and hence also mathematics, is strongly determined by economic factors. (E here paraphrases the argument of B. Hessen in a famous article, "The social and economic roots of Newton's *Principia*." Ever since it was presented (as a lecture by one of the Soviet delegates to the second international congress on the history of science and technology, London 1931), the article has been a constant source of inspiration, and a constant cause of dissent and debate.)

Second episode: ballistics

"One of the things you mentioned," says the visitor after E's reaction, "was trajectories of projectiles. That is part of the science of ballistics and the story of ballistics is, I think, relevant to the points you have just raised." And he goes on to relate a second episode.

The first studies on the paths of projectiles fired from guns date from the 16th century but one can safely say that ballistics (or rather exterior ballistics, i.e. what happens to the projectile outside the gun) became a theory only after Galileo's discovery (published 1638) that projectiles describe parabolas. The theory was worked out; it provided tables and instruments by which the range of the projectile could be calculated from the angle of elevation and the range of one test shot. The theory became a teachable subject called "parabolic ballistics;" it was taught in military schools in the 17th and 18th centuries. Throughout that time it was of virtually no use to the practice of artillery and all artillery men knew this. The reasons were obvious: the theory took no account of air resistance and assumed that in a series of shots from the same gun the initial velocity would be constant. Neither assumption held in practice.

By the 17th century scientists were already turning their attention to the influence of air resistance on the path of projectiles. Huygens, Newton, and others derived differential equations for the trajectories on the basis of various suppositions about the relation of the resistance to the velocity of the projectiles. The most fruitful supposition (in that it led to differential equations that were solvable and interesting) was to assume the resistance to be proportional to the square of the velocity. Thus a new theory called "quadratic ballistics" was developed; it became a teachable subject at the end of the eighteenth century. This theory also provided tables. It was taught to military engineers and artillery technicians. It still was of little or no practical use because the guns, the projectiles and the powder that were used were not standardised in such a way that the initial velocity of the projectile could be considered constant in a series of shots. By this time, the second half of the eighteenth century, the top mathematicians had lost interest in the subject. The theory was later worked out mainly within the army by the artillery engineers themselves.

During the nineteenth century ballistics theory gradually became useful for artillery in the field. Several developments made this possible. First of all, bullet-shaped projectiles were introduced that were fired from guns with spiral grooves in their barrels. The motion of such projectiles is much more stable and therefore more predictable. Moreover, the projectiles and guns were standardised so that the initial velocity of the projectile was reasonably constant. It is noteworthy that this process of standardisation of the material (one might call it mathematisation of the material) was as necessary for the success of the theory as the improvement of the theory itself. Experimental research on air resistance led to a better understanding of resistance as a function of velocity. Thus more accurate differential equations could be set up and after some readjustment these turned out to be integrable. Thus tables could be calculated and by the end of the century these proved to be fairly reliable, at least for projectiles with rather flat trajectories.

World War I (1914-1918) made new demands on ballistic theory; there were the long-distance guns (with highly curved trajectories) and there was ground-to-air artillery. In both cases the density of the air along the trajectory could no longer be considered constant and this led to complications in the differential equations, which meant that they were no longer directly solvable. Now the compilers of tables had to resort to numerical integration. By then this was a well known method for dealing with otherwise intractable differential equations; it had been developed in astronomy. The disadvantage of the method was that it was very time-consuming. Still, it was the only possible way and so, in the period between the world wars, ballistic tables were produced in what can be called ballistic computational laboratories, where groups of calculators, with help of simple adding and multiplication machines, produced the entries in the ballistic tables The need for the automation of this calculating process was keenly felt and this led to the development of the electronic computer. Indeed the two most important motivating forces behind the construction of the first computers in the U S A during World War II were the need to automate ballistic computation and the need to perform scientific calculations in connection with the development of the atomic bomb

So much for the ballistics episode, which is a clear example of the way in which mathematics is interrelated with (at least one part of) society. But in which direction does the "influence" operate? In the early stages one can speak of inspiration; mathematicians were inspired by the practice of artillery to develop theories of ballistics. But there was hardly any influence in the other direction: the theories were of no use in practice This was, indeed, a very common feature of mathematical physics in the 17th and 18th centuries: however brilliant, deep and theoretically fruitful the theories (of mechanics, hydro-mechanics, electricity, heat, etc) were, they could hardly be applied in practice. The situation is well illustrated in a quotation from a pamphlet published by J. Arbuthnot in 1701:

> The great objection that is made against the necessity of mathematics in the … great affairs of navigation, the military art, etc. is that we see those affairs carry'd out and managed by those who are not great mathematicians: as seamen, engineers, surveyors, gaugers, clock-makers, glass-grinders etc , and that the mathematicians are commonly speculative, retir'd, studious men, that are not for an active life and business, but content themselves to sit in their studies and pore over a scheme or calculation " [10]

This non-effectiveness of the early theory in practice is one of the main obstacles to accepting Hessen's picture of the development of science; it seems difficult to rely exclusively on economic explanations in a case where science could develop for such a long time without evident practical results.

It is noteworthy that in the case of ballistics, the theory became effective only after a period of rather strong antithesis between theory and practice The synthesis which brought success came about in the nineteenth century when theorists were working within artillery and when guns and ammunition were standardised and thus adjusted to the requirements of theory

The episode may serve as a reminder that interrelations between theory and practice, between pure mathematics and applications, and between theoretical development and social utility, can be very complex and many-sided. [11]

Reaction: irrelevance for core-mathematics

Teacher E does not immediately react to this episode, but F is rather upset and says to the visitor: "Now you are doing the same as those writers of books on applications, you are avoiding the most important issue! Ballistics, war, the atomic bomb, and all you say is that interrelations are complex! What about the responsibility of the scientists involved? That's what I'd like to hear discussed!"

F's reaction, however, is brushed aside by C who says that the episode has not convinced him at all that mathematics has anything to do with society: "You've been talking about some uses of mathematics, not about mathematics itself, core-mathematics, mathematics as a science. I still don't see how, for instance, integration is different whether you use it for ballistics, for astronomy or for analytic number theory In all cases it is the same integration, a purely mathematical operation independent of what you can use it for, and we as mathematicians should restrict ourselves to that mathematics "

Third episode: pure mathematics in the nineteenth century

The visitor says: "I take it that you are referring to the Riemann integral, or the Lebesgue integral, well defined with ϵ-δ-precision; integration as part of pure analysis in which the real numbers **R** are introduced arithmetically with the same precision, namely constructed from the natural numbers **N**, through the integers **Z** and the rationals **Q**, by means of sets, equivalence relations and Cauchy sequences or Dedekind cuts." C agrees, and the visitor remarks that that style of pure analysis originated in the nineteenth century, and that there is a story to tell about that development which has some relevance to the discussion The story can best be introduced with a quotation from the booklet in which Dedekind explained the construction of the real numbers by means of Dedekind cuts The booklet, *Stetigkeit und irrationale Zahlen* was published in 1872. In the introduction Dedekind tells why he came to adopt this new approach:

> My attention was first directed toward the considerations which form the subject of this pamphlet in the autumn of 1858. As professor in the Polytechnic School in Zürich I found myself for the first time obliged to lecture upon the elements of the differential calculus and felt more keenly than ever before the lack of a really scientific (*wissenschaftlich*) foundation for arithmetic. In discussing the notion of the approach of a variable magnitude to a fixed limiting value, and especially in proving the theorem that every magnitude which grows continually, but not beyond all limits, must necessarily approach a limiting value, I had recourse to geometric evidences.

Such geometric evidence is didactically useful, Dedekind continues, but it cannot serve as a "wissenschaftlich" introduction to analysis.

> For myself this feeling of dissatisfaction was so overpowering that I made the fixed resolve to keep meditating on the question till I should find a purely arithmetic and perfectly rigorous foundation for the principles of infinitesimal analysis " [12]

The theorem to which Dedekind refers is now usually stated in terms of functions: a bounded increasing function has a limit

There are two clues in the quotation which I want to take up. The first is the teaching situation. Around 1800 calculus and analysis became a teaching subject for comparatively large groups of students at institutions of high education, universities and technical universities. Dedekind describes an experience that other teachers of analysis in that period had also: teaching a subject brings extra requirements for the foundation of the subject; it must be possible to present the principles, the fundamental concepts and the arguments of the subject in a convincing way to students. Indeed the new movement in the nineteenth century towards a rigorous foundation of analysis was closely linked with teaching. Cauchy presented the fundamental concepts (differentiation, integration) as based on the concept of *limit*, in textbooks for students at the Paris *Ecole Polytechnique*. And the further development towards rigour in analysis, with precise proofs in ϵ-δ-style, took place primarily in textbooks written for university courses. Dedekind's words, therefore, illustrate how strongly a new social situation of a mathematical subject can influence its concepts and its style.

The second clue in the quotation is the term "wissenschaftlich," "scientific." Dedekind stated that mathematics should be "wissenschaftlich." This meant that it should be studied as a pure, autonomous science, as search for knowledge per se, and not by reason of its usefulness or applicability. The fundamental criteria are truth, elegance and generality. Proofs by recourse to geometric evidence are not strictly convincing, they should therefore be replaced by logically impeccable mathematical proofs.

For Dedekind, and for most German university mathematicians of his time, it was self-evident that mathematics as studied and taught at universities should be "wissenschaftlich" in the above sense. But historically speaking this was a recent conviction, one that developed in Germany in the first half of the nineteenth century. Indeed, why should teaching and research at universities be "wissenschaftlich?" As it so happened, in Germany in the first half of the nineteenth century there was a combination of social and economic forces that proved very favourable for the scientific style at universities. This is a fascinating chapter in the social history of science; the main lines of which are as follows [13].

The period around 1800 witnessed the French Revolution and the beginning of the industrial revolution, both of which affected Germany. The wars against France and especially the defeat of German forces against Napoleon, strengthened German national feeling and gave Germans a pride in their own culture. These ideas became combined in a cultural movement known as "New Humanism" ("Neu-Humanismus") which stressed the value of philosophy, humanist studies and science as they had developed in Germany. A free and independent study of "Wissenschaft" was seen as the best way to cultivate the mind, preserve the morals and develop the personality.

At the same time the beginning of industrialisation changed the power structure and political thinking in Germany. The rising class, the liberal bourgeoisie, advocated free enterprise and required the state to provide adequate technical education suitable for the growing industries.

So, from the New Humanist and from the liberal side there came different demands, the one for pure and scientific education, the other for utilitarian and technical education. In Germany the educational system divided along these two styles. On the one hand there were the lower and higher Technical Schools which provided utilitarian teaching and trained technicians and engineers. And on the other hand there were the Gymnasia and Universities, where pure science was taught to provide students with a general education in the New Humanist style. The government actively encouraged the universities to become institutions of pure "wissenschaftlich" research and teaching, sometimes against the wish of the universities themselves.

The new universities were very favourable for the development of the pure style of mathematics which is now so familiar to us: analysis in the strict ϵ-δ-style, and algebra as an axiomatic study of structures. This is certainly core-mathematics, pure mathematics, but the episode shows that even for this type of mathematics there was a social context that greatly influenced its development.

Reaction: the mathematical concepts themselves

G is not convinced. "You've been talking only about circumstances," he says, "but that does not tell us much. Of course if you create more positions for pure mathematicians you get more pure mathematics, but that is not the point. I claim that circumstances do not touch the core of mathematics. In favourable circumstances it takes less time to find the true foundations of analysis or the true concept of integral, and if circumstances are not favourable it takes longer. But ultimately the same foundations and concept will be found; they themselves are not affected by circumstances, social or otherwise.

Fourth episode: correlation

The visitor says: "Yes, now you raise one of the deepest questions: Are the very concepts of mathematics subject to social influences? This question has recently been raised explicitly by a historian of mathematics, and as a fourth episode I shall say something about his argument." The visitor explains that he is referring to recent work by Donald MacKenzie [14] of Edinburgh University, who has studied the beginnings of mathematical statistics in the period around 1900.

Mathematical statistics is the, now somewhat old-fashioned, name for the kind of statistics that involves more than just collecting numbers and calculating percentages and averages. Mathematical statistics can also calculate such things as regression and correlation between statistical variables, and it can draw conclusions about large populations on the basis of samples. This kind of statistics was developed around 1900 in England by combining the older statistical techniques with the mathematical theory of probability.

The great names in the creation of mathematical statistics were Francis Galton (1822-1911), Karl Pearson (1857-1936) and Ronald A. Fisher (1890-1962). All three did their research in statistics in close connection with their interest in society, in particular in what was called eugenics. The background of these ideas was nineteenth-century English society. England was a rich, industrial and imperialist country, with a comparatively small ruling class and a large industrial working class living in very poor circumstances.

The ruling class consisted of the landed nobility and the high professions consisting of doctors, lawyers and others who had enjoyed an advanced professional education.

During that period exciting new scientific ideas arose, particularly those of Darwin. Darwinism explained the order in the living nature, the evolution of species through natural selection and survival of the fittest. Some people also took over these ideas and applied them to human society. This was called "social Darwinism;" it was mainly used to propagate measures to keep British society strong and stable. It was thought that strength and stability were best protected by ensuring that the socially "fit" (namely the ruling classes) kept the characteristics that made them "fit" (to rule). And these characteristics could be kept, according to social Darwinism, by guiding the natural inheritance of these characteristics within the class. Eugenics was the term used for such genetic guiding policies. Many such policies were proposed, ranging from tax rules that would induce the upper class to produce children while discouraging the working class to do so, to forced sterilisation of the "unfit." One can speak of a "eugenics movement" in the period from about 1880 till the second World War, which was quite influential in social thinking, especially during the beginning of that period.

The term "eugenics" was coined by Francis Galton. He was strongly convinced of the value of eugenics, but, being a positive scientist, he thought that the arguments should be proved scientifically by experiment and measurement. Thus he started an extensive research programme to measure and prove heredity of characteristics. In the course of that research he collected masses of data about the characteristics of parents and their children. The question he now had to answer was how strongly the occurrence of characteristics in children was correlated with the occurrence in their parents. Here Galton was confronted with a new kind of problem; there was no mathematical theory of correlation which he could apply. So he worked out a theory, introduced a certain formula for the "coefficient of correlation," and applied it in his research. He published his new theory on correlation and regression in 1889 in his book *Natural inheritance*.

Pearson later adopted Galton's approach and developed it further: through his teaching and writing the theory became generally known. Pearson, like Galton, considered his contributions to statistics always in connection with the study of heredity and evolution.

Thus the concept of correlation and the pertaining mathematical theory emerged in a very complex interplay between social theories, scientific endeavour and mathematical research. MacKenzie has carefully documented this development and he uses it to argue that mathematical concepts and techniques themselves are influenced by social factors and consequently are not ideologically neutral. His argument can be summarized as follows: Galton chose a certain mathematical formula for the coefficient of correlation, thus mathematising and fixing the concept of correlation in one particular way. His choice of formula is natural and evident in connection with inheritance problems. It is not so natural when related to other situations where certain correlations occur; in these cases alternative formulas would be preferable. In fact such formulas were suggested in the period shortly after Galton, but in the end Galton's formula became generally accepted. Statistical theory, so to speak, canonized the concept of correlation which that formula expressed. MacKenzie claims that this course of events "demonstrates the influence of eugenics on the development of statistical theory as a system of knowledge." [15] In other words, he maintains that social influences on mathematics go deeper than the use of techniques or the rate of development of the theories, and indeed affect the core of mathematics itself as a system of knowledge. MacKenzie's work illustrates that the question which G raised is taken seriously by some historians of mathematics and that, although it certainly cannot be considered settled, some evidence for a positive answer is being put forward.

Parting

Now the bell rings, the classes are waiting, the teachers have to leave the staff room, there is no time for further reactions. G says in passing that the last story has not convinced him. D—what he thinks I do not know, I suppose he is still wondering about mathematicians. E is still convinced of his own point of view. F is very disappointed; why was the problem of responsibility avoided? The visitor has only a feeble excuse. There was no time. No time because he got caught up in one particular problem, namely the question of the influence of society upon mathematics. All episodes were about that, they gave examples of how the social context influenced mathematics, the direction of research in mathematics (the study of the Platonic solids, the development of computing), the style of mathematics (the rigorous style of nineteenth century pure mathematics) and the mathematical concepts themselves (MacKenzie's argument). That programme was full enough for one discussion in the staff room. But there is of course much more to be said about the social aspects of mathematics. To illustrate this the visitor produces a list of books and articles on the social function of mathematics [16] and he shows F some items which may be of interest in connection with the question of responsibility, notably Heims' double-biography of von Neumann and Wiener, two mathematicians who were intensely confronted with questions of responsibility in their professional life.

Leaving the room teacher A puts a final question to the visitor. He says: "You've only been telling stories. But isn't history more than a collection of stories, isn't it a science? Don't historians find answers to the questions we have been discussing? Don't they find *laws* about the development of mathematics?"

And so the historian is forced out of his safe position as story-teller. And at this point I think I should also leave my safe position with my imaginary school and teachers A, B, C, etc., and answer that question directly. My answer is: No, history does not produce laws. Some historians sometimes write down laws they think they have recognized, but they are always very debatable (if not trivial or vacuous). The reason why history does not produce laws is that the questions to which these laws should be the answers are too complicated. They are the questions such as those that were discussed in the staff room So you might say: what is the point of asking these questions if they cannot be answered? I think they are very useful questions; they are inspiring, they help to give direction to historical research (they suggest where one should look for stories) and they link the work of the historian of mathematics with an important debate: the debate on the social function of mathematics. That debate is part of a very necessary reflection of mathematicians about their work. The debate also goes on among pupils in schools (perhaps the question "what's the use of mathematics?" is unanswerable, but it is very crucial) and among teachers. I think the debate is worthwhile. And I have done my best to show how the history of mathematics can help people to decide what position they hold in that debate.

Notes

1. The books on applications in teaching mathematics which A has in mind are:
   Noble, B. *Application of undergraduate mathematics in engineering* (Mathematical Association of America) n p. 1976. Sharron, S. and Reys, R. *Applications in school mathematics, 1979 yearbook* (National Council of Teachers of Mathematics) Reston (Virginia) 1979 [contains very extensive and handy bibliography]. Bell, M. et al *A sourcebook of applications of school mathematics.* (N C T M.) Reston (Virginia) 1980 Steur, H. *Levende wiskunde, toepassingen geordend naar wiskundig onderwerp* Culemborg 1980 The traffic example is on pages 184-185 of Steur's book Compare also: Griffiths, H.B. and Howson, A.G. *Mathematics: society and curricula* Cambridge 1974
2. [Cockcroft, W.H.] *Mathematics counts, report of the committee of inquiry into the teaching of mathematics in schools, under the chairmanship of Dr. W.H. Cockcroft* London (H.M.S.O.) 1982 See the section on the "mathematical needs of adult life", pp. 5-11
3. Hardy, G H *A mathematician's apology* (foreword by C P Snow) Cambridge 1967 (first published 1940); pp 119-120
4. See, for instance, Wussing, H. *Mathematik in der Antike. Mathematik in der Periode der Sklavenhaltergesellschaft,* Leipzig 1962, pp. 53-56
5. Plato *Timaeus* 55 (tr. H.D.P Lee) (Penguin) 1965; p. 75
6. Proclus *A commentary on the first book of Euclid's Elements* (tr. intr. G.R. Morrow) Princeton 1970; p.4
7. On the three traditions, compare for instance: Kearney, H. *Science and change 1500-1700* London 1971
8. Kepler published his findings in *Mysterium Cosmographicum* (1596), from which the picture of the nested orbs and polyhedra is also taken (figure 3); the picture is often reproduced in books on history of science
9. Hessen, B. The social and economic roots of Newton's 'Principia' In: P.G. Werskey (ed.) *Science at the crossroads* London 1971 (first publ. 1931) pp.149-212
10. The passage is quoted in Taylor, E.G.R. *The mathematical practitioners of Tudor and Stuart England,* Cambridge 1954; p 3
11. On ballistics and practical mathematics in general, compare: Bos, H.J.M. Was lehren uns historische Beispiele über Mathematik und Gesellschaft? *Zentralblatt für Didaktik der Mathematik 10* (1978), and Schneider, I. Die mathematischen Praktiker im See-Vermessungs- und Wehrwesen vom 15. bis 19. Jahrhundert. *Technikgeschichte 37* (1970); pp. 210-242
12. Dedekind, R. *Stetigkeit und irrationale Zahlen* Braunschweig 1872; in *Gesammelte Mathematische Werke* (ed. H Fricke et al, Braunschweig 1930-32 *3*: pp. 315-334 I quote from the English edition: Dedekind, R. *Essays on the theory of numbers* (tr. W W Beman, first published 1901) (Dover reprint) 1963; pp 1-2
13. H. Mehrtens has given a more detailed exposition in a hitherto unpublished article: Mehrtens, H Mathematik als Universitäts-wissenschaft - zur Herausbildung und gesellschaftlichen Funktion der "reinen Mathematik", im 19 Jahrhundert (text of a lecture 6-1-1976). (Copies obtainable from the author or from me - H.B.)
14. Mackenzie, D. Eugenics and the rise of mathematical statistics in Britain. In: J. Irvine et al (eds.) *Demystifying social statistics* London 1979; pp. 39-50, and Mackenzie, D. *Statistics in Britain 1865-1930 the social construction of scientific knowledge* Edinburgh 1981
15. Mackenzie "Eugenics ..." (see note 14); p. 47

Appendix

Some books and articles on the social function of mathematics

A. *Books*

Davis, P J. and Hersh, R *The mathematical experience* Boston 1981 [Very readable survey of "external" aspects of mathematics]

Steen, L A (ed.) *Mathematics today twelve informal essays* Berlin 1978. [The essays review recent major developments in mathematics]

Forester, T (ed.) *The micro-electronics revolution* Oxford 1980 [Collection of articles on micro-electronics and its economic and social consequences]

Booss, B. and Krickeberg, H (eds) *Mathematisierung der Einzelwissenschaften* Basel 1976 [Articles on the use of mathematics in natural and social sciences; contains extensive bibliographies]

Irvine, J et al (eds.) *Demystifying social statistics* London 1979 [Collection of critical articles on the social use of statistics]

Fox, L.H. et al (eds) *Women and the mathematical mystique* Baltimore 1980 [Lectures presented during a conference on women and mathematics]

Elton, M (ed) *Mathematics development in the third world countries* Amsterdam 1979 [Proceedings of a conference]

Otte, M (ed.) *Mathematiker über die Mathematik* Berlin 1974 [Collection of essays by mathematicians about mathematics]

Hardy, G H *A mathematician's apology* (with foreword by by C.P. Snow] Cambridge 1967 (first published 1940) [Perhaps the most famous formulation of a pure mathematician's vision of mathematics]

Heims, S J *John von Neumann and Norbert Wiener, from mathematics to the technologies of life and death* Cambridge (Mass.) 1980 [Biography of von Neuman and Wiener, who were both deeply involved in the development of mathematical techniques that had great effects within society]

B *Articles*

Struik, D J. On the sociology of mathematics. *Science and society 6* (1942) pp. 58-70 [Short historical survey of social factors that have influenced mathematics]

Bos, H.J.M and Mehrtens, H. The interactions of mathematics and society in history; some exploratory remarks. *Historia Mathematica 4* (1977) pp 7-30 [A survey, discussing the different social forms of mathematics; contains extensive bibliography]

Schneider, I. Der Einfluss der Praxis auf die Entwicklung der Mathematik vom 17. bis 19. Jahrhundert *Zentralblatt für Didaktik der Mathematik 9* (1977) pp. 195-205 [Detailed survey of developments in mathematics in connection with requirements of practice]

Gross, H. Das sich wandelnde Verhältnis zwischen Mathematik und Produktion In: P Plath et al (eds.) *Theorie und Labor* Köln 1978; pp. 226-269 [A historical survey, containing much information about mathematics in industrial production up to recent times]

Weisglass, J Higher mathematical education in the People's Republic of China. *Amer Math. Monthly 86* (1979) pp. 440-447 [Report of a visit to Chinese mathematical institutions; gives some information on the changes after Mao's death]