Intuition and Logic*

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Intuitive seems the opposite of rigorous, or logical, or formal. There appears to be an immediate contrast implicit in the theme of this paper even if we only try to characterize intuition and logic. Logic seems to be clearly understood. Intuition appears elusive. In a recently published book Fischbein has tried to overcome this difficulty by characterizing intuition or intuitive perception, not descriptively but functionally. Intuition, according to the understanding of the author, has the function "to create the appearance of certitude, to attach to various interpretations or representations the attribute of intrinsic, unquestionable certitude" [Fischbein, 1987: 12] The author justifies his approach with numerous and contradictory descriptions of intuition which he had found in the literature. "Things become much more clear if one admits that the concept of intuition, though apparently vague and inconsistent, expresses a fundamental, very consistent tendency of the human mind: the quest for certitude. In evaluating chances, in predicting outcomes, in making decisions, one naturally tends to produce representations (either conceptual or pictorial) which offer a high level of direct credibility." [Fischbein, 1987: 14]

This is essentially a Cartesian attitude. Descartes knew two modes of cognition, intuition and deduction. They appeared to him not in opposition to each other, because God's infinite mind guaranteed their connection. Intuition and logic seem to be in opposition only because of the very limited powers of the human mind. More appropriately, from Descartes' view, they appeared as separate or complementary.

In his "Rules for the Direction of the Mind", Descartes wrote: "We have now indicated the two operations of our understanding, intuition and deduction, on which alone we have said we must rely in the acquisition of knowledge. . . Truly we shall learn how to employ our mental intuition from comparing it with the way in which we employ our eyes. For he who attempts to view a multitude of objects with one and the same glance, sees none of them distinctly; and similarly the man who is wont to attend to many things at the same time by means of a single act of thought is confused in mind. But just as workmen, who are employed in very fine and delicate operations and are accustomed to direct their eyesight attentively to separate points, by practice have acquired a capacity for distinguishing objects of extreme minuteness and subtlety; so likewise do people who do not allow their thought to be distracted by various objects at the same time, but always concentrate it in attending to the simplest and easiest particulars, are clear-headed.

"But it is a common failing of mortals to deem the more difficult the fairer; and they often think that they have learned nothing when they see a very clear and simple cause for a fact, while at the same time they are lost in admiration of certain sublime and profound philosophical explanations, even though these for the most part are based upon foundations which no one has adequately surveyed — a mental disorder which prizes the darkness higher than the light." (Rule IX).

It is quite clear that for Descartes intuitive insight is the goal and logical deduction a means. Truth is founded on intuition, not on logic, not on proof. Since the 19th century intuition has been more and more found to be deceptive and more and more considered to be an instrument of mathematical invention rather than a foundation of evidence. Finally, modern intuitionists like Poincaré have distinguished between different kinds of intuition.

Poincaré essentially identified mathematical intuition with inductive reasoning, which seemed to him absolutely certain because it is an expression of the structure of the human mind itself. It seems as if the Cartesian "cognito ergo sum" has been radicalized by taking away from intuition any objective relationship. This radicalization was caused by progressive formalization of mathematics, and especially of geometry. "Unlike algebra, geometry was able to free itself from a narrow limitation of subject matter with comparative ease" [Nagel, 1979: 197] and it played a major role in the evolution of formal logics. Poincaré for instance accused Hilbert on the ground that he had made geometry fit only for *blind* machines.

Frequently in today's discussions about computers the priority of concepts and intuitive ideas over information and rules in human thinking is underlined. This is more or less what T. Roszak writes in his book on the myths of the computer age, *The cult of information*. He thinks it a great mistake for data dealers, futurologists and all the others who teach in schools to believe that computer competency must be the quintessence of future education. According to Roszak they lose sight of a fundamental truth, namely that the human mind thinks in ideas not in data or information. This view affirms once again that man thinks intuitively or conceptually; however, such an affirmation does not necessarily have any objective truth. Thinking can better be seen now as a characteristic of the human being, as if it were an expression of his social character or of his "immortal soul". Only if we could simultaneously understand concepts as instruments for the reconstruction of reality would the situation change.

Perhaps one could claim that this complementarity of the descriptive and constructive aspects of theoretical concepts belongs to different levels of thinking, just as Hilbert's

approach to the foundations of mathematics combines *formal* theory with *intuitive* meta-theory In this sense Hilbert and Poincaré seem essentially not too far apart from each other, and in the light of Gödel's incompleteness theorem both would have to acknowledge that intuition (like consistency) is achievable only *locally* not globally.

This brings us back, in fact, to the views of Descartes. We should perhaps accept his other insight too, namely that the tension of immediate perception and of discursive procedure is real insofar as the cognitive subject has only limited power, is finite: *time* being the source of that tension

The positivists of the Vienna Circle in fact combined this view with the radicalized version of Cartesianism mentioned above and concluded that mathematics, as well as logic, contains no real knowledge whatsoever. Both are nothing but immense systems of tautological transformations Poincaré has, according to Hans Hahn, in his judgement of the role of intuition, overlooked "un petit point: le fait que nous ne sommes pas omniscient (*allwissend*). Certainement un esprit omniscient saurait instantanément tout ce qui se trouve simultanément affirmé dans un groupe de propositions Un tel esprit n'a besoin de logique, ne de mathématique Ce n'est pas notre cas; pour nous en rendre compte, nous sommes obligés de poursuivre toute une cascade de transformations tautologiques; c'est pourquoi nous éprouvons une notable surprise à constater finalement que, en affirmant quelques énoncés, nous affirmions quelque chose qui paraît en être du tout au tout différent " [Hahn, 1935: 33/34]

Whereas the mind of God, according to Descartes, would grasp at a glance the truth of all mathematical axioms as well as the truth of all the theorems of mathematics he would, according to Hahn and the Vienna Circle, find no truth at all but would only be able to detect immediately the tautological character of logic and mathematics.

What an impoverishment compared to Cartesian rationalism or compared to Laplace's demon! Nevertheless a new idea comes up from it: namely, *functionalist* conceptions of knowledge. The God of the positivists would notice that all knowledge is just a huge machinery, a set of means toward an undefined end, and you may call it intuition or logic or whatever. To gain anything from the situation requires that you personally choose your perspectives and your goals

Leaving aside for the moment this line of argument, I now want to sharpen the *contrast* between intuition and logic by means of a thought experiment

The world as a labyrinth

Diametrically opposed to *conceptual* thought and intuitive evidence, it would seem, is *algorithmic* thought. If conceptual thinking identifies the object with its mental image, then its polar opposite would seem to be the topic of algorithmic thought, whose reality is set against its cognizing subject. Algorithmic thought does not aim at evidence but at success and it cannot know beforehand whether it will succeed or fail. One could say that registration is possible only *a posteriori*. Therefore algorithmic thought is knowledge without perception. Algorithms are only *functionally* related to objective reality, they do not explain anything. In general, when we consider the question of the substance or the subject matter of our thought, we are confronted with this *duality* of the object: it is both the content of thought and the opposing external reality.

What follows is a *thought experiment* which is meant to demonstrate how this assertion works in concrete terms. Let's imagine that we are in the middle of a wood. If we want to get out of this wood, the simplest method is to choose a direction and follow it consistently To avoid any doubt we need a compass If we bump into a tree, we will turn left and walk around the tree always keeping it on our right until we can continue in our originally chosen direction

But imagine that we are in a complicated labyrinth instead Now this simple algorithm will not always work and quite possibly we will end up walking around in circles. In addition to the compass, we now need a second instrument with which we can count our complete revolutions. The improved algorithm can be formulated in the following manner:

1. Select an arbitrary initial direction, call it "North" and face that way.
2. Go "North" in a straight line until you hit an obstacle.
3. Turn left until that obstacle is on your right
4. Follow the obstacle around, keeping it on your right until the total turning (including the initial turn in step 3) is equal to zero

This algorithm is currently known as the "Pledge Algorithm" (See Abelson & DiSessa [1981]) With its help one can find one's way out of any labyrinth regardless of its construction By using it one doesn't come to know the labyrinth, however; one doesn't develop a map of it The algorithm solves the problem but does not offer any insight into the description of the labyrinth One is no wiser from the experience, nor would one recognize the spot if one were to come across it a second time

It seems problematic to ask: what do we actually learn from the solution to a problem? Equally problematic is the question: what problems does a theory solve? Of course the thought experiment does not explicitly address the latter question And we are, in fact, generally inclined to identify the theoretical perspective with that of winged Icarus However to understand something principally by conceptual means, and in theoretical terms, does not at all imply in general that we are able to effectively solve a related problem In mathematics, in particular, the gap between a conceptual approach on the one hand and a constructive one on the other has been growing during the course of its history.

Computer science, artificial intelligence and their associated theories of knowledge operate on descriptions rather than on "the things themselves". They even define themselves as theories of descriptions. This is related to their exclusive interest in problem solving In fact, problem solving abilities depend very much on how a problem is represented. Solving a problem quite often means representing it so as to make the solution transparent: this is even more true for human problem solvers than for the computer. And the computer makes a procedural representation of knowledge, which stresses its functional organization, possible in a very effective manner

Theoretical mathematics tries to relate to "things them-

selves", because a theoretical idea may serve in the solution of many and rather different kinds of problems, and it will therefore be linked to many quite different types of representations. No theoretical concept exists like a Platonic idea apart from its representations But it may never be identified with any particular one of its names or representations, and, in addition, any particular representation of a theoretical concept is derived from an abstract relational understanding of its properties and not the other way around Theoretical thinking presupposes a *variability* in the distance between the knowledge level and the objective reality about which the knowledge speaks.

In the case of our labyrinth, a bird's eye perspective seems to solve everything and to let one see why a particular solution works. How completely opposite is the situation of the "blind" machine, which looks along the paths of the labyrinth in the manner of the Pledge Algorithm, from that of the flying Icarus. The labyrinth is for him an immediate part of his consciousness, whereas for those without wings it presents an incomprehensibly difficult situation.

Overcoming the labyrinth has led in a certain sense to a division of the subject, an Icarus with and without wings, a blind person looking along the pathways and a winged one whose "sighted" intuition makes up a plan of the maze. The discourse of the problem-solving algorithmic method and that of descriptive theory is parallel to the opposition of "blind" algorithm and "seeing" intuition However, as sharp as the opposition between algorithm or logic and intuition seems to be, it is as impossible to separate the two completely as it is to put them under the same umbrella.

Needless to say, the wings of Icarus, with the possibility they offer of immediately having a map of the labyrinth, are my ideal. But why is that? Isn't the map of the labyrinth as insignificant as the labyrinth itself to those with wings: to birds, for instance? Is not the one who needs the map the one who is caught and lost in the labyrinth? And lastly, isn't the map of the labyrinth which the latter may design actually imagined and fictitious? And even if it isn't fictitious, isn't such an ideal dubious? If the map were really inclusive, then life itself would be determined by it and replaced by it. The unknown aspect of the future and its openness to formation would disappear completely. Then, however, the human subject itself would be turned into a mere thing or an object

When applied practically, the seemingly absolute opposition of (descriptive) theory and (algorithmic) method becomes relative, it becomes fluid.

To the method the object appears at first in the form of a problem or a resistance. The application of the algorithm effects the transformation of the problem into an idea. But what now is the topic that is transformed? The answer in our case is the "labyrinth", understood as a mental entity (as the topic). The contents of cognitive activity with this as a given end become clear to us when we attempt to determine why or under what circumstances the given algorithm functions Actually this already began to become clear when we set up the algorithm, at any rate when we determined that a compass was sufficient to keep us from walking around in circles in the woods but that something more was necessary in a labyrinth.

First of all, we need to have a general idea of what a labyrinth is. The labyrinth must exist in a "flat" world: on a spherical surface the fundamental argument of the algorithm doesn't function any more Paths in the labyrinth must not intersect with bridges. The labyrinth must be fair: it must really have an exit. The "labyrinth" changes in this way from a wilderness into the contents of theoretical thought.

Only when we find ourselves in a real labyrinth and proceed according to the proposed algorithm (of which we have knowledge but no experience), only then is the above affirmation valid: namely, that the algorithm solves the problem but does not in any sense describe the world of labyrinths. At the moment in which we begin to think about the "universal" character of the algorithm there is a theoretical object, the universe of such ideal "labyrinths", to which this algorithm is related in the stated way At this moment our method wanders off into a theory, but into a theory which nonetheless has a construction as its basis and which is not based on the "direct" observation of facts

The image of the labyrinth is still missing, and with it the possibility of picturing oneself in it Perhaps we would have no desire to leave the labyrhith On the other hand, perhaps the desire would be very strong and we should want to get out of it as quickly as possible The Pledge Algorithm is not of much use in determining the human factor. And when one applies it and does not succeed quickly enough, is that because of the nature of the application or because of the way in which the labyrinth was constructed? Is it possible that there is no exit whatever? Such questions (and these are questions that have to do with the goals of the subject and with his independence in the labyrinth world) are questions that are not solved by optimizing the algorithmic method.

Each new application of the algorithm is a leap into the dark. First of all because the procedural perspective relates only functionally to objective reality. Reality is only seen as producing problems and problems are only viewed with respect to their solutions. Enlarging the functionalist point of view, applying it to the reality as a whole, including the algorithm itself, one comes to see the latter as a certain input-output behaviour. Two procedures are identical as soon as equal inputs result in identical outputs But how about a new labyrinth? In fact each labyrinth is new, as I don't know any of them, even when I have managed to escape one. In short, I have not even the possibility of finding out whether I follow the rules of the Pledge Algorithm or whether I in fact use quite a different function If the idea of meaning in the sense of reference makes no sense, then the concept of intending one function rather than another will make no sense either

So at least the algorithm itself has to be characterized descriptively to secure the identity of the operations For this I need some objective reference. I was told, for instance, to "go North". The algorithm must somewhere be explicitly described and cannot be completely conceived of only in functional terms like Fischbein's characterization of intuition There is a fundamental difference involved here, however, insofar as intuition is functionally related to the subject, and the machine to the object (the subject, in contrast, needs some description of it). It cannot be the use as such which gives those algorithmic procedures identity and coherence As was said above, algorithmic thinking need

descriptions or representations in order to operate on them.

The whole problem is summarized in the following quotation from Wittgenstein's *Remarks on the foundations of mathematics*: "How should we get into conflict with truth, if our footrules were made of very soft rubber instead of wood and steel? — "Well, we shouldn't get to know the correct measurement of the table " — You mean: we should not get, or could not be sure of getting, that measurement which we get with our rigid rulers. So if you had measured the table with the elastic rulers and said it measured five feet by our usual way of measuring, you would be wrong; but if you say that it measured five feet by your way of measuring, that is correct. — "But surely that isn't measuring at all!" — It is similar to our measuring and capable, in certain circumstances, or fulfilling "practical purposes". (A shopkeeper might use it to treat different customers differently.)"

The shopkeeper might in fact after a while lose control completely, even with regard to where and what his advantage might be.

On the other hand what is of interest with respect to a machine or an algorithm, even more so than with respect to intuition, is not its structure but the function it fulfills. For instance a student once objected to the above thought experiment: "But the algorithm is false . . one does not really move to the North!" The overall direction of motion within a labyrinth does in fact depend on the particulars of the situation. Nevertheless one must, as an answer, repeat once more: The algorithm or machine is only functionally related to the object, not descriptively. And this is even more important in the case of more complicated algorithms. Take for instance a computer program to play chess. You have to play against it in exactly the same manner and with exactly the same attitude as against a human player. You cannot analyse the structure of its algorithms to decide your own strategy of play.

Finally, structure and function are not tied to one another strictly in a one-to-one relationship. The same holds for theories or theoretical foundations on the one hand and procedures, rules, or algorithms, on the other. Both sides appear in a sort of complementarity which is represented in our cognition by means of *models*. Models have the function of bringing this complementarity to bear.

The Soviet philosopher E. Judin has represented this necessity in the following diagram of human cognitive activity:

Theoretical substantiation

↑
↓

Modelling conceptions

↑
↓

Procedures and algorithms

Judin himself draws three conclusions from his diagram: "First, the emphasis is on the objective content of the scientific matter, which are connected with the process of realizing the activity", for example, insight, text, check, etc. Second, this diagram of activity "without question requires a human subject as its bearer . . .". Third, the diagram permits the establishment of a certain typology of the acts of creative activity.

The vital point of the diagram is not the hierarchy but the correlations indicated by the arrows. This is already evident in Judin's second point: "If we introduce the connection between models and theoretical foundations, we show how theory functions for the acting subject " [E. Judin, 1978]

The cognitive view comes down in general to the principle that any form of cognitive activity requires a model or representation of the "territory" in which it operates. These models are functional with respect to an essential complementarity in human cognition. This complementarity has many expressions, and models fulfill mediating functions with respect to quite a number of these expressions. They help to mediate between the universal and the singular or between the description and the construction of objective reality. Models are primarily to be considered in such a functional perspective if one is interested in the practice of knowledge. To substantiate such claims empirically one should study the knowledge and cognitive behaviour of experts. A particular finding of such research relating to the question of the role of models is what Kline has called the establishing of "design equations" as a connecting link between engineering theory and practical work [cf. Kline, p. 283-313]. A "thought experiment" is another type of model in this sense.

Intuition and time

Let us return to our problem and continue by giving a description of the phenomenology of intuition.

The philosophical tradition of the 17th to 19th centuries developed a conception of thinking as a constructive activity and of intuition as this activity's supreme expression.

W. S. Bibler has proposed a "phenomenology of intuition" that he sums up into a system of four definitions. According to this system, the intuitive movement of thought can be defined as follows:

1. Seeing the essence of an object as an object or form itself.
2. By the self-evidence of the new knowledge. The knowledge and the knowledge about the truth of the knowledge appear identical and elliptical.
3. In intuition, a certain spontaneousness and immediacy of the transition from not understanding to knowledge can be observed: there is no chain of rational links, intuition is not discursive.
4. Intuitive knowledge is characterized by an unawareness: "I don't know how I got this knowledge . . ."

The first two definitions now also appear as determinations of formal logic and are summed up in the so-called immediacy claim for formal systems. Only the text, the sign, the formal can "in the last instance, be given in a pure state and thus does not entail any further problems of meaning and justification" [Kambartel, 1968: 192]. On this basis, Hilbert called formal logic a material logic. "The problem of logic is

a very direct one: how can a proposition tell something about itself?" The starting point of this problem is the assumption that every proposition immediately implies itself. If I say "p is true" this means "p" is true. Nothing is added to the proposition "p" by the words "is true".

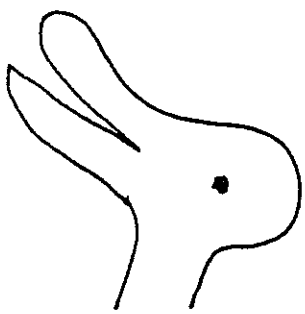
The difference between intuition and logic must thus be sought in other determinations. But what is the resulting difference? It seems as if the intuitive were nothing but a compressed and unresolved logic. So the difference is a matter of *time,* and so a matter of the finite or infinite character of the subject.

If this hypothesis about the importance of "time" for the difference of intuition and logic is true, then this points to the *social* character of cognition and to the idea that this contrast may be relativized on the social level. I want to illustrate the reasoning I have in mind by another example. For instance, the sudden "Eureka!" is experienced as a transformation of the subject self, as a switch in her being. This experience therefore points to the social level because the identity of human subjects cannot be conceived outside their social relations.

Intuition is in fact a way of seeing the world. In this it resembles a "paradigm" in the sense of Thomas Kuhn. I. Scheffler has criticized Kuhn by claiming that the two fundamental and controlling metaphors of Kuhn's theory, *vision* and *revolution,* are contradictory or incongruous with one another.

"Take first the topic of *vision.* Kuhn assimilates theoretical change to a gestalt reorganization of vision. "What were ducks in the scientists' world before the revolution are rabbits afterwards." The patterned alteration of thought and experience concomitant with theoretical change is plausibly compared with the intuitive and spontaneous shift in perception of a reversible figure, and resembles it better than it does the piecemeal articulations associated with deliberation. Anomalies and crises, says Kuhn, "are terminated, not by deliberation and interpretation, but by a relatively sudden and unstructured event like the gestalt switch . . ."

"The metaphor of *revolution* is employed in *conjunction* with that of the gestalt switch. "What were ducks . . . before the revolution are rabbits afterwards." But the notion of revolution is ramified in further ways. Competition is visualized as combat, with victory the prize. The conflict is a matter of "techniques of persuasion, or about argument and counterargument in a situation in which there can be no proof." Progress always accompanies victory because the winning camp is in a position to rewrite the textbooks and the implicit history of the subject. . .



"Now a closer look at these two controlling metaphors reveals a critical incongruity between them. While there is, both in the case of the reversible figure and the case of revolutionary conflict a certain mutual exclusiveness of elements, the notion of an opposition of *claims* applies only to the latter." The two views on the duck-rabbit are, as a matter of natural fact, exclusive at any given moment. But "we can flip from one to the other and may even come to shuttle back and forth between them with a certain amount of familiarity. However, they cannot both be seen simultaneously. Such exclusivity may be superficially assimilated to that of the revolutionary situation in which our allegiance must, at any time, be given to one side or the other — where we may shift loyalties but where it is impossible to be loyal to both sides at once or to join segments of each in some form of compromise. However — and this is the critical point — a revolution is a matter of opposed *loyalties and allegiances,* of conflicting *judgments and claims,* whereas there are no analogous questions of loyalty or allegiance or of conflicting claims in the case of alternative views of a reversible figure.

"Accordingly, having appreciated the reversibility of the duck-rabbit, there is no question of *arguing* over the relative merits of the duck or the rabbit as the *proper and exclusive* view of the duck-rabbit figure, nor is there any clash of opposed loyalties in the interpretation of the reversible cube. . .

"The case of revolution is quite different. Each side seeks victory, demands exclusive allegiance, claims superiority, expresses commitment, propounds arguments, engages in interpretation and persuasion, formulates its rationales, rebuts the arguments of the opposition. Nor is each party totally enclosed in its own conceptual and rhetorical box. It expresses its own view, to be sure, but it attacks the views of its opponents, claiming to understand them well enough to refute them. . . To reduce the combat of revolutionary parties to a gestalt switch is to *leave out* the critical aspect of *advocacy and opposed loyalties,* . . ." [Scheffler, 1986: 263-266].

Knowledge has no definite structure

Scheffler essentially asserts the incompatibility of the terms *vision* and *revolution* because of the supposed antagonism inherent in the latter. Whereas the act of conceiving of an "A" as a "B" (a duck as a rabbit for instance) claims only temporal exclusivity and, in general, the fascinating thing about such a metaphorical connection lies exactly in the possibility of symmetrically flipping forth and back from one perspective to the other. As we shall see below when criticizing Wertheimer's foundationalist conception of knowledge, exactly this example of the ducks and rabbits may be employed in the argument. Knowledge has neither absolute foundations nor a definite hierarchical structure. From the point of view of cognitive activity one notices a symmetry between the means and the objects of cognition. What were means in one context may be objects of investigation within another, and vice-versa. Now the reversibility of the gestalt switch as well as the symmetry between means and objects exist in an unrestricted sense only when the epistemic subject is infinite and in principle unlimited (if,

for instance, we take society in history as the epistemic subject)

As the individual human subject is finite and limited there is always an asymmetry involved here It may take a whole individual life to work out the statement "This A is essentially a B" (for instance, force is essentially a vector) in all its details and in its complete scope Intuition and logic have a quite different status with respect to this problem of time, and from this difference the opposition between the two results

Revolutions are gestalt-switches to which people have committed themselves in order to elaborate all their possible consequences Thinking is acting and not hypothetical play. Programs may take a lifetime or more to elaborate. And, in addition, scientific revolutions are accompanied by gestalt switches; a person belongs to one camp or another because (s)he has made the commitment to work out the consequences of one perspective or another.

"One need only compare the spirit of, and approach to analysis in Lagrange and Cauchy, — who after all were in a common French tradition — to see how radically analysis was reoriented. Lagrange and Cauchy simply do not speak the same mathematical language, whatever the similarity of vocabulary and grammar. For instance, to single out only one point, no matter how much Lagrange may assert and insist that a function is for him an "abstract" mathematical object, in his thought patterns it somehow is residually a mechanical orbit or perhaps a physical function of state; whereas in Cauchy orbits and forces and pressures are always functions, as they are for us today " [Bochner, 1974: 837]

This A is a B. This statement (or metaphor) may be fundamental and definite for a person within a certain situation As a means of development it must not be conceived of objectively; there has to be a decision, an intuition, an element of personal commitment based on the flexibility to see either rabbits or ducks.

Intuition is forceful. On the other hand an absolute insight or intuition does not exist This is very often misunderstood

For example, the well-known Gestalt psychologist Max Wertheimer (1880-1943) comments on the presentation and solution of Zeno's paradoxes by means of a geometric series that is current in present day mathematics Rather, he comments on the current proof of the convergence of that series, which is accomplished by multiplying the series by a and subtracting afterwards. Set $S = 1 + a = a^2 + \ldots$ Then $S - aS = 1$ or $S = 1/(1 - a)$.

"It is correctly derived, proved, and elegant in its brevity A way to get real insight into the matter, sensibly to derive the formula is not nearly so easy; it involves difficult steps and many more. While compelled to agree to the correctness of the above proceeding, there are many who feel dissatisfied, tricked. The multiplication of $(1 + a + a^2 + a^3 + \ldots)$ by a together with the subtraction of one series from the other, gives the result; it does not give understanding of how the continuing series approaches this value in its growth. Real understanding proceeds by considering what happens in the growth of the series and derives the law of this growth, leading to the limit. Many do not bother really to understand They are satisfied to have the result.

"There are theorems in mathematics for which we have at this time only "external" solutions because the problems are still too complicated for constructive understanding. Extreme are certain cases of the so-called negative, indirect proof in which the principle of the excluded middle is used, showing that the opposite assumption is impossible, leads to contradictions, yet without any possibility of seeing how the positive solution comes about constructively." [Wertheimer, *Productive thinking,* 1945]

As an appendix to his book Wertheimer presents his solution of the problem, together with a train of thought leading to that solution. The essential characteristic of these consists in his relying on the *meaning* of some relevant concepts (fraction, etc.) "If I want to understand", he says, "I must realize from the beginning what the first term $1/a$ means as a part of its whole " [218]

It thereby seems that a conceptual argument is being contrasted with symbolic manipulations, which however also demand experience and understanding of the entities employed Wertheimer considers this second type of insight "not sufficient to reach the solution by way of structural understanding." [221] I believe that the psychological differences between the two different solutions are of minor importance They are just questions of taste and style and, in fact, demand the same kinds of cognitive capabilities The essential difference is to be sought elsewhere: in the fact that Wertheimer's approach is "foundationalist", insofar as it reduces a problem concerning one concept A ("series") to the meaning of another one B ("fraction"), whereas a complementarist approach would stress the symmetrical aspects of the conceivable relations between A and B. We can, for instance, by interpreting a periodic decimal fraction (B) as a geometric series (A), prove directly that these decimal fractions are rational numbers.

Wertheimer by focussing too much on the phenomenology of intuition, employs a psychologistic conception of "meaning" and thereby encapsulates the process of knowing within an individual meaning experience. In this manner the essential complementarity of meaning (reference) and operation is torn apart.

Intuition in its double-edged character illustrates well the problematic relationship between individual and social thinking.

I add one final remark with respect to the history of philosophy

If the statement that the opposition between intuition and logic, or between direct and discursive thinking, stems from the problem of limited time is true, then this opposition must have come into sharp focus in the very period of history when cognition was no longer founded on the infinite mind of God, and when the relationship between the individual and the general (or social) became problematic This historical period began alongside the Industrial Revolution and with respect to the history of philosophy it is marked by Kant's "Critique of Pure Reason" It is in accordance with this observation when C S. Peirce says that Kant's greatest merit "lay in his sharp discrimination of the intuitive and the discursive processes of the mind. The distinction itself is not only familiar to everybody but it had long played a part in philosophy. Nevertheless, it is on such obvious distinctions

that the greater systems have been founded, and (Kant) saw far more clearly than any predecessor had done the whole philosophical import of this distinction.

"This was what emancipated him from Leibnizianism and at the same time turned him against sensationalism It was also what enabled him to see that no general description of existence is possible, which is perhaps the most valuable proposition that the Critic contains " [Peirce, 1 35]

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