"The Language of Mathematics": Towards a Critical Analysis of Mathematics Texts

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1 Introduction

"The language of mathematics" has been a concern of the mathematics education community for some time, several substantial efforts having been made to describe its characteristics and, in particular, the ways in which it may support or cause difficulties for learners of mathematics. There, are, however, a number of aspects of mathematical language that have been largely neglected and yet can, I believe, be of use and interest to teachers and learners, and to research in mathematics education In this paper I will suggest an approach to the analysis of mathematical texts[1] that makes use of linguistic theory to provide insights into how the language of a text may influence the ways in which its readers make sense of it, in particular the ways in which teacher-readers may make sense of texts produced by students. Within the context of mathematics education the approach also allows us to consider the relationships between the conventional forms of academic mathematical genres and the characteristics of texts that may be produced and used within the local classroom.

Descriptions of mathematical language by mathematicians and mathematics educators have tended to focus on vocabulary and symbolism and some limited areas of specialist grammatical structures not commonly found in everyday language such as *the sum of the series to* n *terms*. One grammatical structure that has received some attention, because of its relevance to the formation of new mathematical objects and concepts, is the extensive use of nominalisation (forming a noun from a verb and hence an object from a process, e.g. *permutation, relation, rotation*) This focus, however, has largely concentrated on the naming of mathematical objects rather than on the nature of relationships between them and the participants in the mathematical activity or on the formation of mathematical arguments[2] In particular, little attention has been paid to the grammatical structure of mathematical texts. The fact that school students have generally been involved in extremely little production of their own texts probably contributes to this neglect; if all that is required from them is a single word, number, or algebraic expression, or at most a short sentence, there is little need to examine the structure of longer texts. However, recent interest in the use of writing-to-learn in mathematics (e.g Waywood [1992], Borasi & Siegel [1994]), an increase in the use of open-ended problem solving and extended projects, and moves towards "authentic assessment", mean that students have found themselves involved in producing longer texts In the UK, the introduction of the assessment of extended investigative tasks[3] for students at age 16+ has placed unprecedented linguistic demands on students whose teachers themselves have, on the whole, little experience of producing lengthy mathematical texts and few supportive strategies available to them While some students succeed in learning the implicit rules of the game and produce texts that are judged to be well written, many others, particularly those from less advantaged backgrounds, may not have sufficient linguistic awareness to recognise and produce texts that conform to particular academic genres. There is thus a case for developing more explicit knowledge of the characteristics of mathematical texts to help teachers and students to address the issue of what constitutes an "appropriate" text in a given context

A further issue that deserves attention is the diversity of mathematical texts The phrase *the language of mathematics* that I have used in my title is a misleading one that nevertheless indicates the way in which many authors have attempted to address issues of language in mathematics The use of the definite article *the* carries with it an assumption of uniqueness, suggesting that *one* description of this language will be sufficient to characterise any text that arises within the practice of mathematics. The focus on vocabulary and symbolism identified above may indeed reflect part of what is common to a wide range of such texts. It is, however, inadequate to characterise the similarities and differences between, say, an academic mathematics research paper, a primary school text book, and a 16 year old student's report of her work on a mathematical investigation.[4] These texts are likely to differ, not only in their vocabulary, degree of symbolism, and specific content matter (although this may in all cases be categorised as "mathematics"), but also in the ways in which arguments are made and in the relationships constructed between the author and his or her readers.

The development of the means of analysing mathematical texts that will be outlined in this paper originally arose from my concern with the difficulties that secondary school students appear to have in producing written reports of mathematical investigative work using language that their teacher-assessors consider to be "appropriate" [Morgan, 1995a]. The analytic tools used in studying such reports have, I would claim, a much wider application, both in providing a means for researchers in mathematics education to interrogate linguistic data, and in providing a language for teachers and students to talk and think about mathematical language and so to enhance their linguistic power.

2 A linguistic approach to mathematical texts

Mathematical texts differ in ways related to their subject matter, in the relationships between author and readers, and in the formation of argument These aspects correspond to the three metafunctions of the language posited by Halliday: the ideational, interpersonal and textual.

(i) The *ideational* or *experiential* function refers to the way in which language expresses "the categories of one's experience of the world" [Halliday, 1973: p 38] and one's interpretation of that experience. The aspect of mathematical language that has been most frequently addressed — the naming of mathematical objects — contributes to the ideational function by influencing the types of objects that may be participants in mathematical activity.

(ii) The *interpersonal* function expresses social and personal relations between the author and others, "including all forms of the speaker's intrusion into the speech situation and the speech act" [ibid., p 41]. Pimm's [1987] discussion of the use of *We* is a rare example of concern with the interpersonal in mathematical texts Given the recent growth of attention to the social in mathematics education generally, it would seem that further attention to the social as it is constructed in mathematical texts would be appropirate

(iii) The *textual* function is what makes language "operationally relevant" in its context and "distinguishes a living message from a mere entry in a grammar or a dictionary" [Halliday, 1973: p. 42] The formation of argument is one obvious way in which the textual function is fulfilled in mathematical texts but other types of "messages" may also be communicated, including, for example, reports that define, describe, or classify, and narratives relating the progress of mathematical activity

Halliday argues that every text fulfills each of these functions and his systemic-functional grammar [1985] indicates the ways in which grammatical features of language serve to fulfill them A key concept in this view of language that I have found very useful in considering the interpretation of mathematical texts is that of *choice* Whenever an utterance is made, the speaker or writer makes choices (not necessarily consciously) between alternative structures and contents. Each choice affects the ways the functions are fulfilled and the meanings that listeners or readers may construct from the utterance. As Kress argues, these choices are not arbitrary but are motivated by the writer's "interests" and reflect her "place in the world, physically, cognitively, socially, culturally, conceptually" [1993, p. 172]. The writer has a set of resources which constrain the possibilities available, arising from his or her individual social and cultural history but also from her current positioning within the discourse in which the text is produced. Similarly, the meanings constructed from a text by its readers will vary with the resources of individual readers and with the discourse(s) within which the text is read. The text itself may provide a reading position from which the text is unproblematic and "natural", but readers do not necessarily take up this "ideal" position and may resist the text by interpreting it in a different discourse [Kress, 1989]. Any analysis of texts must therefore take account of the situation of texts within their contexts of production and interpretation. The methods of Critical Linguistics and Critical Discourse Analysis (CDA), as seen in the work of such authors as Hodge & Kress [1988; 1993], Fowler & Kress [1979] and Fairclough [1989], make use of grammatical tools, based largely on Hallidayan linguistics, to provide descriptions of texts which are then interpreted in the light of a theoretical perspective on the social context in which they are situated. These methods have generally been applied to texts arising in social contexts that highlight the strength of power relationships within society, including, for example, advertising, newspaper articles, and job interviews The ways in which language is used by the various participants within such contexts is critical to the exercise of power. The recognition that power relationships are also at play within educational and academic settings makes CDA a useful and appropriate means of interrogating and interpreting mathematical texts In particular, for the student writer, one of the key concerns must be to produce texts which satisfy the teacher's or examiner's expectations, including their (usually implicit) expectations about the type of language used.

In the next sections I shall summarise the linguistic tools that may be used in this approach to mathematical texts (grouped according to their ideational, interpersonal, or textual functions) and to illustrate the ways in which they may be applied to a text and interpreted in light of the contexts of production and consumption of the text itself. The illustrations I will use are extracts taken from two rather different sources: a published academic research paper in mathematics [Dye, 1991] and a set of texts produced by secondary school students as reports of their work on investigative tasks. The academic paper, while not "representative" of academic mathematical writing in any formal sense, displays linguistic characteristics which make it commonly recognisable as "mathematical", i.e. specialist vocabulary, symbolism, etc. The student texts formed part of the data in my study of the discourse of investigations [Morgan, 1995a]. I find it particularly interesting to consider the language in these texts for a number of reasons:

- the investigative activity itself is supposed to involve students in "doing mathematics" in a way that is claimed by some of its advocates to be close to the way in which "real" mathematicians work — it must therefore be asked what resemblance the features of the writing produced bear to those of academic mathematical writing;
- students are expected to produce written texts that are substantially longer and more complex than other types of text produced within their experience of the mathematics curriculum — it is therefore important to ask whether they possess the linguistic skills necessary to produce effective texts;
- the texts are assessed and, for the older students (14–16 years), form part of a "high stakes" external examination. The power relationships mentioned above are therefore very strong and it is a high priority for the students to produce texts that will be acceptable to their teacher-assessors.

Having said this, the tools themselves may be applied to a far wider set of texts, the interpretation of their frustrations being highly dependent on knowledge of the contexts within which the texts have been produced and consumed.

2. The ideational function: Presenting a picture of the nature of mathematics

The central question to be addressed by using the analytical tools discussed in this section is "What is mathematics (as it appears in the text being analysed)?" This general question, which makes the assumption that the text under consideration is in some sense about mathematics, may include the following more specific issues:[5]

- What sorts of events, activities and objects are considered to be mathematical?
- How is "new" mathematics brought about (or created or discovered?
- What is the role of human beings in mathematics?

The significance of answers to each of these questions must be considered in the light of existing differences and debates within mathematics and mathematics education. For example, the absolutist/social constructivist divide in the philosophical basis of mathematics might be considered, as well as educational questions about the relative importance of mathematical processes or content matter.

In analysing the picture of mathematics and mathematical activity presented in a text, a significant role is played by examining the transitivity system, that it, the types of processes and the types of participants that are active in them. Halliday identifies six main types of processes: material, mental, relational, behavioural, existential, and verbal, of which the first three types are the most common. A high proportion of material processes may be interpreted as suggesting a mathematics that is constructed by doing; mental processes (e g. seeing, thinking) may suggest that mathematics is a pre-existing entity that is sensed (discovered) by mathematicians; relational processes present a picture of mathematics as a system of relationships between objects or between objects and their properties. It is of particular interest to note whether a generalisation is expressed as a relation:

> (TOP LENGTH + BOTTOM LENGTH) × SLANT LENGTH = NUMBER OF TRIANGLES

or as a material procedure:

> If you add the top length and the bottom length, then multiply by the slant length, you get the number of unit triangles.

When read by a teacher-assessor, the procedural formulation is likely to be less highly valued than the relational one as it may be seen as representing an earlier stage of development of algebraic thinking.[6] Relational processes, particularly those which serve an identifying function, are very frequent in both mathematical and scientific academic writing [Halliday, 1985]. In mathematical texts the identifying function is frequently fulfilled by an equals sign. Given the evidence of children's "misuse" of equals signs to fulfil material (operator) functions [Kieran, 1981], its role may often be of interest when considering the ways in which teachers are likely to respond to students texts

In examining the picture of the nature of mathematics presented in a text it is clearly significant to ask not only what types of process take place but also what kinds of objects are participants in the text, and hence what sorts of objects are the actors in mathematical processes or are affected by these processes. As has been noted above, many discussions of mathematical language have focused on the naming of mathematical objects. My concern, however, is not only with the forms of the names but with the influence that such linguistic forms have on the nature of the objects themselves.

The extent of the use of nominalisations in mathematical text, transforming processes into objects, such as *rotation, permutation* or *relation*, has already been mentioned. Such nominalisation has a number of effects on what it is possible to say with and about such process-objects. Firstly, it brings the process into immediate relation with another verb and hence, where this verb is relational, with other nominals; it allows the process to act as the theme of a clause; it also allows the process to be presented as a cause or effect. The power of the use of such nominalised process-objects may be illustrated by this example taken from a student's text. The author starts by describing a number pattern:

> As you can see the unit number increases by two every time the top length increases by one.

He is then able to extend his generalisation to a wider range of situations:

> This can be done by using any slant number, but if you change this you may find that *the unit increases* may be different

By expressing the process of increasing as a nominal, he is able to describe not only a particular pattern but also a more general relationship between a range of patterns:

> This time *the unit increase* is by 4 instead of 2
>
> On the next one when you increase the slant *it increases to 6*

The subject matter of the passage thus shifts from a focus on numbers and measures to a focus on rates of change, allowing the author to address a further level of mathematical ideas

Another consequence of the use of nominalisations is the obscuring of agency; the transformation of process into object removes the grammatical need to specify the actor in the process. The use of, for example, *rotation* or *permutation* without any indication that these processes are actually performed by anyone fits in with an absolutist image of mathematics as a system that exists independently of human action. A similar function is performed by the use of representational objects (i e. tables, graphs, diagrams, etc.) as actors in verbal processes, e.g. *the table shows that* ... rather than *I have shown in the table that*..., which obscures the writer's presence as author as well as mathematician. The use of passive rather than active forms of verbs is a further way of obscuring agency that is much used in academic writing and is even seen as the "correct", "objective" way of writing. As Halliday and Martin [1993] point out, there is a difference between objectification and objectivity but, in the rationalisations for their practices provided by scientists and other academic writers, the two are often confused.

Having noted the ways in which human agency may be suppressed, it is also of particular interest to examine the place of human beings in the text and in the doing of mathematics when they do appear. Is the main role of human beings to "see" or "discover" (perhaps suggesting a Platonist view of mathematics), or do they manipulate shapes and symbols (the main activity of pupils in the mathematics classroom)? The interpretations offered here are only illustrative of the possible significance of different roles.

In considering the portrayal of mathematical activity, it is also important to determine how causal relationships are represented in the text: that is, what types of objects cause or are caused. Here again the presence or absence of humans as causal agents is significant in the extent to which mathematics is seen as an autonomous system. For example, in an academic mathematical paper previously established facts (labelled by numbers and hence further distanced from the activity which originally established them) are presented as causes of other facts without any intervening activity:

> By (4), (6) the other Brianchon point of the former edge is (1, −1), 1)

In contrast, a Year 9 student's rough work shows mathematical facts and relationships to be dependent upon human action:

> whenever there is one dot inside and you count up the perimeter and the area will be exactly half it

The importance of explanation and proof in mathematics is also to be seen in the frequency with which expressions of causality occur in a text.

In summary, the image of mathematics and mathematical activity presented in a text may be considered through examination of the types of processes, in particular the uses made of the equals sign and the types of processes used in the expression of generalisations; the types of participants in these processes; the portrayal or suppression of agency through nominalisations, non-human actors, the non-active forms of verbs; the nature and extent of the expression of casual relationships. Another powerful feature of many mathematical texts is the symbolisation of mathematical objects which allows them not only to act and to be acted upon but to also be combined and manipulated to form new objects. While I am not concerned here with the grammar of mathematical symbolism, its presence or absence is relevant to the image of mathematics that is presented in a text and, for some readers, the extent to which the text itself is considered to be "mathematical".

2.2 The interpersonal function: the roles and relationships of the author and reader

The concern of this section is not only with the relationship between the author and her reader(s) but also with the ways in which the author and the reader are constructed as individuals. In asking "Who is the author of this text?", the areas of interest include her attitude and degree of authority towards mathematics and the particular mathematical task being undertaken. The analysis may also consider how the reader's relationships to mathematics and to the task are constructed within the text. This includes asking the question: "Why is the constructed reader reading this text?" which may itself involve considering the relationship between author and reader. An important aspect of this relationship is its symmetry or asymmetry: to what extent are the participants "equal" members of a community of mathematicians or is there a greater authority ascribed to one or to the other? How intimate is their apparent relationship?

One of the most obvious ways in which interpersonal relationships are expressed in a text is through the use of personal pronouns. This has been discussed by authors concerned with ideological aspects of language use in general [Fairclough, 1989; Fowler & Kress, 1979] and by those specifically concerned with the nature of academic scientific writing [Tarone *et al*, 1981; Bazerman, 1981] and the language of mathematics education [Pimm, 1987]. The use of first person pronouns (*I* and *we*) may indicate the author's personal involvement with the activity portrayed in the text. It may also indicate an expectation that the reader will be interested in this personal aspect as, for example, a teacher might be concerned to know to what extent the mathematics in a piece of student writing was the product of work done by the individual student. For example, a student who had investigated a problem in a small group and then written a report of his activity to be submitted for examination, introduced his original problem thus:

> The problem that we were given was

and used the plural pronoun to refer to the group as a whole throughout the first part of his text. When he started his "extension", however, he claimed individual ownership of both problems and solution:

> For my extension I am going to

In the particular context in which text was situated it was important that the author should make this personal claim because of the weight given by the assessment criteria not only to understanding and performing adequately, which could be achieved and evaluated in a group setting, but also to posing an appropriate, original, extension question, which can only be done as an individual. In contrast, the academic mathematics paper quoted here uses the first person plural throughout in spite of the fact that it was written by a single author; claiming, for example, that:

> We shall show that

and thus suggesting that the author is not speaking alone but with the authority of a community of mathematicians.

While such uses of the first person may draw attention to the activity and authority of the author, *we* may also be used in an inclusive way to imply that the reader is also actively involved in the doing of mathematics. For example, from the same academic paper:

> We saw in section 2.2 that

and

> By Theorem I we may assume that H is H*

gives the reader a share in the responsibility for constructing the argument. Not all readers, however, may be happy with accepting this responsibility; Pimm comments on similar uses of the first person plural in a mathematics text book:

> The effect on me of reading this book was to emphasize that choices had been made, ostensibly on my behalf, without me being involved. The least that is required is my passive acquiescence in what follows. In accepting the provided goals and methods, I am persuaded to agree to the author's attempts to absorb me into the action. Am I therefore responsible in part for what happens? [1987: pp. 72–3]

The particular way in which *we* is read will depend on how the individual reader is oriented towards the text In the context of school mathematics it is important to ask how a teacher-reader might respond to a student's use of *we* in circumstances that do not refer to joint activity by a group of fellow students. Either authoritative or inclusive uses, although in common use in a range of texts written by adult mathematicians, might well be considered socially inappropriate if used by a child

The ways in which the second person pronoun is used are also of interest. Addressing the reader as *you* may indicate a claim to a relatively close relationship between author and reader or between reader and subject matter For example, one boy wrote in his coursework:

> On this grid you will notice that it has coloured boxes around the numbers.

By including the words *you will notice* it appears that the author is addressing an individual reader personally and directing her attention with a degree of authority; it also suggests that the reader ought to be interested in the details of the mathematics presented in the text. On the other hand, some uses of *you* appear to be attempts to provide expressions of general processes rather than being addressed to individual readers This seems to be the case particularly where children are struggling both to formulate generalisations and to communicate them For example, the generalisation by a collaborating pair of Year 9 students (age 13–14):

> the area would be half of the perimeter if you add one to the area

contains a mixture of relational and procedural forms as well as a combination of a general relationship between two properties of a shape and an action by a human agent. While it is possible to separate the grammatical features related to the ideational function (types of process, agency) and those related to the interpersonal (personal pronouns, conditional modality), it is less easy to separate out the effects of these on the way a reader might make sense of the text. In this case, the combination of contrasting features is likely to be significant. Martin [1989] points out that "mature" writers will use one form consistently; such lack of consistency of expression is thus likely to be interpreted negatively by a teacher-assessor as a lack of maturity or a mathematical deficiency. This example illustrates the difficulty of separating form from content; there is no evidence in the text that allows the reader to know whether the form of this generalisation arose from a mathematical or a linguistic weakness

While considering the significance of different ways of using personal pronouns it is also relevant to mention their absence As was discussed in the previous section, constructions such as the use of the passive voice obscure the presence of human beings in the text This not only affects the picture of the nature of mathematical activity but also distances the author from the reader, setting up a formal relationship between them rather than an intimate one

One common characteristic of academic mathematics texts (and some school texts) is the conventional use of imperatives such as *consider, suppose, define, let* x be. Like the use of *we*, these implicate the reader, who is addressed implicitly by the imperative form, in the responsibility for the construction of the mathematical argument. The use of imperatives and of other conventional and specialist vocabulary and constructions characteristic of academic mathematics marks an author's claim to be a member of the mathematical community which uses such specialist language and hence enables her to speak with an authoritative voice about mathematical subject matter. At the same time it constructs a reader who is also a member of the same community and is thus in some sense a colleague (although the nature of this relationship may vary according to the type of action demanded) [7] In academic writing this assumption of mutual membership of the mathematics community is to be expected In the school context, however, the relationships between writers and reader are more asymmetric The teacher or textbook may use such conventional forms at least part of the time to induct the student into such a mathematical community[8] but for the student-writer there are tensions between the need to display her familiarity and facility with conventional mathematical language (and hence to have the right to be considered part of that community) and her need to satisfy other classroom demands. For example, the student may be expected to demonstrate her thinking processes, to show detailed "working out" or to "explain" without using symbols A student who addresses the teacher-reader with authority as a colleague may even be perceived as arrogant. When a set of children's investigation reports were read by teachers and researchers at a meeting of the British Society for Research into Learning Mathematics, several expressed negative reactions to the text of a student who had adopted an authoritative position in his writing.

> An interpersonal feature of one child's work was commented upon negatively He wrote:
>
> > When I had finished writing out this table I had seen another pattern. Can you see it?
>
> This way of addressing the reader was seen as inappropriate It is, however, typical of the way that children are themselves addressed either by the writers of mathematics text books and work cards or (usually orally) by their teachers.
>
> This boy seems to have identified and copied one of the features of the mathematical texts provided for him without realising that it might be considered inappropriate in his own writing. [Morgan, 1992: p 12]

Adoption by a less powerful participant of the language of those who hold more powerful positions in the discourse may not be considered "appropriate" by those more powerful participants

Relations between author, reader, and subject matter may also be seen in the modality of a text: "indications of the degree of likelihood, probability, weight or authority the speaker attaches to the utterance" [Hodge & Kress, 1993: p. 9] This may be expressed through use of modal auxiliary verbs (*must, will, could,* etc.), adverbs (*certainly, possibly),* or adjectives (e g. *I am sure that*. .) Again, expressions of certainty are particularly sensitive in the relationship between pupil and teacher-assessor where they may be interpreted as inappropriate claims to knowledge or authority.[9]

In summary, the roles and relationships of author and reader may be considered through: examination of the use of personal pronouns; the extent of specialist mathematical vocabulary and conventional forms of language such as imperatives; the expressions of certainty and authority in the modality of clauses.

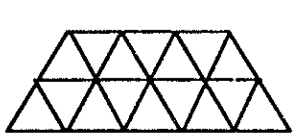
2.3 The textual function: The creation of a mathematical text

In this section, the way in which the text is constructed as a coherent, meaningful unity is considered: what sort of text is it? This may be addressed by examining internal features which contribute to the way in which the text is constructed as well as the overall structure of the text as a whole. Answers to this question also contribute to the ideational and interpersonal functions of the text. By constructing a particular kind of text as a part of mathematics the nature of the discourse of mathematics is implicated, as are the expectations of the participants about what constitutes appropriate writing within the given context

By examining the types of theme that a writer has chosen to use, a picture can emerge of what sort of things the text as a whole is about. The theme of a clause is an indication of its main subject matter; in English it is not only the starting-point of the message but is also realised by being positioned at the start of the clause. Van de Kopple [1991] citing Fries, for example, contrasts two descriptions of houses, one of which presents a picture of movement through the house with a progression of clauses whose themes refer to location, while the other orients the reader's attention on the house as a set of components by thematising the contents rather than on their locations Given the high status of deductive reasoning in the mathematics community, we might expect to find expressions of logical reasoning thematised, focusing the reader's attention on the progression of the argument For example, the presence in a report of mathematical activity of a large number of themes expressing reasoning (e.g., *hence*, *therefore*, *by Theorem 1*, etc.) would serve to construct the text as a deductive argument. This type of thematic progression occurs repeatedly in the academic paper, particularly in those sections explicitly labelled as proofs of theorems, e.g.

> **By (4), (6),** the other Brianchon point of the former edge is (1, -1, 1). **By (4)** this is also on the edge $jx^2 + jy - z = 0$ joining $(1, -j, 0)$ to $(0, 1, j)$ and the edge $x + j^2y + jz = 0$ joining $(-j, 0, 1)$ to $(0, 1, -j)$. **Hence** the other sides of the triangle Δ_1 of H with $jx - y - j^2z = 0$ for one side are the edges joining $(1, -j, 0)$ to $(0, 1, -j)$ and $(0, 1, j)$ to $(-j, 0, 1)$, namely $j^2x + jy + z = 0$ and $x - j^2y + jz = 0$. **Now** (4) gives the following display of the vertices of
>
> $$\Delta_1: (1, -j^2, j), (-j, 1, j^2), (j^2, j, 1) \qquad (13)$$

On the other hand, a predominance of temporal themes (e.g., *first*, *next*, *then*, etc.) would construct a story or report recounting what happened, e.g.,

> **Once Suzanne and I had completed tasks 1 and 2,** we set out to discover if there was any connection between the triangular area and the lengths of the sides. **First** we lettered the sides:
>
> 
>
> <u>a = slant line</u>
> <u>b = base line</u>
> <u>c = top line</u>
> <u>d = area</u>
>
> **In a very short time** we had discovered a relationship between the lengths of the sides and the area (triangular)

or, if used with imperatives, would construct an algorithm. These are clearly not the only alternatives that may arise in an analysis of the thematic progression of mathematical texts. They do, however, reflect important aspects of the mathematical writing that students may produce, any of which may be valued in some circumstances, but which may be considered inappropriate or less valuable in others

A number of cohesive devices serve to construct the whole text as a meaningful unit These include the repeated use of the same or related vocabulary, connectors making links between sentences, and the use of words which refer backwards or forwards within the text. It is particularly worth examining the way in which reasoning is constructed in the text. Martin [1989] points out that reasoning can be expressed through the use of conjunctions (*because*, *so*), nouns (*the reason is...*), verbs (*X causes Y*) or prepositions (*by*, *because of*) It may also be expressed less explicitly through the simple juxtaposition of casually related statements. Such lack of explicitness is discussed by Swatton [1992] in the context of assessment of processes in science; he points out the difficulty in determining whether a hypothesis has been formulated (a specified assessment criterion) unless explicit causal language is used Juxtaposition and the use of simple conjunctions are characteristic of what Altenberg describes as "the unpremeditated, rambling progression of conversational discourse" [1987: p. 61], whereas "mature" writing tends to use other devices to express causal relationships [Martin, 1989] Where students' writing is to be assessed this difference is significant because writing that uses forms of language characteristic of speech may be judged to be expressing less "mature" forms of thought as well

As well as looking at internal characteristics of the text it is important to consider the overall structure of the text as a whole Do various parts of the text fulfill different functions and how clearly are these defined? Such sections may be signalled by explicit labelling, by paragraphing or other lay-out devices, or only by changes in content matter or style. In interpreting an element of a text, readers are influenced by its position and by what they expect to see at that point in the text If the structure is unclear or unconventional this is likely to affect the meanings that the reader ascribes to the text, her evaluation of it and of its author.

The way in which a mathematical text is constructed as a coherent whole and the type of "message" it attempts to convey may, in summary, be investigated by considering thematic progression, the cohesiveness of the text, particularly the ways in which reasoning is expressed, and the overall structure of the text.

3. An example of the application of the linguistic approach

The examples of extracts from mathematical texts provided so far have merely explicated the grammatical features that may be significant to an analysis of mathematical texts. In this section, I shall offer an illustration of the way in which these tools may serve to illuminate issues of significance to students and teachers of mathematics.

An experienced mathematics teacher, Dan, was asked to read extracts of students' texts (containing similar correct generalised statements but written in different styles) and to say which he preferred.[10] His comparison of extracts No. 2 and No. 3 (see Figure 1), while acknowledging the similarity of their "content", indicated the strength of the influence of the different choices of language made by the two student-writers.

> Number 2 gives me the impression they obviously know what they're talking about whereas this one, although it says almost the exactly the same thing in different words, er, it doesn't give me the same impression

Dan had already dismissed the obvious structural differences between the two extracts (No. 2's numerical examples and the use of verbal or symbolic variable names), claiming that they did not greatly affect his assessment of the students. An analysis of the verbal descriptions of the procedure allows us to speculate about what might be contributing to Dan's "impression" of difference between the two students

Extract No. 2:

> If you add the top length and the bottom length, then multiply by the slant length, you get the number of unit triangles.
>
> For example:
>
> 3 + 5 = 8 and 2 + 4 = 6
> 8 × 2 = *16* 6 × 2 = *12*
>
> This , therefore is the formula:
>
> (TOP LENGTH + BOTTOM LENGTH) × SLANT LENGTH = NUMBER OF TRIANGLES

Extract No 3:

> If you add together both the top length and the bottom length and times it by the slant length, you will end up with the number of unit triangles in that trapezium.
>
> You can write this as $S(T + B)$.

3.1 Ideational aspects

Processes and participants No. 3's procedure uses *add together* rather than simply *add*, and *times it by* rather than *multiply by*. The number of unit triangles is also qualified as being *in that trapezium*. These additional words include reference to the concrete lengths, numbers or shapes. The procedure may thus be read as being at a lower level of abstraction

Causal relationships No 3's introduction of the final formula by *You can write this as* ... presents the symbolic formula merely as an alternative to the verbal procedure. No. 2's announcement *This therefore is the formula*, on the other hand, displays the formula as a product in its own right which follows logically from the procedure rather than merely being equivalent to it. Mathematics is thus presented as an autonomous system rather than being dependent on human action. It may also suggest that No 2 has a better understanding of the importance of the relational formula in mathematics, even though she has not used algebraic symbols to express it.

3.2 Interpersonal aspects

Specialist vocabulary The use by No. 3 of *times* rather than *multiply* is less formally "mathematical" and the use of such vocabulary may be read as a remnant of the early years of mathematics schooling and hence a sign of immaturity

Cohesion No. 2 presents the variables and operations in the same order in both procedure and formula, whereas No. 3 changes her order from $(T + B) \times S$ in the verbal procedure to $S(T + B)$ in the symbolic formula. This disjunction further reinforces the lack of logical structure in No. 3's text and may even be taken by a teacher-assessor as a suggestion that the symbolic formula was copied from another source rather than "belonging" to the student herself.

Any of these features might have contributed to Dan's impression that No. 3 is less competent mathematically. While it is not possible to say precisely which aspects contributed to his assessment, there is clearly a mismatch between her text and Dan's expectations which appears to have affected his evaluation of the whole of No. 3's performance and even of her general level of intellectual "ability". Unfortunately, Dan himself was unable to identify the features of the two texts which gave rise to his impressions. As a consequence, it seems unlikely that he would be able to provide advice to a student on how to produce an acceptable text.

4 Implications for research and teaching

The analytic tools I have outlined above provide a means of describing the main features of the various genres of mathematical text and of describing the functions a particular text may fulfill within a given discourse. By stressing the idea of *choice* between different forms of language and studying the effects of various choices, it is possible to take into account the diversity of mathematical activity and of mathematical texts and thus to move away from the idea of a unique *language of mathematics*. By going beyond the traditional focus on vocabulary and symbolism it becomes possible to interrogate both written and oral texts produced within mathematical contexts in order to address a wider range of questions about the nature of the mathematical activity, about the relationships between the participants and the activity and about the forms of reasonings involved. It is, however, important to remember that, while linguistics can provide means of describing mathematical texts, their interpretation is highly dependent on knowledge of the discourses in which a text is produced and consumed and on the analyst's theoretical perspective on the activities and social relationships within those discourses.

My original concern in studying the language of mathematical texts arose from an awareness that student appeared to find it difficult to produce written texts that were acceptable to their mathematics teachers and which would be judged to display their mathematical attainment to best advantage. I must therefore ask whether the description of particular mathematical genres made possible by the use of these linguistic tools could help students and teachers to improve this situation. There is currently some debate in the domain of literacy education about whether students should be explicitly taught the characteristics of specific genres of writing (see, for example, Reid [1987]). Most of the examples drawn upon by both sides in this debate seem to be from the earlier years of education when, it might be argued, the consequences of deviations from the expected genre may not be so significant. Moreover, it appears to be largely assumed that this is an issue for language teachers, concerned with general language development, rather than for teachers in other curriculum areas, concerned with communication within in their subject area. An exception to this is Kress [1990], who illustrates his argument in favour of teaching specific genres with examples of student writing in school leaving examinations in economics. In such high-stakes contexts the use of forms of language that will be judged "appropriate" by the teacher and that are likely to serve to construct meanings that conform to the expectations of the particular academic discipline has great significance for the individual student, for whom success or failure in the examination may have life-long effects. A teacher-assessor's judgement that a text does or does not give "the impression they obviously know what they're talking about" is not a trivial judgement in such circumstances.

As Kress points out, subject teachers are unlikely to be aware of the ways in which their judgements are affected by students' use of particular forms of language. Moreover, the focus of existing descriptions of mathematical language on vocabulary and symbolism suggests that, while mathematics teachers are likely to be aware of a student's use of mathematical words, they are less likely to be able to identify other grammatical structures contributing to the mathematical nature and the "appropriateness" of a text. This seems to be confirmed by the example discussed above in that Dan, even though aware that the differences in the language used by the two students were affecting his judgement, was unable to explain or even point explicitly to these differences. Where student-writers are unaware of such effects it is likely to be impossible for a teacher-assessor to make a distinction between those "weaknesses" arising from a lack of linguistic awareness and those arising from a lack of mathematical understanding. Linguistic analysis of specific mathematical genres in which students are expected to participate, in particular those that are likely to be highly valued in assessment situations, could provide knowledge for teachers to use in supporting their students' development of mathematical language.

I am not, however, suggesting that lists of linguistic rules and appropriate forms to be learnt and copied would be either adequate for this purpose or desirable. Indeed, one of the effects of using such algorithmic means to guide students' writing is likely to be the disempowerment of students, stifling any possibilities of creativity, as Dixon [1987] argues in his attack on the idea of "teaching genre". It is in the interpretation of the effects of various linguistic choices within particular discourses that the power of Critical Linguistics lies. Awareness of how using personal pronouns may construct an authoritative or subservient image of the writer, or that, for example, introducing the word *therefore* can signal to the reader that logical connections have been made, together with awareness that logic is highly valued in mathematical discourses, could provide students with the power to manipulate their own use of language to influence their readers and to demonstrate their mathematical understanding to best advantage. This would enable students to conform to the conventional expectations of the genre but would also empower them to make informed choices to break the conventions deliberately in order to achieve specific effects.

Notes

1 The term *text* may be taken to refer to either spoken or written texts. While the illustrations used in this paper are all extracts from written texts the analytic method described is equally applicable to other modes of communication. While I will, in the rest of the paper, refer to *writers* and *readers*, these may be read as *speakers* and *listeners* or, more generally *producers* and *consumers* of texts.

2 A few exceptions to this neglect have arisen from interest in the difficulty students in higher education have with the construction and understanding of proofs (e.g. Leron [1983], Konior [1993]. Here, the construction of argument may be seen as a major part of the mathematical "content" of the text. Such exceptions, therefore, do not really go against the trend to focus on the naming of objects and other "content" issues.

3 Students are expected to explore a mathematical situation or open-ended problem, leading eventually to general conclusions. They produce a written report describing their work on the task. This report forms the primary basis of assessment by the student's own teacher. The assessment criteria for such tasks lay emphasis on the processes used (e.g. working systematically, communicating and reasoning) rather than the "content" knowledge and skills.

4 Mousley and Marks [1991] list a number of different genres of writing produced by mathematicians, ranging from *procedure* and *observation* to the *explanation* and *exposition* of high status academic writing. They indicate the roles of these genres in relation to reporting various types of mathematical activity and hint at the social circumstances in which they may be used, but do not identify the characteristics of the genres as used by mathematicians.

5 These questions reflect my interests; other researchers might generate different or additional questions.

6 Van Dormolen [1986] draws parallels between this distinction between procedural and relational thinking and Freudenthal's [1978] distinction of levels of language. However, whereas Freudenthal is clear that it is the *language* that is at a lower level in the procedural example, van Dormolen slips between referring to levels of language and levels of knowledge.

7 Rotman [1988] draws a distinction between the roles constructed for the reader by the use of inclusive ("Let's go" although the "Let's" may be only implicit) and exclusive ("Go") imperatives in mathematical writing. Inclusive imperatives, which Rotman identifies as mental processes like those above, are addressed to a "thinker" and "demand that speaker and hearer institute and inhabit a common world or that they share some specific argued conviction about an item in such a world" [p. 9]. Exclusive imperatives are addressed to the reader as a "scribbler" who must perform some material action (*integrate, multiply, drop a perpendicular* ...).

8 Dowling's [1991] analysis of texts intended for use by different groups of students suggests that this aim of introduction into a mathematical community is only considered relevant for a minority of students.

9 See Rowland [1995] for a discussion of children's use of "hedges" during mathematical talk to protect themselves against the possibility of being wrong

10 This episode was part of the study described in Morgan [1995a] and its analysis has also been discussed in Morgan [1995b]. I would like to thank the participants in my session at the meeting of the British Society for Research into Learning Mathematics at Birmingham in October, 1995, for their discussion of and contribution to the analysis.

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Lévi-Strauss says somewhere that "the student who chooses the teaching profession does not bid farewell to the world of childhood: on the contrary he is trying to remain within it

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