the child's "potential". "Why should we teach mathematics and to whom?" is thus, in my opinion, one of the researcher's "must-ask" queries. If we want to make sure we do not perpetuate orders that we would rather like to change, we have to be always watchful of how mathematical discourse is used.

Notes

[1] "Anything goes" is the slogan which, in the eyes of objectors, encapsulates the postmodernist stance. The critics derive it from the postmodernist rejection of the idea of "absolute truth". But "anything goes" does not follow from "no story is true in an absolute manner", just as the claim that every dress is equally good for me does not ensue from the fact that there is no dress that fits me in an "absolute" fashion.

[2] Some of these stories are offered in the double special issue (volumes 51 and 52) of the *Journal for Educational Research* titled "Developing mathematical discourse—Some insights from communicational research", published in March 2012.

[3] Storytelling is not an exclusive activity of researchers—it is something everybody does. What makes our research storytelling distinct is our being explicit and meticulous about our tools, and thus also about our assumptions. This, of course, makes us more accountable for our narratives than any lay storyteller

Explosive problems in mathematics education

OLE SKOVSMOSE

Dear Richard,

Thank you for drawing my attention to David Wheeler's collection of problems. It has been fascinating to reconsider them.

Wheeler did ask for problems "whose solution would be likely to advance substantially our knowledge of mathematics education". As examples I consider the following three, formulated by Geoffrey Howson, John Mason and Alan Bishop:

Howson:	It would help me to have a better theoretical framework within which to consider/study/investigate mathematics education [1]
Mason:	Is mathematics really essential? [2]
Bishop:	How is mathematical meaning shared? [3]

These formulations are short and clear. Nevertheless, I have the feeling that it is impossible to reach any solution to be generally agreed upon. Howson might have had the same feeling, as he modified the task by adding: "Here I am for the moment not asking for a theory, just a framework which might help to develop one." But even with such modification, his problem appears gigantic.

Naturally, I came to think of David Hilbert's 23 problems presented in 1900, which, since then, have provided an ongoing challenge to mathematics research. I looked at a web page [4] where the problems are nicely classified: some were solved in a particular year, some are partially solved, one is too vague in its formulation to be solved, while three remain unsolved. Progress in mathematics has, at least partially, been related to the resolution of these problems.

Hilbert's famous second problem was: "Prove that the axioms of arithmetic are consistent." Gödel's second incompleteness theorem, however, showed that no proof of this consistency can be carried out within arithmetic itself. This was a most alarming result considering Hilbert's metamathematical aspirations. When I was young, I spent half a year studying Gödel's proof. I was fascinated by the perspectives it provided on mathematics.

While Hilbert's problems concern mathematics, Wheeler's problems concern mathematics education. Linguistically speaking there is not much difference between "mathematics" and "mathematics education"; epistemically speaking, there is. The difference becomes clear when we consider the difference in nature between Hilbert and Wheeler's problems. The formulation of Hilbert's problems seems be based on some shared conceptions of what a solution could mean, while what to consider a solution to a Wheeler problem appears to be ambiguous.

This brings me to consider the notion of explosive concepts. By this I understand a concept that can be defined only through concepts just as open and vague as itself. As an example, one can think of "democracy". In order to clarify what democracy means, one may consider the meaning of, for instance, equity, justice and inclusion. However, these notions are no easier to define than the notion of democracy. Thus a definition of an explosive concept does not "narrow down" its possible meanings. Instead, it opens into new landscapes of possible meanings.

Hilbert's problems are formulated through concepts that one could call *solid*, like: integer, real number, axiom, function, arithmetic, consistency, polynomial, group, algorithm, prime number, *etc*. Naturally, the definition, of such concepts are not fixed; they develop during history. What might be considered a proper definition in one period might appear inadequate later. Think of the notion of function, for example, which has been part of a fascinating conceptual development. But, although "function" demonstrates a historicity, it is not explosive. Its definitions and redefinitions are based on notions generally accepted in the mathematical community at a given time. In this sense, we can consider "function" a solid concept. (Let me just add that the notion of a proof being "finite", as used by Hilbert in formulating what methods could be allowed in metamathematics, can hardly be called solid. Let this, then, indicate that explosions might also occur in mathematics.)

Contrary to Hilbert's problems, many of Wheeler's, including those presented by Howson, Mason, and Bishop, are formulated through explosive concepts. In order to define a "theory" one needs to address broader questions about knowledge, justification, and power. A discussion of mathematics being "really essential" leads us to consider the possible roles of mathematics in society, in technology, and in people's life-worlds. A discussion of "meaning" brings us into a turbulent trip through the history of philosophy, even before we come to address "shared meanings".

Let us call a problem that is formulated through explosive concepts an explosive problem. Thus many of the problems collected and presented by Wheeler are explosive. Hilbert's problems, however, can be characterized as being solid. They are formulated through solid concepts, and there exists a general agreement, within the mathematical community, of whether a particular problem is solved or not. No such solved/ not-solved duality seems applicable to the Wheeler problems.

What to think of this situation? Should mathematics education try to formulate its fundamental problems in solid concepts? My answer is "no". I find it healthy for mathematics education to grapple with explosive problems. I do not find it promising to follow the trend that tries to introduce operationalized definitions of educational phenomena, say, students' performance and learning gains, and, in this way, try to deactivate explosive concepts. Instead, it is important to acknowledge the explosive nature of crucial problems in mathematics education.

Consider again Howson's problem, which includes the notion of theory. Sure, there have been many attempts to define what a theory is, as well as attempts to try to close the concept, for instance by defining a theory as an instrument for making predictions. However, I find it much more productive to acknowledge the explosive nature of "theory" and explore its relationships to other open concepts. Thus we can consider relationships between knowledge and power as well as relationships between theory and power. We can ask: What interests might a theory in mathematics education serve? What functions might the school mathematics tradition, with all its rituals and exercises, serve? In what way does this tradition accommodate the economic order? Does mathematics education have a disciplining function, as implied by Foucault? All such questions can be related to Howson's problem of establishing some adequate categories for investigating mathematics education.

Mason's problem brings us further: Is mathematics really essential? This question opens up a range of perspectives. We could start with the socio-economic order of today and ask if mathematical knowledge is essential for increasing productivity. We could consider if mathematics education serves a disciplining function, for instance by cultivating what I call a prescription readiness. Naturally, one could also ask to what extent mathematics is essential for human dignity and for human life in general. One could ask if mathematical rationality is essential for developing and maintaining democratic institutions in society. One could consider in what sense mathematics education contributes to the development of citizenship, and also of critical citizenship. All such issues whirl us into different open conceptual landscapes.

The same can be said about Bishop's problem. The very notion of meaning has a long history in philosophy. Many different interpretations of "meaning" have been formulated. And, certainly, no general agreement is in sight. Meaning has been interpreted in terms of reference, while Wittgenstein opened a quite different line of thinking by associating the meaning of a concept with its use. Discourse theories are going further by addressing meaning in terms of actions and power relationships. With respect to mathematics education, one can consider meaning a relational concept: thus the meaning of a classroom activity has to do with its relationships to other practices. I have suggested that students' experiences of meaning have to do with relationships to their foregrounds. The notion of meaning is explosive—and all such considerations are only introductory for addressing Bishop's concern about "shared meaning".

It is important to address the issues raised by Howson, Mason and Bishop. In general, it is important for mathematics education to grapple with explosive problems.

Let me just say a couple of words about theoretical progress in mathematics education. Naturally, we immediately have to recognize that the very notion of progress is explosive. So when we talk about progress in mathematics education we cannot be supposed to know what we are talking about. Let me, however, acknowledge this fact by suggesting that progress in mathematics education is related to a readiness to grapple with explosive problems. So, while progress in mathematics might be related to the solution of (solid) problems, progress in mathematics education, seen as a theoretical discipline, can be related to grappling with explosive problems. This could bring about, for instance, dramatic conceptual changes, new discourses, reconstructions of perspectives, and provocative changes of concerns. This means that mathematics education should not consider solid problems, as formulated by Hilbert, to represent any epistemic ideal. I see instead the explosive nature of problems in mathematics education as being a remarkable strength.

Ah, I almost forgot to say anything about what kind of fundamental problems might be relevant to address in mathematics education, now, almost 30 years after Wheeler collected his problems. I would need more time to think about this, but let me just mention some concepts that I, for the moment, think could be relevant for formulating such problems: globalisation, ghettoising, inclusion, exclusion, democracy, citizenship, social justice, equity, democracy, mathemacy, students' foregrounds, intentionality, imagination, hope, dialogue, empowerment, despair, meaning, action, rationality, discipline, power, capitalism, suppression, exploitation, colonisation, racism, sexism, interest, critique.

Sure, this list needs to be continued. My point, however, is not to try to provide an exhaustive list, but to point out the relevance of explosive concepts in the formulation of important problems in mathematics education. (Naturally, there are many other non-explosive notions that could be relevant to use as well like, say, statistics, probability, algebra, landscapes of investigation, algorithm. But this is another issue.)

I have written this letter to you enjoying an excellent view from my flat, 4th floor, here in Rio Claro. It is warm, more than 30 degrees. I will stop now. Then you can read this letter with a cooler and clearer mind.

PS. Let me just mention that in my book *Travelling Through Education,* I address the notion of explosive concepts, and that prescription readiness is discussed in *An Invitation to Critical Mathematics Education.* In another book, *In Doubt,* I address the related idea of theoretical uncertainties and say a bit more about my flat.

Notes

[1] Research Problems in Mathematics Education – I, *For the Learning of Mathematics* 4(1), p. 41.

[2] Research Problems in Mathematics Education – III, *For the Learning of Mathematics* 4(3), p. 24.

[3] Research Problems in Mathematics Education – II, *For the Learning of Mathematics* 4(2), p. 41.

[4] en.wikipedia.org/wiki/Hilbert's_problems.