# Communications

## Mathematics is not required

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Roy, a young student of fifteen who attends an 'alternative' high school and the son of friends, was a recent visitor to my house. 'Are you taking any math?', I asked him. His answer was sharp and immediate. 'No. Math is not required in my school. Anyway, math is completely useless!'

Intuiting that the student did not really mean that math was entirely useless to him, but piqued by his ignorance and my feelings about the implied inadequacies of his school, I launched into a sermon. A newspaper lay nearby. I opened it to the front page and told him that if he counted carefully, he could probably find about sixty individual numbers (not just digits) on that one page.

Some of the numbers are, mathematically speaking, trivial: e.g. 'continued on page 7'. Others are deeper: the date, for example, which embodies thousands of years of adjustment to arrive at a system that combines astronomic realities and civic and religious traditions. To predict the date of Easter in advance, for instance, cannot be done with one or two simple arithmetic operations. The time of the high tide and its anticipated height, given at the bottom of the front page, require theories of harmonic analysis.

The product stripping on the lower left corner, a recent innovation, has a mathematical underlay that is substantial, involving what are known as 'error correcting codes'. The weather map, on the last page of the paper, is created automatically from data that are analyzed programmatically and plotted up using graphical algorithms; even more so are the dynamic, animated weather maps, shown on TV. The pictures of yourself on your web page are chip-processed, involving mathematical theories of signal transmission.

The front page of the paper may yield sixty or so numbers, but if you turn to the sports page or the financial pages, the high density of numbers is visible to the naked eye. The average investor-gambler (now frequently on-line) uses economic news, his or her own interpretation of trend graphs, brokers' advice, tout sheets and software, hunches, astrological and other oracular tips. The more sophisticated investor uses highly mathematized models (not necessarily better). I could have gone on and on in this manner showing the impact of mathematics on daily life.

The young man and his parents (who were also present) were visibly impressed by the vehemence with which I praised the utility of mathematics. Perhaps they were a bit embarrassed in their realization of how uninformed they had been. I doubt, though, that my ardor did anything to persuade Roy to study some mathematics.

The U.S. public, typified by this young student Roy, largely remains uninformed and indifferent to the extensive mathematical bases of today's life. The educational élite, including many professors in a non-scientific department, is largely indifferent. A professor of the history of science recently published a book that is autobiographical. Her history colleagues said to her, 'I enjoyed your book enormously, but of course, I skipped the mathematics'. She said to me, 'My humanistic colleagues get twitchy even at the mention of high-school algebra'.

We are living in a totally mathematized and chipified civilization. The mathematical base is most often hidden from view: it lies in the hardware and in the software that is built in and which is employed with hardly a thought given (except when something goes awry). The mathematization of life predates the digital computer, itself the most mathematical engine in existence, but the computer has accelerated the pace of mathematization enormously. This process is on the increase for the foreseeable future.

How will Roy make out without algebra, geometry, trigonometry, calculus, etc. as he gets older? If he is headed towards college or towards a profession that requires mathematics (chemistry, physics, etc.) or towards a profession that uses mathematics as a gate to exclude candidates (law, medicine, etc.), he will make out poorly. If he is headed toward jobs that do not require mathematics either to enter or to perform (and most jobs are in this category), he should not be disadvantaged. If higher education will introduce him to the finest creations of human thought, and I believe it ought to, then his education will have a substantial gap because mathematics is among such creations.

Yet the truth is this: the average person apparently does not need to know much math in order to live in today's chipified world. The high degree of mathematizations which are now spreading over the world, created by a relatively small cadre of mathematicians, computer specialists and communication technologists, gives testimony to this fact. Once satellite positioning had been put in place, giving ship captains and pleasure yachtsmen their position to within 30 meters, they can forget all about spherical trigonometry and throw their Bowditch *Practical Navigator* into the ocean.

Many of the skills that would appear to be mathematical are learned by simple experience or by being 'walked through' once or twice. There are millions of travel clerks on terminals doing scheduling, pricing and billing. There are even more millions of people who insert their cards into ATMs. None of these skills needs to be taught in school. We can all tie our shoelaces once our parents show us how, even though there are profound and difficult mathematical theories of knots relevant to physical theorists that are irrelevant to almost all of us. It is true, though, that knowledge of more mathematics opens up certain opportunities that would otherwise be closed. But this is true for any craft, such as writing or cabinet making, where formal training or apprenticeship is necessary and where auto-didacts are few.

How about the argument that if the young Roys of the country do not learn some math, physics, engineering, then within a few years the U.S. will not be productive in those areas, will not be commercially competitive and will be left defenseless against higher-tech rivals? This is an argument frequently used not only by mathematical educators and testing establishments, but by politicians and newspapers columnists.

The mathematical educational establishment in the U.S. worries itself to death over the shocking public indifference and students' ignorance while the politicians and the media, thinking always in the mode of the International Olympics, worry about the poor relative showing of our students in mathematics U.S. ratings on international tests touch national pride, even as professional arguments grow heated over the interpretation of the results of the tests.

Admittedly, there is a worry here, although – at least in the case of the USA whose students have not done very well on such tests – there has been no shortage here of mathematical or scientific talent. The reason: since the social and economic prospects in this country have been brighter than in many other parts of the world, there has been a steady importation and immigration of highly talented people. This has been going on for almost two centuries In the 1840s, J. J Sylvester came from England to the University of Virginia At the time of writing, the faculty of Applied Mathematics at Brown University lists twenty-six members of whom only about half are native-born to the U.S. And then it does not take many mathematical professionals to saturate the demand. From time to time over the past fifty years the market for mathematicians has been glutted.

Many claims have been made that the average person will need more mathematics to survive in a mathematized civilization. This may be the case, but I tend to doubt it. (See also Keitel, 1989.) I do not think that the evidence is clear. I think it is not even clear how much of their advanced mathematical training is used by those who have become technologists (see Noss, 2002). A student of mine is currently obtaining a Ph.D. degree in mathematical education at Stanford University by investigating just this question

> The picture of how structural engineers use math seems to be rather more complex than most educators and ethnographers have portrayed it. I'd say that they do use math more than the most cynical want to claim (i.e those who think the engineers just use memorized, low-level routines and punch out results on the computer, without calling upon mathematical and physics understanding) But they probably use it quite a bit less than engineering professors want to claim – though the role of the theoretical fundamentals is unclear to me and is turning out to be the focus of my study

Yet it would seem that there should be a basic core of mathematics knowledge that everyone should know – even those who hate the subject. What is that core? It is hard to pin down, but here is my list: Arithmetic certainly; but who does long multiplication or division now with pencil and paper when a 2 x 3 inch cheap-as-dirt, hand-held, easily-worked computer does it for us? Geometry? Surely the area and volume of a few basic figures. Surely what a graph means and some of the easily and intuitively grasped aspects of basic patterns of time variation. What are the implications of linear, cyclic, the exponential, damped exponential, chaotic variations in time. Then, a bit of elementary probability and statistics; not the formulas, but ideas of what it is all about; what it implies about day-to-day experiences Surely. Every time I make such a list, I find it grows longer and longer. But what I most certainly want – and this is not now core material – is for students to acquire an appreciation of the role that mathematics plays in today's highly mathematized civilization; what it does for us and what it does to us.

There is yet another cause of worry as (or if) fewer of our young people go into science and mathematics. I recently read that many U.S high-school students are investing in the stock market on line. Their conversations one with the other have to do not with sports or dates or 'what answer did you get on the fifth problem in algebra?', but with hot tips on the market. This turn to money in its most abstracted and most degenerate form indicates a serious loss of idealism Is this change from the attitudes of my generation understandable? Yes. We are living in an age that has seen the collapse of dogmatic religions and of political and social idealisms. And the belief that science and technology embodies all that is objective, enlightening and all that is humane and necessary for human happiness, has been shown by selected consequences of technology over the past two hundred years to be false. What has been the result of this partial collapse of the pursuit of science as an ideal form of knowledge and the pursuit of science for its own sake? The mad chase after money and the belief in and the practice of a large variety of irrationalisms.

The general populace does not like mathematics. Enticements are offered to the young by stating that math can be fun. But it is fun only to a negligible fraction of the population because very likely those fortunate brains are hard- wired that way. The higher levels of the subject are hard to learn. They are hard to popularize The popularization of physics, astronomy or medicine is easier. Whether or not the public (or the physicists for that matter) knows what a black hole is, these holes capture the imagination and make the papers regularly.

To some extent, the difficulty of popularization lies in the very nature of mathematics. But the research mathematical establishment must take some of the blame. Its insistence on identifying the hard professional problems (Fermat, Bieberbach, Riemann) as the heart of mathematics professionalism (as what alone is worthy of public attention, its emphasis on related prize awards) misrepresents the nature of the mathematical enterprise and its relation to society.

Yes, there is publicity to be gained in this way; but it is publicity that leaves the public confirmed in its long-held opinion that mathematicians are a bunch of brilliant but crazy nuts who do things that are impossible to understand So why bother even trying? These efforts to attract and gain attention contribute to a public that is indifferent to the inner processes of mathematics.

Yet for all its lack of understanding, the public hardly needs to encourage the processes of mathematization – it has already done that in its immediate, enthusiastic and often uncritical acceptance of the products. But hopefully, an elevated understanding through meaningful and not sensational expositions, through elementary education that leads to a critical attitude towards social mathematizations, will lead to introspection and to a resistance that is ultimately necessary to humanize its ill-conceived products.

## References


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