Considering Dialogue as a Social Instrument in the Mathematics Class

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I present a dialogue between two mathematics teachers of different nationalities as they walk along the streets of a Spanish city. A pleasant walk and a little conversation provide a context where ideas and different arguments about mathematics arise through the exchange of ideas However, this innocent example of engaging with mathematics makes the two teachers reflect on the possibility of using a similar dialogue in the classroom, whether between students or between students and the teacher.

In this article, I show how many opinions arise, sometimes lively and at other times more sceptical Some research on the topic is mentioned This dialogue is formulated around three points: the role of the students, the role of the teacher and how dialogue may be understood as a medium for teaching. Finally, I offer some conclusions on which this form of teaching may be based

1. A dialogue of two mathematics teachers

Bill and José are going for a walk, as they do every Thursday In this way, they also maintain the friendship they have been developing for some time. But, as on every Thursday, the subject of mathematics always crops up at some time or other If not mathematics itself, something related to it and its teaching is always discussed. They are, after all, both mathematicians and teachers of mathematics

José: I don't know what to do with my students any more They seem to be completely lacking in motivation I know that you can always do more, but I think I'm doing all I can. I look for attractive problems; I try to talk to them and adapt to their learning rhythms. Sometimes I get serious but usually there's a relaxed atmosphere; and we often laugh in class because I prepare things that they find amusing But when they hear the word 'mathematics', they seem to turn their backs on it straightaway without even thinking of considering the specific content being dealt with.

Bill: Don't exaggerate That has always happened. And at the same time there have always been people interested, usually enough to stimulate a few more students and also the teacher You can't get all of them to learn everything.

José: I know that, but things are getting worse and worse. And those who are 'interested', as you call them, are becoming fewer and fewer in such an alarming way that I very often wonder if there are any in my class.

Bill: You're right about that. Since they are a minority, and a decreasing minority, they let themselves be led by the majority instead of stimulating them And at the age of fourteen, well, you know what teenagers are!

José: I think that it's partly to do with the problems we put out for them Usually they don't find them interesting. They can't see any future use We're still asking them to find the height of a castle, knowing a certain angle of vision. But not many of them live in castles! Some of them haven't even seen one.

Bill: Or we make them do routine exercises.

José: The other day a colleague in Science asked me to help him find a square root because his son had to do it I really couldn't remember how to do it because I hadn't done one for years.

Bill: But I'm sure you knew how to solve the problem

José: Of course! With the challenge of a colleague's question, I couldn't leave him in the lurch. I did it by a series of approximations between limits. It didn't take me long and he was really satisfied with the way it was solved Although at the end he told me that he didn't know whether his son would like that way of doing it because, perhaps, the teacher wouldn't accept that particular method

Bill: Why shouldn't he accept it if the answer is right! It may not be the most common way of doing it or the one that the teacher likes, but if it's right, it's right.

José: You know that things sometimes aren't quite as clear as that

Bill: OK But, going back to what we were talking about, I think you should work with aspects closer to your students' experiences.

José: I try to. I know that the official guidelines recommend working on mathematics from situations close to daily life. That's why sometimes, for example, we take an electricity bill, work out how to interpret it and calculate the amount spent by a family over a period of time. That seems to

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interest them Or else we deal with mathematics in the supermarket. But, of course, many of them never go to the supermarket and others have never seen an electricity bill.

Bill: But they will some day.

José: That's what I think But the biggest problem I find is that many mathematical problems can really be taken from ordinary life, but this is not enough. Many of them are going to need mathematics at a higher level.

Bill: Right. But, isn't it important to prepare students for ordinary life?

José: I agree with you. That is important. However, we can't neglect the fact that some will be heading for work needing a high level of mathematical understanding. For many, this will not be the case. You talk about ordinary life. Where, for example, are the geometrical problems in the street?

Bill: Well, there must be some geometry out here!

José: Yes, of course. There are geometrical aspects everywhere But it's not very inspiring asking them to look out the window and tell me how many geometrical shapes they can see.

Bill: You don't need to be so pessimistic. Agreed, it's not easy, but geometrical problems can be identified in our surroundings

José: Yes, of course. I've already tried that. Add the number of squares you can see, multiply it by the number of circles and then divide the result by the number of rectangles Now, what do you call the plane figure which has as many sides as the remainder from that division?

Bill: Everything really does seem gloomy to you today. I don't think things are quite so bad Look, what can you see over there? (See Figure 1)

José: A triangle. And beneath that a rectangle. And circles, semicircles, parallel lines, perpendicular lines . The same as always.

Bill: Pull yourself together. Let's look at that triangle. What can we say about it?

José: What is there to say about it? Well, it has three sides and three angles. Now it's you who's driving me mad!

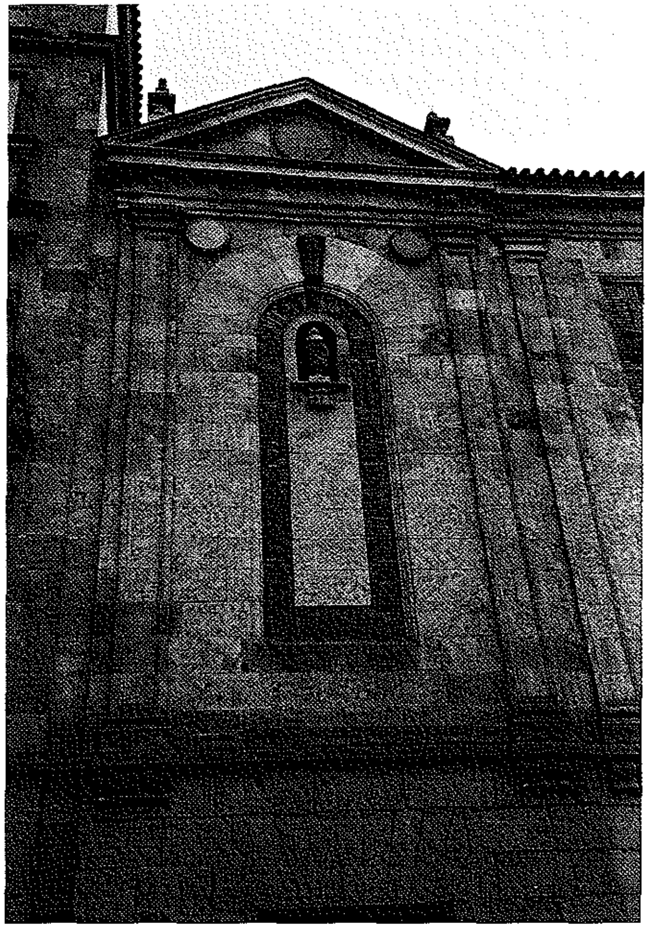

Figure 1:

Bill: Well, that's something It has three sides, two seem equal and three angles, two of which also seem to be equal So, it seems to be an isosceles triangle. But it's certainly not right-angled Are you sure it's not right-angled? It certainly doesn't look as though it is. If we wanted it to be one, we would have to raise the upper vertex a bit

José: Yes, but we don't know how much What we do know is that if we raise or lower the upper vertex, the triangle would still be isosceles as it is now.

Bill: That would be if we moved the vertex perpendicularly. If we moved it a little to one side, it wouldn't be isosceles.

José: Of course.

Bill: Of course nothing. We haven't explained it It would have to be raised following the perpendicular bisector of the segment that coincides with the base of the triangle

José: Oh, of course! Because the points of the perpendicular bisector of a segment are equidistant from its ends.

Bill: Is that really so? How can it be proved? Let's think. Ah! It can be seen, well, with a piece of paper where the segment whose perpendicular bisector we want to find is drawn. To find it, all we have to do is fold the paper so that the ends of the segment coincide exactly and so that one part of it is exactly on top of the other. The perpendicular bisector is precisely the fold formed.

José: Then, how can it be proved that the points of the perpendicular bisector are equidistant from the ends of the segment?

Bill: Do it and you'll see right away. Take this paper and draw a segment.

José: Let's see. Any segment, right?

Bill: That's right. Fold the paper like I told you and when it's like that and the two ends of the segment are one on top of the other, we would just have to take any point of the perpendicular bisector and join it to an end. You can see that it is at the same distance from the other one because that point is the same in both cases and one end is on top of the other.

José: Of course, how silly I am. I'm not used to handling paper.

Bill: What? You say you look for attractive problems and you try to achieve a good atmosphere and you're not used to handling paper? You should always have paper, pencil and scissors handy. That's what Pedro Puig Adam said in the prologue to his *Geometría Intuitiva* (1928):

> Here, dear reader, we introduce you to those who will be your workmates: some scissors, a ball of string, a ruler, a couple of set squares, a very large pile of paper. On not one single day should you start the Geometry lesson without having at hand these good companions, or finish it without leaving your table literally full of paper cuttings with shapes.

José: I'd like to see Puig Adam with my students.

Bill: You're not very optimistic today, are you! Look at the triangle. We said that if we raised or lowered the upper vertex following the perpendicular bisector of the lower side of the triangle, it would still be an isosceles triangle, if it really is isosceles to start with.

José: Let's suppose it is.

Bill: Right, let's suppose that it is. And that there's only one point that would make the triangle right-angled.

José: Only one point? Let me think. . . . Right. No. There are two points: One above the base, and another below, symmetrical with the previous one with respect to the axis that forms the base.

Bill: You're right. Two points. But let's fix our attention on the one at the top.

José: How can we if we don't know which one it is?

Bill: We can easily find it by merely seeing when its angle measures ninety degrees. But we can't measure it from here.

José: There's a device for measuring angles at a distance.

Bill: Let me have it and we'll clear it up right away.

José: You don't think I've got one here!

Bill: Well if we haven't got one, let's try to discover it without those instruments. We may not even be able to do it from here just by looking.

José: What exactly are we trying to find?

Bill: How to prove whether the upper angle of the triangle measures more than ninety degrees or less, and at what point it would measure ninety degrees.

José: Let's see. It seems clear that the angle is greater than a right angle. How would we get it to be a right angle? If we rotate the triangle. . . How silly! By rotating everything stays the same. Let's put the triangle inside a square to see what happens. The only possibility is a square with sides that measure the same length as the unequal side of the triangle. But I still can't see anything.

Bill: Look! If we put the triangle inside a square like you say, and we draw in the diagonals, these are above the other sides of the triangle. It seems that we could get something from there, but I can't quite see what.

José: Wait a minute! If the diagonals are above the sides of the triangle . . let's see . . . if the diagonals come to coincide exactly with the sides of the triangle, then the angle would be a right angle!

Bill: Why?

José: Because the diagonals divide the square into four equal parts and, as we know that a complete turn is three hundred and sixty degrees, each of those parts will measure ninety degrees.

Bill: Of course! And, in this case, the sides would be under the diagonals, then the upper angle of the triangle would be greater than a right angle, so that the nearer the base the greater it is For example, at the limit, when the two sides become fused with the side of the base it would measure the maximum, one hundred and eighty degrees

José: And if the two sides were above the diagonals, the angle would measure less than ninety degrees.

Bill: And to obtain a right-angled triangle, we would just have to put the triangle in question into a square with a side the same measurement as the unequal side of the triangle, so that its diagonals would indicate exactly where the other sides of the triangle should go

José: Well, now we've got it It wasn't easy. But, did you know how to solve it?

Bill: I didn't even know that problem. Besides, you did it.

José: I don't think so. What is true is that the knowledge we used was minimal. Our students know how to do this perfectly. Any of them could have solved it The problem is it needed some thought And that's what they're not ready to apply

Bill: You really are a pessimist. Perhaps it's because we don't ask them to do it. Look here The cross over the chemist's (see Figure 2) There we do have right-angled triangles.

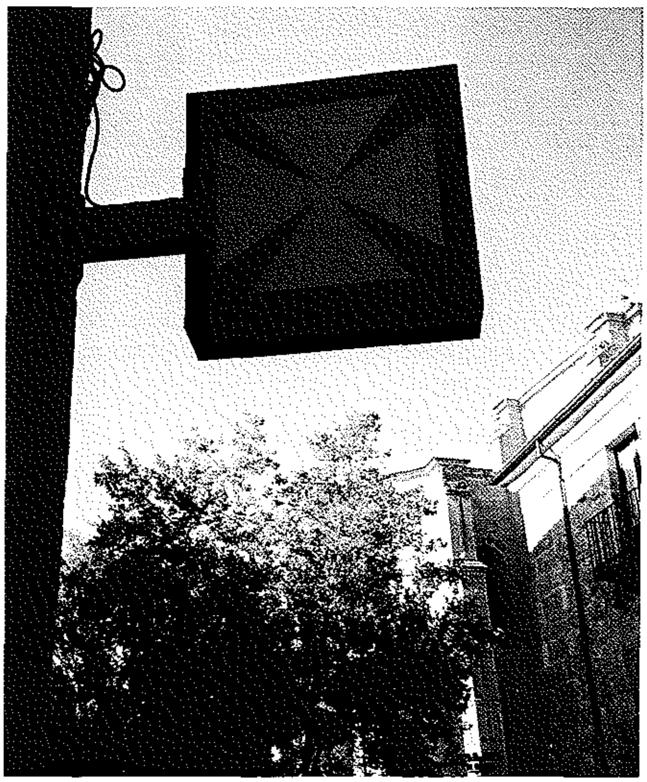

Figure 2:

José: Equilateral triangles you mean. You can see they have three equal sides.

Bill: That's it. Equilateral I was thinking of the previous right-angled triangles!

José: And you can see that all the angles are equal They all measure sixty degrees

Bill: Right But, how could we prove that those triangles are equilateral?

José: With a ruler: measure them and that's that.

Bill: Give me one and a ladder too.

José: I haven't got one.

Bill: Well then ... Let's try it in the situation we are in, from here. Although we may not be able to prove it

José: I don't know. If they were equal, we could join them all, considering the centre as the vertex of all of them and each triangle as having a side common to the triangle beside it Well, what about it?

Bill: Nothing. Ah! But if we join them all, for them to be equilateral, we would have to be able to put six in to fill the whole circumference.

José: Why six?

Bill: Because six would make a regular hexagon And a regular hexagon can be inscribed in a circumference where the radius is equal to the side of the hexagon.

José: And you can see it's not possible to inscribe six in there. There's not enough room It's impossible to put two triangles similar to those in the spaces there are between the green. One, perhaps.

Bill: So, those triangles are not equilateral.

José: Well, they looked as if they were. Appearances are sometimes deceiving. We've solved this problem straightaway It's a very good one. It seemed much harder.

Bill: That's because we've warmed up. Anyway, there'll be quite a few that can't be solved So problems can be posed which very often will have no solution or, perhaps, we won't be able to solve them

José: All right But we'll keep the good ones Look there, on the pavement (see Figure 3) There are squares, some inside others! It's the same as the problem I put on the exam the other day!

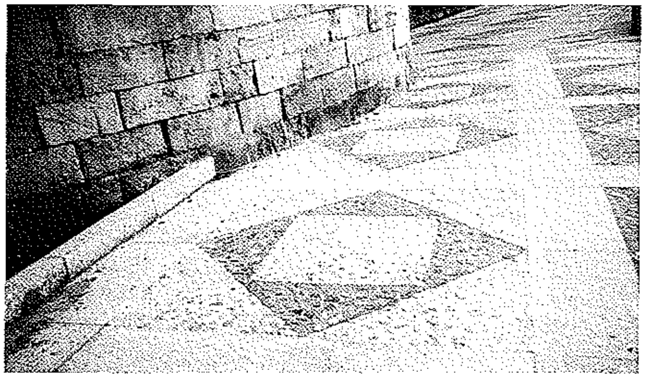

Figure 3:

Bill: And how did they get on?

José: The same as always: they nearly all got it wrong

Bill: What did they have to calculate?

José: The ratio between the areas of the largest square and the one immediately smaller.

Bill: Well, that's easy. You can see it just by superimposing on the smaller square the triangles that are formed in the corners of the largest square. The area of the largest square is exactly double that of the next largest square.

José: And with the next smallest square the same thing occurs

Bill: It's not only the same, but you can see and almost feel what happens

José: They must not walk by here like we do. They've got other concerns. Besides, I remember that I also asked them about the ratio between the perimeters.

Bill: That's a bit more difficult. It doesn't seem easy to do it visually, but it can be done right away by Pythagoras.

José: Yes, I know how to do it. You have to calculate the hypotenuse of one of those triangles, because they're all the same and it coincides with the side of the smaller square. Then that hypotenuse will be the square root of two times half of the side of the largest square, because the triangles are right-angled isosceles triangles and that solves it.

Bill: It's solved for those who can solve it, because, according to what you say, not many of them solved it correctly

José: The trouble is, when a problem is posed, they immediately start calculating without stopping to think for a moment. And, very often, they get carried away with their calculations

Bill: Yes, that's right, they don't think too much They work in the way we taught them So we're to blame.

José: Well, with all the time I spend preparing classes and reading new things looking for ways to motivate them!

Bill: I know, but sometimes it's not a question of learning and knowing a lot, but rather the way of applying the knowledge you have.

José: Yes, the same as always. Give me good examples and you'll see how they like them.

Bill: But it's not a question of their liking it, rather of them learning and of it serving for something. Although if they like it, of course, that's better.

José: Listen, I'm really getting to like this discussion The ideas that each of us suggests, even if they're not accurate, are at least considered. By linking some ideas with others, we have found that we eventually come to a solution I'm sure that I wouldn't have achieved much this afternoon without your help.

Bill: But you're the one who has solved nearly all of the problems this afternoon.

José: Rather the opposite Let's say it was half and half

Bill: Look at that window arch (see Figure 4) How could we find the centre from where it should be traced?

José: There's a way to do it with a ruler and compass: knowing three points, it is easy to calculate the circumference that goes through them just by ... I know what you are going to say: we haven't got a ruler and compass

Bill: But it can't be difficult. If we imagine the chord that joins both ends of the arc, would anything happen?

José: I know what you're getting at!

Bill: I don't understand.

José: Of course, if we consider the square which has that chord as its side and we imagine that it is just under the arc, then the intersection point of its two diagonals is the centre of the circumference to which the arc belongs. This is because the distance to one end of the arc is the same as the distance to the other end. At the same time, it is half of the diagonal and the radius of the circumference!

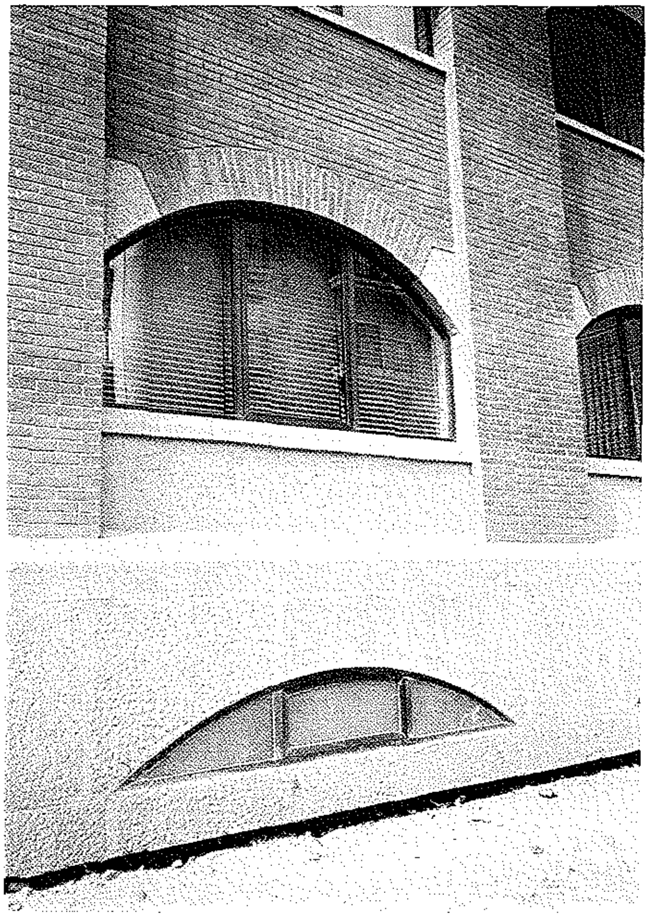

Figure 4.

Bill: It's not clear to me. It's true that the centre is on the perpendicular bisector of that chord, from what we saw before. And that bisector passes through the intersection point of the diagonals of the square. I don't see what you are saying, though. I don't think it can be done from here, although it needs a bit more thought.

José: You're right. With three known points it's easy. You trace the perpendicular bisectors that join them and the centre is their intersection point. But under these circumstances, I don't think this problem can be solved.

Bill: That's enough for me for today. I think we've earned a beer. Come on, it's on me. Look, an octagon. What an ugly octagon (see Figure 5).

José: It's irregular.

Bill: It certainly is. It's quite obvious that its sides are unequal. It's not possible to construct a regular octagon inside a rectangle. It would have to be inscribed in a square.

José: What do you mean, it's not possible? All you have to do is take the sides so that ...

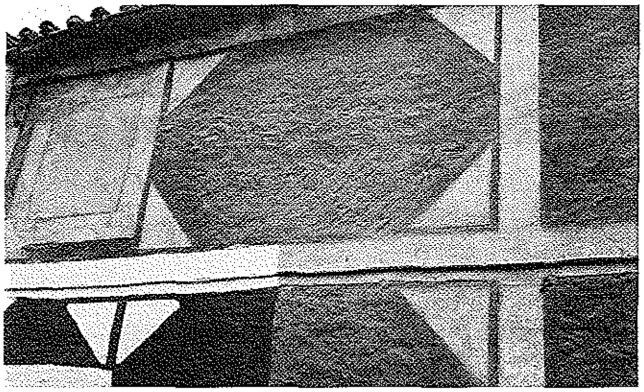

Figure 5.

Bill: Ah, yes. Maybe. For homework for the next time we go for a walk.

José: You'll see how I prove it. It can't be difficult. You'll have it tomorrow.

They both go into the bar across the road.

Bill: Look at the poster advertising the beer: for each beer you drink, you get a number and see if it comes up! (See figure 6.)

Figure 6.

José: Isn't it what they say about maths: numbers are everywhere? Look at the list of possible prizes. They're arranged according to the table of twelve. All kinds of congruence can be done there in twelves: for instance, it can be seen right away what time it will be after a certain number of hours or which month corresponds to a certain number of months after the one we're in. Or even more exercises with numbers

Bill: But, weren't we doing geometrical problems? Forget about numbers and drink your beer.

José: Does the theorem of Pythagoras seem like a geometrical aspect? Look at the mirror over there. Pythagoras' theorem can be demonstrated just by counting the squares (see Figure 7).

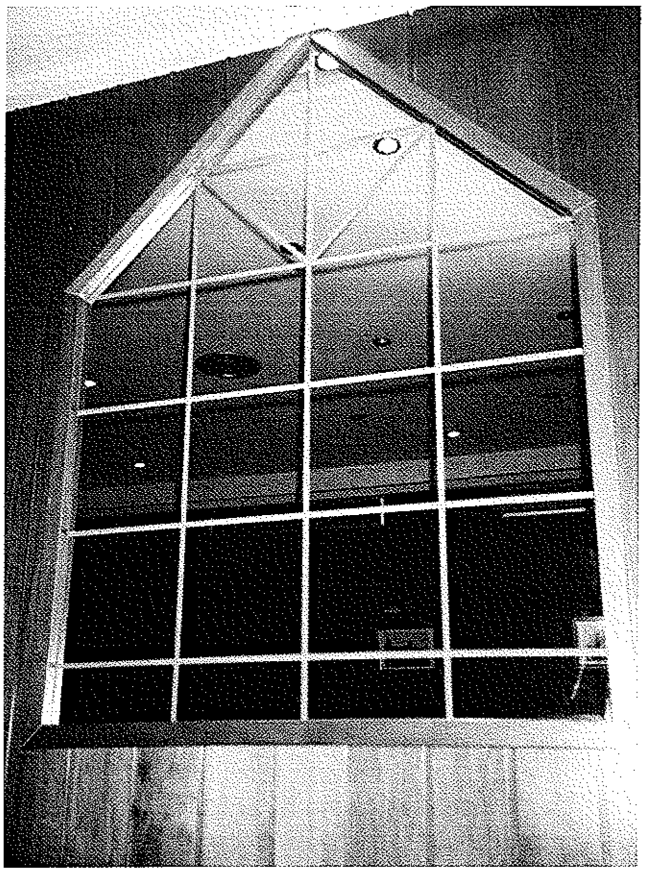

Figure 7:

Bill: Oh, yes! At the bottom, to make a square on the hypotenuse of the triangle you need four by four ... sixteen squares. Look at the other sides of the triangle, on the sides adjacent to the right angle: one, two, three, four. ... With eight, it would be complete. Naturally, eight and eight are sixteen. Then the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the sides adjacent to the right angle. The thing is, it's not a general case. The triangle is isosceles

José: It's a special case, but if my students had that image in their heads, I'm sure they would never forget one demonstration at least of Pythagoras' theorem.

Bill: Finish your beer and let's go. But close your eyes, because otherwise, besides taking up all my afternoon, you'll take up all night as well

2. The next Thursday

José was impressed by the dialogue between the two mathematicians during their walk the previous Thursday. He had been thinking about it all weekend. He had read books and articles on the topics they had discussed. What is more he had also tried to transcribe what they had talked about. To do this, he returned to the places they had visited. The following Thursday, when he had again arranged to meet Bill for a walk, he took several sheets of his notes with him. This surprised Bill.

Bill: Why have you written all that?

José: Because we had a good time, and we were learning. Couldn't teaching be like that?

Bill: Like what? Talking?

José: Exactly. With students talking to each other and the teacher as well.

Bill: That's very different from the teaching we had in our schooldays. The teacher seemed to do nothing but lecture

José: That's a long time ago now. What we experienced last Thursday arose from our own environment and interests. That's a constructivist perspective, isn't it?

Bill: But does it follow the official guidelines?

José: I can see you don't like what I've written

Bill: I didn't say that. I think it may be an idea, but I don't know if it's possible to put it into practice

José: Bill, all week I've been going over and over this matter and I have some ideas. We often hear said that students are thought to understand mathematics better if they are involved in their own learning. So teachers must be aware that the students have to be allowed to think, speak and express their opinion.

Bill: The truth is that doing it that way helps to reveal the originality of their thought and the diverse ways in which they reason in order to understand mathematics.

José: It also helps us to understand that students have different views of the world in which we live.

Bill: But it won't be easy to get students to express their opinions and ideas freely For one reason or another, not everybody is used to doing that in their daily life Many adults feel too intimidated to express their opinions in public.

José: That's true. And also that fear of being ridiculed is usually worse in young students, especially when they feel they are being judged severely and critically by their teacher.

Bill: But that would be the teacher's 'mission statement'. Let's say: 'to create an atmosphere in which students can develop their knowledge through the free exchange of their ideas'.

José: The teacher's work is fundamental Don't you agree it's to establish social norms during problem-solving sessions and support their understanding of mathematical explanations

Bill: To do that requires lots of patience and knowledge, as well as a keen awareness and sensitivity

José: Undoubtedly. That is not easy You're talking about ensuring harmony between all the diverse components of the educational process It's about fostering a democratic atmosphere in the classroom providing the stimulus for active and free participation.

Bill: But that should always be the case, shouldn't it? Surely each of us in the teaching process should respect the point of view of others, even though we may strongly disagree on occasions

José: I'm glad you said that We shouldn't suppress disagreement, confrontation of opinions and ideas Sometimes these are accompanied by a little heat too. However, we have to be careful that such challenges are not directed personally.

Bill: So in general, we are looking for a spontaneous flow of ideas, yet at the same time ensuring that they keep to the subject.

José: You have to take into account that although all these responses are acceptable if we take a constructivist viewpoint, this doesn't mean that they are all correct. Do you know, I've read a great deal about this subject this week. (Yackel (1995), Turner and Rossman (1997) and Fraivillig, Murphy and Fuson (1999).)

Bill: Actually, I've also been reading about dialogue, although I didn't do it for the purpose you indicated For example, Clements (1997) pointed out that the teacher's efforts were both important and fundamental in identifying why the student had given one answer and not another.

José: Of course. If mathematics were treated like that in the classroom from an early age, students would experience it as a discipline in which reasoning passes through the validation of ideas. This can be seen as a forerunner of the development of mathematical argumentation and even of formal proof

Bill: I remember some research by Wood (1999) that corroborates that It was carried out with seven-year-old pupils during their study of arithmetic in the classroom. Disagreements among the pupils were solved by dialogue and debate starting from each pupil's point of view During this time, a process of negotiation was carried out by the teacher.

José: Consequently, dialogue is an important social instrument. I have read how Ernest (1991) defined the basis for describing mathematical knowledge as a social system revolving around three points:

(a) mathematical knowledge, besides being knowledge, is agreement and linguistic rules - language is a social construction;

(b) by means of interpersonal social processes, one goes from individual subjective mathematical knowledge to accepted objective knowledge;

(c) objectivity in itself will be understood as social

Bill takes a deep breath. He looks seriously at José and says:

Bill: All right. It seems that what you want to do aligns with a constructivist methodology. Consequently, with this understanding as a way of teaching, you'll have to reconsider what is expected of the students, the role the teacher has to play and the character of the dialogue within the classroom.

José: That's what we've been talking about. But let's take a closer look at those three points We'll focus on the first one. The students are expected to take advantage of the opportunities for debating, criticising, explaining and, when necessary, defending their interpretations and solutions while they interact with each other or with the teacher. In order to do so, the students must work with problematic situations, so that they can explain and defend their reasoning until they come to an agreement among themselves as to which is the right route to take towards reaching a solution. They can do this without the constant presence of the teacher.

Bill: But, do you think that work in this sense really promotes the students' individual learning processes?

José: I've found references to this question. For example, Dekker and Elshout-Mohr (1998) created a model which reflected the work of two students on the same mathematical problem tackled in different ways. When they talked to each other about what each of them was doing, different things happened: they demonstrated, explained, defended and reconstructed their own work.

Bill: But, did all this arise from the questions and comments of the other students? Actually, this mutual exchange and help usually favours mental operations of self-criticism, expression, refining of judgement, making logical connections, arranging and structuring of thought and working out new hypotheses....

José: Besides, when the students talk they internalise the meaning of what they're saying

Bill: And there's an increase in motivation to achieve. I should think that this favours the development of the personality.

José: What's more, this social aspect helps to establish norms for working democratically. Also, the recognition of other people's opinions helps to correct one's own opinions. It can make them more flexible, so that the students learn to work out and put into practice collective plans of action responsibly

Bill: This could help the students to achieve a comprehensive grounding for everyday life.

José: Do you remember what we said in an article we once wrote? We've mentioned it on a number of occasions:

> Really, we want our students to know a lot about mathematics. But rather than learning a few specific concepts, we seek to give them a comprehensive grounding as persons. We want to form people that can face problems, overcome difficulties, know how to persist when they have to and when to give up And to be humble enough to recognise when they have made a mistake We should not forget that very often we can learn more from a mistake than a right answer. (Rawson and Chamoso, 1998, p 2)

Bill: Wait a minute. Do you know about Goos and Galbraith (1996) and their work on two students working together? They got satisfactory results but found that, on certain occasions, social interaction hindered progress.

José: I know it well They suggested further research into the matter. If I remember rightly, they understood that it didn't fulfill Forman's (1989) three conditions for there to be a two-way and effective close development zone between students. These conditions were: mutual respect for each other's opinions when carrying out the work, a similar distribution of knowledge and a similar distribution of power? And, for example, as Dunne and Bennett (1990), Bennett and Dunne (1992) and Steele (1999) all say, it must be done differently according to whether the students work in a similar or different way.

Bill: You're optimistic. The thing is, working in that way, one would be following a way of teaching different from the way in which many of us were taught Let's go on to the second point of what we were talking about. What would the teacher's role be?

José: There would have to be a change in the usual way of thinking, since no longer would the teacher be the one who has to make all the relevant decisions regarding how and when to teach and evaluate. This seems to imply that the teacher becomes just another element in the students' learning process and sometimes not even the one who decides the content that interests the students, their way of working and the subsequent presentation of their work.

Bill: For this to be possible, are you saying that the teacher would have to accept as reasonable whatever the students say and do, even if it is not immediately clear what they mean?

José: Exactly. On hearing the students' answers, the teacher becomes aware of the direction the flow of ideas is taking. This allows for connection with the students' way of thinking rather than to try simply to lead them towards the teacher's way of reaching the right answer.

Bill: This way, you're suggesting that the normal development of learning would follow its own path which the teacher should not change according to any prior conceptions that may be held of what that path should be. Doesn't this mean that the teacher loses authority in the classroom?

José: Far from it. The teacher establishes a methodology which aims at getting students to construct knowledge Cobb, Wood and Yackel (1991) refer to this.

Bill: I can see a positive side to what you're saying. The teacher is constantly learning while being immersed in a process of constant research In that way, an ability to ask, answer and listen is improved This seems to imply that everybody would benefit, both student and teacher

José: I agree with you. But there are also negative aspects. For example, there can be constant change, sometimes strange and often unpredictable.

Bill: That would be more difficult since you wouldn't always be able to have the answer ready beforehand

José: Which also means more work as well as further insecurity and tension in one's teaching

Bill: Besides, when the students are constructing a certain element of knowledge, it would be very difficult for the teachers to know when and how to intervene.

José: And we mustn't forget the difficulties of giving instructions to students when explaining an intended activity, as Brown (1997) and Battista (1999) say.

José and Bill carry on walking in silence. José looks toward the horizon while Bill has his eyes on the ground Suddenly, José stops again and says:

José: Bill. Isn't the idea that students should construct their own methods for problem solving and understand the associated mathematical concepts with a problem through dialogue among themselves under the teacher's supervision?

Bill: And you also think that the teacher should be the one who encourages students to find meanings through their own thought and reasoning.

José: but without reducing their autonomy.

Bill: I understand that what you are referring to is that he or she should look for Vygotsky's (1978) zone of proximal development in each student.

José: We've often talked about that!

Bill: We're not the only ones to have talked about it. A lot of people do, but it's not easy to discover either how or when work is carried out in this zone

José: Perhaps it can be done with suitable dialogue

Bill: I imagine that it is even more difficult in large classes Besides, Steffe and D'Ambrosio (1995), Simon (1995b) and Fraivillig, Murphy and Fuson (1999), for example, both refer to the fact that this zone of development varies from one student to another.

José: The same task varies while different students do it. This is understandable, because we're working with human beings, complex systems formed by subjective and inter-subjective mechanisms and not with programmes of activities. So, a task will never be the same if the social conditions change in some way.

Bill: Thus, the teacher's work is built around looking for the students' resources and the students learning from the teacher's. That's one of the paradoxes of teaching: children must learn to do division from a complete ignorance of it but, as Newman, Griffin and Cole (1991) say:

> for the class to work, it must be supposed that anything the children do can become a way of doing a division! (p. 81)

José: That's why it's so difficult to intervene.

Bill: Sometimes a good question may be a suitable answer

José: That's right. We've often talked about that. But not from an inquisitorial point of view, rather by the teacher becoming involved in the dialogue with a discourse similar to that of the students in the class: "How did you do that?", "And that, why is it like that?", "How can that be seen to be true?" or even "Oh, of course!" That is, by trying not to change the conversation and normal debate in the classroom Brown and Wragg (1993) refer to this

Bill: But that isn't possible in a class of thirty students

José: It presents the teacher with a problem, that's for sure. Consequently, a context must be created where the students can listen to each other while they explain their ideas and offer their solutions, as well as express their different points of view

Bill: We seem to be continually returning to this expression as a solution to our dilemma.

José: I read how Schifter and O'Brien (1997) described the teacher's action when two students who couldn't agree with each other about how to solve a certain problem asked for help. The teacher asked one of them to explain to the other the reasoning that had been applied This gave them the opportunity to validate each other's conjectures. It also helped them to identify their mistakes until they reached the right answer which, curiously, did not coincide with their original answer.

Bill: In that case a happy ending but it doesn't mean it'll always be like that.

José: Of course not. In a research study, Clarke (1997) reported the experiences of two teachers with completely different results, although they were in the same school, the same year, similar students, working together and, even, sometimes team-teaching.

Bill: You also have to take into account that, as Brown and Borko (1992) and Simon (1994) say, research on the development of the teacher is just beginning.

José: I wonder: if we teachers like to talk about things, why don't we let the students do the same?

Bill: Because you know what would happen? They would talk about other things, sport, television programmes, their friends ....

José: Just like us. But we also talk about maths. Look, I've taken a few notes from Steffe (1991), Simon (1994, 1995a) and Simon *et al.* (2000). No doubt we will pursue these thoughts further. Here's what I wrote:

1. The main area for the development of instruction is the thought and understanding of the students.

2. The mathematical tasks posed should be problematic situations

3. Knowledge is not received passively, but actively and socially, constructed through interactive discourse

4. The students give mathematical activity a personal meaning, explain and defend their personal methods and try to improve their ability to give sense to them

5. The teacher does not have to tell the student how to think, but rather has to create a context where the students generate powerful ideas. In this way, the teaching objectives have to be modified continually in interaction between the teacher and the students, since it is understood that the mathematical thought comes from the students.

Bill remains thoughtful. He is very pleased with the interest his colleague shows in looking for ways to improve his teaching. Why does he try to improve it? Has he not already had a great deal of success in the classroom? The reactions of his students seem to endorse this. However, the desire to improve is there and what more can a colleague do than to encourage this

Bill: All right, let's work on it. What is the first point we have to think about?

José: I've just told you! We've taken part in many meetings with other teachers where we have seen that they like to talk things over. Why don't we let the students do the same?

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